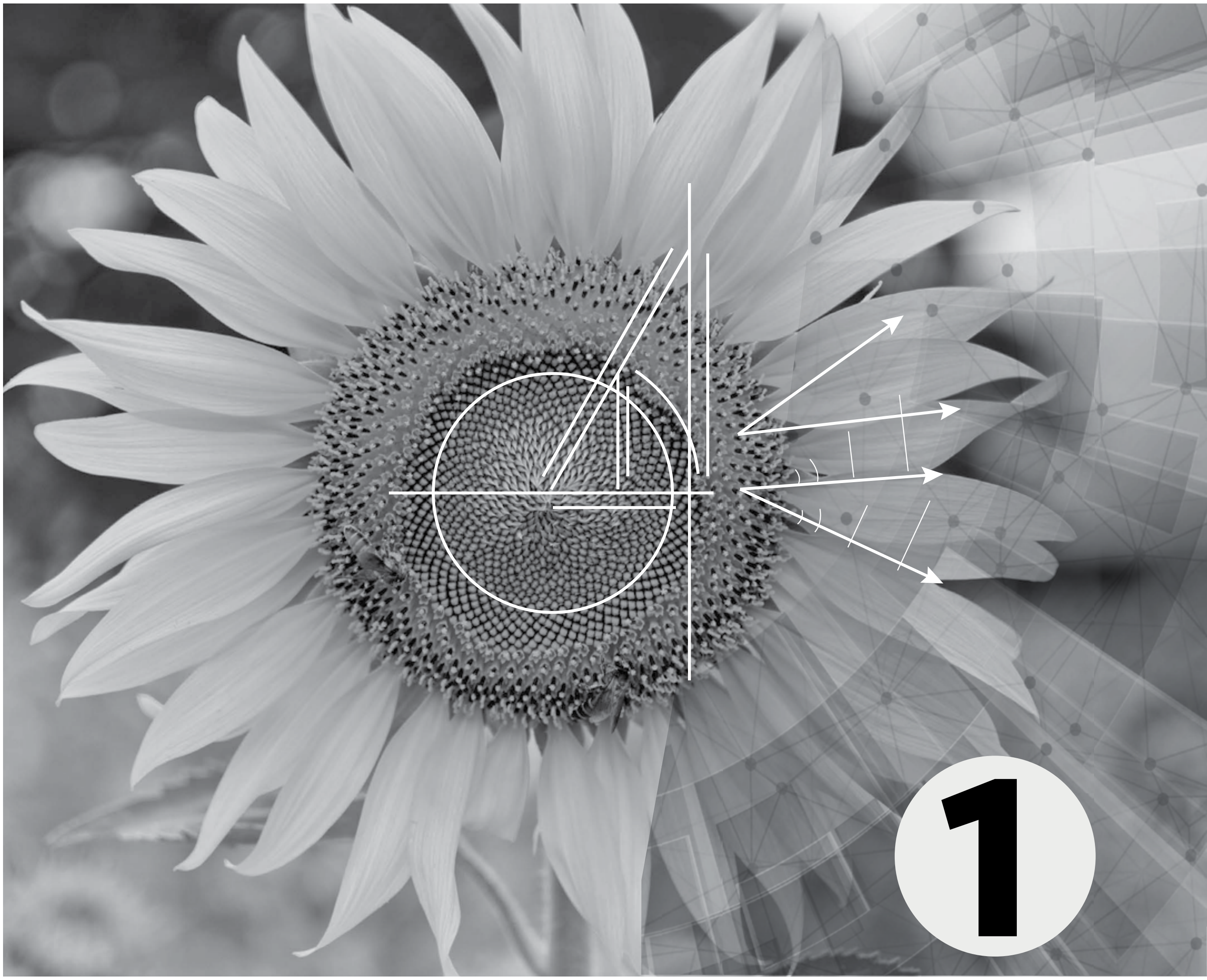
7th EDITION

NEW SYLLABUS MATHEMATICS

TEACHER'S RESOURCE BOOK



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CONTENTS

Syllabus Matching Grid

Cambridge O Level Mathematics (Syllabus D) 4024/4029. Syllabus for examination in 2018, 2019 and 2020.

