Topic Study Group No. 30: Mathematical Competitions

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Our TSG 30 worked during four of the six days of ICME-13. Sessions were well attended by delegates from all over the world. Each talk was followed by a constructive and productive discussion. New relationships were forged, new collaborative projects envisioned. One of these projects was kindly offered to us by the Convenor of the Congress, Prof. Dr. Gabriele Kaiser: to compose a book of high quality papers on Mathematical Competitions, which may be published by Springer. The program of TSG 30, which follows, may convey the flavor of our study group and its international breadth. The titles of plenary talks are followed by quotations from them that impressed me the most.

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The Program

July 26, Tuesday, 12:00–1:30
Plenary Talk Alexander Soifer (USA):
Beyond *Lăozĭ*: The Goals of Mathematics Instruction.

Give a man a fish, and you will feed him for a day.
Teach a man how to fish, and you will feed him for a lifetime.
—老子 (*Lăozĭ,* VI century BC)

Before we address the purpose of mathematics instruction, it is instructive to ask ourselves, what is the purpose of life itself? It seems to me that the purpose of life is to discover and express ourselves, and in so doing contribute to high culture of our planet. The ultimate purpose of instruction is therefore to aid our students in their quest for self-discovery and self-expression.

Lăozĭ proposes to teach a man fishing as a method of solving the problem of survival. This does go further than giving a man a fish. However, is it good enough in today's world? Not quite, dear Sage, not in today's rapidly changing world. What if there is no more fish? What if the pond has dried out while your man has only one skill, fishing? A problem solver will not die if the fish disappears in a pond—he'll learn to hunt, grow crop, solve whatever problems life puts in his way. And so, we will go a long way by putting emphasis not on training skills but on creating environment for developing problem solving abilities and attitudes. This is the state-of-the-art. The proverb for today's world ought to be:

Give a man skills, and you will feed him in the short run.
Let a man learn solving problems, and you will feed him for a lifetime.

Every day we confront and solve a myriad of problem. Life *is* about solving problems. And mistakes in solving life's problem can be quite costly. This is where mathematics comes in handy. Mathematics allows us to learn how to think creatively, how to solve problems. And once our student masters problem solving in mathematics, s(he) will be better prepared to confront problems in any human endeavor. Moreover, one *cannot teach* mathematics, or anything else for that matter. State-of-the-art in mathematics instruction is about creating an atmosphere where students can learn mathematics by doing it, with a gentle guidance of a teacher.

1:10 Iliana Ivanova Tsvetkova (Bulgaria):

Mathematics Competitions as a Tool for Development of Gifted Students

July 27, Wednesday, 12:00–1:30
Plenary Talk María Falk De Losada (Colombia):

Are Mathematics Competitions Changing the Way Mathematics Is Being Done and the Mathematics that Is Being Done?

Mathematical problem solving competitions, as a branch of mathematics education, have a feature that distinguishes the work being done from every other initiative in the field. And this has its roots in Hungary at the Eötvös and Kürschák competitions and the journal of problems in mathematics and physics, Középiskolai Matematikai Lapok or KöMaL. With common roots in these pioneering competitions, a school was formed that produced outstanding figures in mathematics, in methodology and in epistemology. Beginning with the work and leadership of Lipót Fejér (Leopold Weiss) who grew up solving problems from KöMaL and who placed second in the Eötvös competition of 1897, a school was formed that came to include, in varying degrees, Paul Erdős, George Pólya and Imre Lakatos, the great mathematician and collaborator with mathematicians around the globe, the influential thinker on problem solving and method, and the philosopher–epistemologist who dared to question formalist mathematics proposing an alternative interpretation of the character, origins, structure and justification of mathematical knowledge and its historic evolution. These three stand out among the many great Hungarian mathematicians whose mathematical formation began in or was intimately related to the competitions, especially because they migrated to England and the United States and worked and published in English, thus opening their ideas and results and bringing them to bear on the worldwide community of mathematicians and mathematics educators…

Timothy Gowers, IMO gold medalist and winner of the Fields Medal, states:

> Loosely speaking, I mean the distinction between mathematicians who regard their central aim as being to solve problems, and those who are more concerned with building and understanding theories…. consider the following two statements.

i. The point of solving problems is to understand mathematics better.
ii. The point of understanding mathematics is to become better able to solve problems.