Theme or Topic	Subject Content	Reference
1. Number	Identify and use: • Natural numbers • Integers (positive, negative and zero) • Prime numbers • Square numbers • Cube numbers • Common factors and common multiples • Rational and irrational numbers (e.g. π, $\sqrt{2}$) • Real numbers	Book 1: Chapter 1 Chapter 2
2. Set language and notation	• Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets • Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \leqslant x \leqslant b\}$, $D = \{a, b, c, \ldots\}$	Book 2: Chapter 14 Book 4: Chapter 2
2. Squares, square roots, cubes and cube roots	Calculate • Squares • Square roots • Cubes and cube roots of numbers	Book 1: Chapter 1 Chapter 2
4. Directed numbers	• Use directed numbers in practical situations	Book 1: Chapter 2
5. Vulgar and decimal fractions and percentages	• Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts • Recognise equivalence and convert between these forms	Book 1: Chapter 2
6. Ordering	• Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, <, >, \leqslant, \geqslant$.	Book 1: Chapter 2 Chapter 5
7. Standard form	• Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leqslant A < 10$.	Book 3: Chapter 4
8. The four operations	Use the four operations for calculations with: • Whole numbers • Decimals • Vulgar (and mixed) fractions including correct ordering of operations and use of brackets.	Book 1: Chapter 2
9. Estimation	• Make estimates of numbers, quantities and lengths • Give approximations to specified numbers of significant figures and decimal places • Round off answers to reasonable accuracy in the context of a given problem	Book 1: Chapter 3
10 Limits of accuracy	• Give appropriate upper and lower bounds for data given to a specified accuracy • Obtain appropriate upper and lower bounds to solutions of simple problems given to a specified accuracy	Book 3: Chapter 3

11. Ratio, proportion, rate	• Demonstrate an understanding of ratio and proportion • Increase and decrease a quantity by a given ratio • Use common measures of rate • Solve problems involving average speed	Book 1: Chapter 9 Book 2: Chapter 1
12. Percentages	• Calculate a given percentage of a quantity • Express one quantity as a percentage of another • Calculate percentage increase or decrease • Carry out calculations involving reverse percentages	Book 1: Chapter 8 Book 3: Chapter 5
13. Use of an electronic calculator	• Use an electronic calculator efficiently • Apply appropriate checks of accuracy • Enter a range of measures including 'time' • Interpret the calculator display appropriately	Book 1: Chapter 2 Chapter 4 Book 2: Chapter 11 Book 3: Chapter 10 Book 4: Chapter 4
14. Time	• Calculate times in terms of the 24-hour and 12-hour clock • Read clocks, dials and timetables	Book 1: Chapter 9
15. Money	• Solve problems involving money and convert from one currency to another	Book 3: Chapter 5
16. Personal and small business finance	• Use given data to solve problems on personal and small business finance involving earnings, simple interest and compound interest • Extract data from tables and charts	Book 3: Chapter 5
17. Algebraic representation and formulae	• Use letters to express generalised numbers and express arithmetic processes algebraically • Substitute numbers for words and letters in formulae • Construct and transform formulae and equations	Book 1: Chapter 4 Chapter 5 Book 2: Chapter 2 Book 3: Chapter 1
18. Algebraic manipulation	• Manipulate directed numbers • Use brackets and extract common factors • Expand product of algebraic expressions • Factorise where possible expressions of the form: $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$ $a^2 + 2ab + b^2$ $ax^2 + bx + c$ • Manipulate algebraic fractions • Factorise and simplify rational expressions	Book 1: Chapter 4 Book 2: Chapter 3 Chapter 4 Chapter 6
19. Indices	• Understand and use the rules of indices • Use and interpret positive, negative, fractional and zero indices	Book 3: Chapter 4

20. Solutions of equations and inequalities	• Solve simple linear equations in one unknown • Solve fractional equations with numerical and linear algebraic denominators • Solve simultaneous linear equations in two unknowns • Solve quadratic equations by factorisation, completing the square or by use of the formula • Solve simple linear inequalities	Book 1: Chapter 5 Book 2: Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 3
21. Graphical representation of inequalities	• Represent linear inequalities graphically	Book 4: Chapter 1
22. Sequences	• Continue a given number sequence • Recognise patterns in sequences and relationships between different sequences • Generalise sequences as simple algebraic statements	Book 1: Chapter 7
23. Variation	• Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities	Book 2: Chapter 1
24. Graphs in practical situations	• Interpret and use graphs in practical situations including travel graphs and conversion graphs • Draw graphs from given data • Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and deceleration • Calculate distance travelled as area under a linear speed-time graph	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 7
25. Graphs in practical situations	• Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer • Interpret graphs of linear, quadratic, cubic, reciprocal and exponential functions • Solve associated equations approximately by graphical methods • Estimate gradients of curve by drawing tangents	Book 1: Chapter 6 Book 2: Chapter 1 Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 7
26. Function notation	• Use function notation, e.g. $f(x) = 3x - 5$; $f : x \mapsto 3x - 5$, to describe simple functions • Find inverse functions $f^{-1}(x)$	Book 2: Chapter 7 Book 3: Chapter 2
27. Coordinate geometry	• Demonstrate familiarity with Cartesian coordinates in two dimensions • Find the gradient of a straight line • Calculate the gradient of a straight line from the coordinates of two points on it • Calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points • Interpret and obtain the equation of a straight line graph in the form $y = mx + c$ • Determine the equation of a straight line parallel to a given line • Find the gradient of parallel and perpendicular lines	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 6

28. Geometrical terms	• Use and interpret the geometrical terms: point; line; plane; parallel; perpendicular; bearing; right angle, acute, obtuse and reflex angles; interior and exterior angles; similarity and congruence • Use and interpret vocabulary of triangles, special quadrilaterals, circles, polygons and simple solid figures • Understand and use the terms: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment	Book 1: Chapter 10 Chapter 11 Book 2: Chapter 8 Book 3: Chapter 9 to Chapter 13
29. Geometrical constructions	• Measure lines and angles • Construct a triangle, given the three sides, using a ruler and a pair of compasses only • Construct other simple geometrical figures from given data, using a ruler and protractor as necessary • Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary • Read and make scale drawings • Use and interpret nets	Book 1: Chapter 12 Chapter 14 Book 2: Chapter 8 Book 4: Chapter 8
30. Similarity and congruence	• Solve problems and give simple explanations involving similarity and congruence • Calculate lengths of similar figures • Use the relationships between areas of similar triangles, with corresponding results for similar figures, and extension to volumes and surface areas of similar solids	Book 2: Chapter 8 Book 3: Chapter 11 Chapter 12
31. Symmetry	• Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions • Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone) • Use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length	Book 2: Chapter 13 Book 3: Chapter 13
32. Angles	• Calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines (c) angles formed within parallel lines (d) angle properties of triangles and quadrilaterals (e) angle properties of regular and irregular polygons (f) angle in a semi-circle (g) angle between tangent and radius of a circle (h) angle at the centre of a circle is twice the angle at the circumference (i) angles in the same segment are equal (j) angles in opposite segments are supplementary	Book 1: Chapter 10 Chapter 11 Book 3: Chapter 13
33. Loci	• Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point (b) at a given distance from a given straight line (c) equidistant from two given points (d) equidistant from two given intersecting straight line	Book 4: Chapter 8
34. Measures	• Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units	Book 1: Chapter 13 Chapter 14

35. Mensuration	• Solve problems involving: (a) the perimeter and area of a rectangle and triangle (b) the perimeter and area of a parallelogram and a trapezium (c) the circumference and area of a circle (d) arc length and sector area as fractions of the circumference and area of a circle (e) the surface area and volume of a cuboid, cylinder, prism, sphere, pyramid and cone (f) the areas and volumes of compound shapes	Book 1: Chapter 13 Chapter 14 Book 2: Chapter 12 Book 3: Chapter 10
36. Trigonometry	• Interpret and use three-figure bearings • Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or an angle of a right-angled triangles • Solve trigonometrical problems in two dimensions involving angles of elevation and depression • Extend sine and cosine functions to angles between 90° and 180° • Solve problems using the sine and cosine rules for any triangle and the formula area of triangle = $\frac{1}{2}$ $ab \sin C$ • Solve simple trigonometrical problems in three dimensions	Book 2: Chapter 10 Chapter 11 Book 3: Chapter 8 Chapter 9
37. Vectors in two dimensions	• Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or $\mathbf{a}$ • Add and subtract vectors • Multiply a vector by a scalar • Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • Represent vectors by directed line segments • Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors • Use position vectors	Book 4: Chapter 7
38. Matrices	• Display information in the form of a matrix of any order • Solve problems involving the calculation of the sum and product (where appropriate) of two matrices, and interpret the results • Calculate the product of a matrix and a scalar quantity • Use the algebra of 2×2 matrices including the zero and identity 2×2 matrices • Calculate the determinant $\lvert\mathbf{A}\rvert$ and inverse $\mathbf{A}^{-1}$ of a non-singular matrix $\mathbf{A}$	Book 4: Chapter 5
39. Transformations	• Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations • Identify and give precise descriptions of transformations connecting given figures • Describe transformations using coordinates and matrices	Book 2: Chapter 9 Book 4: Chapter 6
40. Probability	• Calculate the probability of a single event as either a fraction or a decimal • Understand that the probability of an event occurring = 1 – the probability of the event not occurring • Understand relative frequency as an estimate of probability • Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate	Book 2: Chapter 15 Book 4: Chapter 3