Most mathematicians would say that there is truth in both (i) and (ii)….

> So when I say that mathematicians can be classified into theory-builders and problem-solvers, I am talking about their priorities, rather than making the ridiculous claim that they are exclusively devoted to only one sort of mathematical activity.

Gowers considers himself to be a mathematician whose priority (in the tradition of Paul Erdős) is problem solving.

1:10 Chen Donglin and Frederick K.S. Leung (Hong Kong)

China Mathematical Olympiad School: A Case Study

July 29, Friday, 12:00–1:30
Plenary Talk Peter J. Taylor (Australia):

Some Reflections, some Suggestions

Competitions have a unique value in the education system. Because the questions are normally set externally, independently of class activity, they can test a

student's ability to apply known mathematics in new situations, and as such they can well help equip a student to be more useful in their later career.

Many features of everyday life are competitive, and in particular the development of mathematics and mathematical research have a competitive nature. These have sometimes been driven by tangible challenges such as the 23 problems set by Hilbert in 1900, the offering of million dollar prizes by the Clay Foundation in the 1990s, or necessity, such as the need to tighten financial security and the need to understand genetic structure.

The modern existence of mathematics competitions for school students dates back to 1894 in Hungary, Olympiads started in the early 1930s in the Soviet Union, and large, inclusive competitions commenced in the US in 1950. By 1984 there were many competitions held nationally and internationally and a need had developed for the organisers to form a learned society to enable information exchange…

The World Federation of National Mathematics Competitions (WFNMC) was founded at a meeting attended by about 20 people attending ICME-5 in Adelaide in 1984. It grew into a respectable organisation, an Associated Study Group (ASG) of ICMI, and now has its own refereed Journal, its own web site (http://www.wfnmc.org/), system of awards and conducts Conferences.

The Conferences have been particularly important, not only having allowed people in different countries to meet each other and establish lines of communication, but the first Conference was one of the exciting I have attended. There have been opportunities to meet Paul Erdos and John Conway, and some of the lectures, such as by Erdos, Conway, Robin Wilson, and those by Alexander Soifer on the chromatic numbers, axiom of choice and van der Waerden, have been most memorable. The Journal, offering opportunity to exchange information via refereed papers, has also been useful.

1:10 Luis F. Caceres Duque, Jose H. Nieto Said, and Rafael Sanchez Lamoneda (Puerto Rico, USA):

The Mathematical Olympiad of Central America and The Caribbean: 17 Years Supporting Math Contests in the Region

July 30, Saturday, 12:00–1:30
Plenary Talk Kiril Bankov (Bulgaria):

Numbers on a Circle

The intellectual treasure of every mathematics competition is the set of the problems given to the participants. Competitions present variety of problems: from these that are closely connected to the school curriculum to those that deal with "non-standard" situations. The latter usually stimulate creative thinking and thus remain in the minds for a long time. Many of these problems give rise to numerous mathematical ideas because finding their solutions develop the mathematical abilities. This paper discusses such problems: some are taken from mathematics competitions, others are inspired from competition problems. In both cases, being

among the best examples of the beauty of mathematics, they provoke an interest in mathematics that often begins with the consideration of attractive problems.

The life is full of operations. Many times in a day we take decisions about the series of operations to be done in order to obtain particular results. The correctness of these decisions depends on the ability to estimate the final results. Mathematics helps in modeling this reality by tasks using a particular admissible operation to transform a given situation to a different one. These problems lead to interesting generalizations by changing either the admissible operation or the initial/final situations. This part presents such examples taken from mathematics competitions in the context of arrangement of numbers on a circle.

1:10 Borislav Yordanov Lazarov and Albena Vassileva (Bulgaria):

Age Factor in Performance on a Competition Paper