41. Categorical, numerical and grouped data	• Collect, classify and tabulate statistical data • Read, interpret and draw simple inferences from tables and statistical diagrams • Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used • Calculate an estimate of the mean for grouped and continuous data • Identify the modal class from a grouped frequency distribution	Book 1: Chapter 15 Book 2: Chapter 17 Book 4: Chapter 4
42. Statistical diagrams	• Construct and interpret bar charts, pie charts, pictograms, simple frequency distributions, frequency polygons, histograms with equal and unequal intervals and scatter diagrams • Construct and use cumulative frequency diagrams • Estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams • Calculate with frequency density • Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram • Draw a straight line of best fit by eye	Book 1: Chapter 15 Book 2: Chapter 16 Book 4: Chapter 4

Secondary 1 Mathematics Scheme of Work

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1	**1 Primes, Highest Common Factor and Lowest Common Multiple**	**1.1 Prime Numbers** (pp. 3 – 9)	• Explain what a prime number is • Determine whether a whole number is prime • Express a composite number as a product of its prime factors	Identity and use prime numbers	Investigation – Classification of Whole Numbers (pp. 3 – 4) Thinking Time (p. 4) Investigation – Sieve of Eratosthenes (p. 5) Journal Writing (p. 5) Investigation – Interesting Facts about Prime Numbers (p. 7) Thinking Time (p. 8)	Investigation – Interesting Facts about Prime Numbers (p. 8)		Thinking Time (p. 4) Investigation – Sieve of Eratosthenes (p. 5) Journal Writing (p. 5) Worked Example 2 (p. 7) Practise Now 2 Q 1 – 2 (p. 7) Thinking Time (p. 8)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1		**1.2 Square Roots and Cube Roots** (pp. 9 – 14)	• Find square roots and cube roots using prime factorisation, mental estimation and calculators	Identify and use square numbers and cube numbers Calculate squares, square roots, cubes and cube roots of numbers	Thinking Time (p. 12)			Attention (p. 10) Attention (p. 11) Thinking Time (p. 12) Main Text – 'Since $\sqrt{997}$ = 31.6 (to 1 d.p.), the largest prime less than or equal to $\sqrt{997}$ is 31. To determine whether 997 is a prime, it is enough to test whether 997 is divisible by 2, 3, 5, 7, ... or 31 (only 11 prime numbers to test). We do not have to test all the 167 prime numbers. Why?' (p. 13) Ex 1A Q 11 – 12 (p. 14)
2		**1.3 Highest Common Factor and Lowest Common Multiple** (pp. 14 – 24)	• Find the highest common factor (HCF) and lowest common multiple (LCM) of two or more numbers • Solve problems involving HCF and LCM in real-world contexts	Identify and use common factors and common multiples				Practise Now 11 Q 3 (p. 18) Ex 1B Q6, 9(a) – (e), 10, 11(a) – (d), 13(i), 14(ii) (pp. 21 – 22)
2		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
2	**2 Integers, Rational Numbers and Real Numbers**	**2.1 Negative Numbers** (pp. 27 – 30)	• Use negative numbers, rational numbers and real numbers in a real-world context • Represent real numbers on a number line and order the numbers	Identify and use natural numbers and integers (positive, negative and zero) Use directed numbers in practical situations	Class Discussion – Use of Negative Numbers in the Real World (p. 27) Main Text (pp. 28 – 29)			Class Discussion – Use of Negative Numbers in the Real World (p. 27) Thinking Time (p. 28) Main Text – 'The number is marked out on the number line. Explain how the point on the number line is obtained.' (p. 29)
				Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, <, >, ≤, ≥.	Thinking Time (p. 28)			
3		**2.2 Addition and Subtraction involving Negative Numbers** (pp. 30 – 37)	• Perform operations in real numbers, including using the calculator	Use the four operations for calculations with whole numbers including correct ordering of operations and use of brackets.	Main Text (pp. 31 – 32) Class Discussion – Addition involving Negative Numbers (p. 33) Main Text (p. 34) Class Discussion – Subtraction involving Negative Numbers (p. 35)	Main Text – 'Alternatively, you may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTool™ software.' (p. 30)		Class Discussion – Addition involving Negative Numbers (p. 33) Class Discussion – Subtraction involving Negative Numbers (p. 35)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
3		**2.3 Multiplication and Division involving Negative Numbers** (pp. 37 – 43)			Main Text (pp. 38 – 39) Class Discussion – Multiplication involving Negative Numbers (p. 39)			Class Discussion – Multiplication involving Negative Numbers (p. 39) Thinking Time (p. 41)
					Thinking Time (p. 41)			
				Use of an electronic calculator efficiently	Main Text (p. 42)			
				Apply appropriate checks of accuracy	Practise Now 4b			
4		**2.4 Rational Numbers and Real Numbers** (pp. 44 – 54)		Use the four operations for calculations with decimals, vulgar (and mixed) fractions including correct ordering of operations and use of brackets.				
				Identify and use rational and irrational numbers (e.g. π, $\sqrt{2}$)	Investigation – Terminating, Recurring and Non-Recurring Decimals (p. 50)	Investigation – Some Interesting Facts about the Irrational Number π (p. 51)		Thinking Time (p. 49)
					Thinking Time (p. 49) Investigation – Some Interesting Facts about the Irrational Number π (p. 51)			
4		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
4	**3 Approximation and Estimation**	**3.1 Approximation** (pp. 59 – 62)		Make estimates of numbers, quantities and lengths	Class Discussion – Actual and Approximated Values (p. 59)			Class Discussion – Actual and Approximated Values (p. 59) Practise Now 1 Q 2 (p. 60) Practise Now 2 Q 2 (p. 61) Ex 3A Q 5 – 7 (p. 62)
5		**3.2 Significant Figures** (pp. 63 – 67)	• Round off numbers to a required number of decimal places and significant figures	Give approximations to specified numbers of significant figures and decimal places	Investigation – Rounding in Real Life (p. 67) Journal Writing (p. 67)			Practise Now Q 2 (p. 64) Practise Now 4 Q 2 (p. 66) Investigation – Rounding in Real Life (p. 67) Journal Writing (p. 67)
5		**3.3 Rounding and Truncation Errors** (pp. 68 – 71)	• Explain the problem of rounding and truncation errors		Investigation – The Missing 0.1% Votes (p. 68)			Investigation – The Missing 0.1% Votes (p. 68) Thinking Time (p. 70) Ex 3B Q 3, 8 – 9, 10(iii) (pp. 70 – 71)
					Thinking Time (p. 69) Investigation – Rounding and Truncation Errors in Calculators (p. 70)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
5		**3.4 Estimation** (pp. 71 – 77)	• Estimate the results of computations • Apply estimation in real-world contexts	Round off answers to reasonable accuracy in the context of a given problem	Worked Example 6 (p. 73) Investigation – Use of a Smaller Quantity to Estimate a Larger Quantity (p. 75) Performance Task (p. 76)			
5		**Miscellaneous**					Solutions for Challenge Yourself	
6	**4 Basic Algebra and Algebraic Manipulation**	**4.1 Fundamental Algebra** (pp. 81 – 91)	• Use letters to represent numbers • Express basic arithmetical processes algebraically • Evaluate algebraic expressions • Add and subtract linear expressions	Use letters to express generalised numbers and express arithmetic processes algebraically Substitute numbers for words and letters in formulae	Class Discussion – Expressing Mathematical Relationships using Algebra (p. 83) Investigation – Comparison between Pairs of Expressions (p. 84)	Investigation –Comparison between Pairs of Expressions (p. 84) Journal Writing (p. 85)		Investigation – Comparison between Pairs of Expressions (p. 84) Journal Writing (p. 85)
				Construct and transform formulae and equations Manipulate directed numbers	Main Text (pp. 85 – 88) Practise Now (p. 89)	Practise Now (p. 89)		

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
6		**4.2 Expansion and Simplification of Linear Expressions** (pp. 91 – 97)	• Simplify linear expressions	Expand product of algebraic expressions	Main Text (pp. 91 – 95) Practise Now (p. 92) Practise Now (p. 92) Practise Now (p. 95) Class Discussion – The Distributive Law (p. 94)	Practise Now (p. 92) Practise Now (p. 92) Practise Now (p. 95)		
					Thinking Time (p. 96)			Thinking Time (p. 96)
7		**4.3 Simplification of Linear Expressions with Fractional Coefficients** (pp. 98 – 99)						
7		**4.4 Factorisation** (pp. 100 – 102)	• Factorise algebraic expressions by extracting common factors	Use brackets and extract common factors	Class Discussion – Equivalent Expressions (p. 101)			
7		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
8	**5 Linear Equations and Simple Inequalities**	**5.1 Linear Equations** (pp. 109 – 119)	• Explore the concepts of equation and inequality • Solve linear equations in one variable • Solve fractional equations that can be reduced to linear equations	Solve simple linear equations in one unknown	Main Text (pp. 110 – 113) Practise Now (p. 110) Practise Now (p. 111) Practise Now (p. 112) Practise Now (p. 113)	Practise Now (p. 110) Practise Now (p. 111) Practise Now (p. 112) Practise Now (p. 113)		Main Text – 'From Table 5.1, discuss with your classmate what a linear equation is.' (p. 109) Journal Writing (p. 113) Thinking Time (p. 115)
					Journal Writing (p. 113) Thinking Time (p. 115)			
				Solve fractional equations with numerical and linear algebraic denominators	Worked Example 2 (pp. 115 - 116) Worked Example 3 (pp. 116 - 117)			
8		**5.2 Formulae** (pp. 118 – 121)	• Evaluate an unknown in a formula					Ex 5B Q 17(ii) (p. 121)
9		**5.3 Applications of Linear Equations in Real-World Contexts** (pp. 122 – 125)	• Formulate linear equations to solve word problems		Internet Resources (p. 122)	Internet Resources (p. 122)		

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
9		**5.4 Simple Inequalities** (pp. 125 – 129)	• Solve simple linear inequalities	Solve simple linear inequalities	Journal Writing (p. 128)			Investigation – Properties of Inequalities (p. 126) Ex 5D Q 7 (p. 129)
9					Investigation – Properties of Inequalities (p. 126)			
		Miscellaneous					Solutions for Challenge Yourself	
10	**6 Functions and Linear Graphs**	**6.1 Cartesian Coordinates** (pp. 135 – 138)	• State the coordinates of a point • Plot a point in a Cartesian plane	Demonstrate familiarity with Cartesian coordinates in two dimensions	Class Discussion – Battleship Game (Two Players) (p. 135) Internet Resources (p. 135)	Internet Resources (p. 135) Story Time (p. 138)		Class Discussion – Ordered Pairs (p. 136)
					Class Discussion – Ordered Pairs (p. 136) Journal Writing (p. 137)			
10		**6.2 Functions** (pp. 139 – 145)			Investigation – Function Machine (pp. 139 – 142)	Internet Resources (p. 139)		Thinking Time (p. 143)
					Thinking Time (p. 143)			
10		**6.3 Graphs of Linear Functions** (pp. 145 – 148)	• Draw the graph of a linear function	Draw graphs from given data	Class Discussion – Equation of a Function (p. 147) Thinking Time (p. 147)			Class Discussion – Equation of a Function (p. 147) Thinking Time (p. 147)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
11		**6.4 Applications of Linear Graphs in Real-World Contexts** (pp. 149 – 153)	• Solve problems involving linear graphs in real-world contexts	Interpret and use graphs in practical situations including travel graphs and conversion graphs Interpret graphs of linear functions	Worked Example 2 (p. 149) Thinking Time (p. 151)			Thinking Time (p. 151) Ex 6C Q 3(ii) (p. 153)
11		**Miscellaneous**					Solutions for Challenge Yourself	
11	**7 Number Patterns**	**7.1 Number Sequences** (pp. 159 – 161)	Recognise simple patterns from various number sequences and determine the next few terms	Continue a given number sequence	Class Discussion – Number Sequences (p. 159)			
12		**7.2 General Term of a Number Sequence** (pp. 161 – 165)	Determine the next few terms and find a formula for the general term of a number sequence	Recognise patterns in sequences and relationships between different sequences	Class Discussion – Generalising Simple Sequences (p. 162)			
12		**7.3 Number Patterns** (pp. 165 – 167)	Solve problems involving number squences and number patterns	Generalise sequences as simple algebraic statements	Class Discussion – The Triangular Number Sequence (p. 167)			
12		**7.4 Number Patterns in Real-World Contexts** (pp. 168 – 177)			Worked Example 5 (p. 170)	Journal Writing (p. 169)		
					Investigation – Fibonacci Sequence (pp. 168 – 169) Journal Writing (p. 169)			
12		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
13	**8** **Percentage**	**8.1 Introduction to Percentage** (pp. 185 – 192)	• Express a percentage as a fraction and vice versa • Express a percentage as a decimal and vice versa • Express one quantity as a percentage of another • Compare two quantities by percentage	Calculate a given percentage of a quantity Express one quantity as a percentage of another	Class Discussion – Percentage in Real Life (p. 185) Class Discussion – Expressing Two Quantities in Equivalent Forms (p. 189)			Class Discussion – Percentage in Real Life (p. 185)
13		**8.2 Percentage Change and Reverse Percentage** (pp. 193 – 200)	• Solve problems involving percentage change and reverse percentage	Calculate percentage increase or decrease Carry out calculations involving reverse percentages	Thinking Time (p. 198) Internet Resources (p. 198)	Internet Resources (p. 198)		Just for Fun (p. 195) Attention (p. 195) Thinking Time (p. 198)
13		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
14	**9** **Ratio, Rate, Time and Speed**	**9.1 Ratio** (pp. 205 – 213)	• Find ratios involving rational numbers • Find ratios involving three quantities • Solve problems involving ratio	Increase and decrease a quantity by a given ratio	Journal Writing (p. 208) Worked Example 5 (p. 210) Class Discussion – Making Sense of the Relationship between Ratios and Fractions (p. 207) Worked Example 4 (p. 208) Worked Example 6 (p. 211) Internet Resources (p. 208)	Journal Writing (p. 208) Internet Resources (p. 208) Performance Task (p. 210)		
					Investigation – Golden Ratio (p. 207) Performance Task (p. 210)			
15		**9.2 Rate** (pp. 214 – 217)	• Distinguish between constant and average rates • Solve problems involving rate	Use common measures of rate	Investigation – Average Pulse Rate (p. 215) Thinking Time (p. 216)			Investigation – Average Pulse Rate (p. 215) Thinking Time (p. 216)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
15		**9.3 Time** (pp. 218 – 220)	• Solve problems involving time	Calculate times in terms of the 24-hour and 12-hour clock Read clocks, dials and timetables	Main Text (pp. 218 – 219)			
16		**9.4 Speed** (pp. 221 – 227)	• Discuss special types of rates such as speed and rate of rotation • Solve problems involving speed	Solve problems involving average speed	Main Text (pp. 221 – 226) Performance Task (p. 225) Thinking Time (p. 224)	Internet Resources (p. 222) Performance Task (p. 225)		Just for Fun (p. 221) Thinking Time (p. 224)
16		**Miscellaneous**					Solutions for Challenge Yourself	
16	**10 Basic Geometry**	**10.1 Points, Lines and Planes** (pp. 233 – 234)		Use and interpret the geometrical terms: point; line; plane	Thinking Time (p. 234)	Internet Resources (p. 234)		Thinking Time (p. 234)
17		**10.2 Angles** (pp. 235 – 266)	• Identify various types of angles • Solve problems involving angles on a straight line, angles at a point and vertically opposite angles	Use and interpret the geometrical terms: parallel; perpendicular; right angle, acute, obtuse and reflex angles Calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines				Just for Fun (p. 235)

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
17		**10.3 Angles formed by Two Parallel Lines and a Transversal** (pp. 244 – 252)	• Solve problems involving angles formed by two parallel lines and a transversal, i.e. corresponding angles, alternate angles and interior angles	Calculate unknown angles and give simple explanations using angles formed within parallel lines	Investigation – Corresponding Angles, Alternate Angles and Interior Angles (pp. 245 – 246)	Investigation – Corresponding Angles, Alternate Angles and Interior Angles (pp. 245 – 246)		Investigation – Corresponding Angles, Alternate Angles and Interior Angles (pp. 245 – 246) Practise Now (p. 246) Ex 10B Q 1(b) – (c) (p. 250)
17		**Miscellaneous**					Solutions for Challenge Yourself	
18	**11 Triangles, Quadrilaterals and Polygons**	**11.1 Triangles** (pp. 259 – 268)	• Identify different types of triangles and state their properties • Solve problems involving the properties of triangles	Use and interpret the geometrical terms: interior and exterior angles Use and interpret vocabulary of triangles Calculate unknown angles and give simple explanations using angle properties of triangles	Investigation – Basic Properties of a Triangle (pp. 262 – 263)	Investigation – Basic Properties of a Triangle (pp. 262 – 263)		Thinking Time (p. 260) Investigation – Basic Properties of a Triangle (pp. 262 – 263)
					Thinking Time (p. 260)			

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
18		**11.2 Quadrilaterals** (pp. 268 – 276)	• Identify different types of special quadrilaterals and state their properties • Solve problems involving the properties of special quadrilaterals	Calculate unknown angles and give simple explanations using angle properties of quadrilaterals Use and interpret vocabulary of quadrilaterals	Investigation – Properties of Special Quadrilaterals (pp. 269)	Investigation – Properties of Special Quadrilaterals (p. 269)		Thinking Time (p. 271) Just for Fun (p. 271)
					Thinking Time (p. 271)			
19		**11.3 Polygons** (pp. 276 – 290)	• Identify different types of polygons and state their properties • Solve problems involving the properties of polygons	Use and interpret vocabulary of polygons Calculate unknown angles and give simple explanations using angle properties of regular and irregular polygons	Investigation – Sum of Interior Angles of a Polygon (pp. 279 – 280) Investigation – Tessellation (pp. 282 - 283) Investigation – Sum of Exterior Angles of a Pentagon (pp. 284 – 285) Main Text (p. 285)	Class Discussion – Naming of Polygons (p. 277) Internet Resources (p. 277) Investigation – Sum of Exterior Angles of a Pentagon (pp. 284 – 285)		Main Text – 'The shapes shown in Fig. 11.13 are *not* polygons. Why?' (p. 276) Thinking Time (p. 277) Journal Writing (p. 278) Investigation – Sum of Exterior Angles of a Pentagon (p. 284 – 285)

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
					Thinking Time (p. 285)			Thinking Time (p. 285) Ex 11C Q 19(ii) – (iv) (p. 290)
					Class Discussion – Naming of Polygons (p. 277) Thinking Time (p. 277) Investigation – Properties of a Regular Polygon (p. 278) Journal Writing (p. 278)			
19		**Miscellaneous**					Solutions for Challenge Yourself	
20	**12 Geometrical Constructions**	**12.1 Introduction to Geometrical Constructions** (pp. 297 – 298)						
20		**12.2 Perpendicular Bisectors and Angle Bisectors** (pp. 299 – 301)	• Construct perpendicular bisectors and angle bisectors • Apply properties of perpendicular bisectors and angle bisectors	Measure lines and angles Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary	Investigation – Property of a Perpendicular Bisector (p. 300) Investigation – Property of an Angle Bisector (p. 301)	Investigation – Property of a Perpendicular Bisector (p. 300) Internet Resources (p. 300) Investigation – Property of an Angle Bisector (p. 301)		
20		**12.3 Construction of Triangles** (pp. 301 – 306)	• Construct triangles and solve related problems	Construct a triangle, given the three sides, using a ruler and a pair of compasses only				Just for Fun (p. 303) Ex 12A Q 15 – 16 (p. 306)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
20		**12.4 Construction of Quadrilaterals** (pp. 306 – 311)	• Construct quadrilaterals and solve related problems	Construct other simple geometrical figures from given data, using a ruler and protractor as necessary		Internet Resources (p. 309)		Worked Example 6 (pp. 306 – 307) Practise Now 6 Q 1 – 2 (p. 307) Practise Now 7 Q 2 (p. 308) Ex 12B Q 1 – 3, 5, 8 – 9, 11(ii), 15 (pp. 310 – 311)
20		**Miscellaneous**					Solutions for Challenge Yourself	
21	**13 Perimeter and Area of Plane Figures**	**13.1 Conversion of Units** (p. 317)	• Convert between cm^2 and m^2	Use current units of mass, length and area in practical situations and express quantities in terms of larger or smaller units	Class Discussion – International System of Units (p. 317)	Class Discussion – International System of Units (p. 317)		
21		**13.2 Perimeter and Area of Basic Plane Figures** (pp. 318 – 323)	• Find the perimeter and area of squares, rectangles, triangles and circles • Solve problems involving the perimeter and area of composite figures	Solve problems involving the perimeter and area of a rectangle and triangle, and the circumference and area of a circle	Practise Now (p. 318)			

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
22		**13.3 Perimeter and Area of Parallelograms** (pp. 324 – 327)	• Find the perimeter and area of parallelograms • Solve problems involving the perimeter and area of composite figures	Solve problems involving the perimeter and area of a parallelogram	Investigation – Formula for Area of a Parallelogram (p. 325) Practise Now (p. 324)	Thinking Time (p. 325)		Investigation – Formula for Area of a Parallelogram (p. 325) Thinking Time (p. 325)
					Thinking Time (p. 325)			
22		**13.4 Perimeter and Area of Trapeziums** (pp. 328 – 333)	• Find the perimeter and area of trapeziums • Solve problems involving the perimeter and area of composite figures	Solve problems involving the perimeter and area of a trapezium	Investigation – Formula for Area of a Trapezium (pp. 328 – 329) Practise Now (p. 328)			Investigation – Formula for Area of a Trapezium (pp. 328 – 329) Thinking Time (p. 329)
					Thinking Time (p. 329)			
22		**Miscellaneous**					Solutions for Challenge Yourself	

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
23	**14 Volume and Surface Area of Prisms and Cylinders**	**14.1 Conversion of Units** (pp. 339 – 340)	• Convert between cm^3 and m^3	Use current units of volume and capacity in practical situations and express quantities in terms of larger or smaller units	Class Discussion – Measurements in Daily Lives (p. 329)	Class Discussion – Measurements in Daily Lives (p. 339)		
23		**14.2 Nets** (pp. 341 – 342)		Use and interpret nets	Investigation – Cubes, Cuboids, Prisms and Cylinders (pp. 341 – 342)			
23		**14.3 Volume and Surface Area of Cubes and Cuboids** (pp. 343 – 347)	• Find the volume and surface area of cubes and cuboids	Solve problems involving the surface area and volume of a cuboid	Class Discussion – Surface Area of Cubes and Cuboids (p. 345)			Class Discussion – Surface Area of Cubes and Cuboids (p. 345) Ex 14A Q 18(ii) (p. 347)
24		**14.4 Volume and Surface Area of Prisms** (pp. 348 – 353)	• Find the volume and surface area of prisms	Solve problems involving the surface area and volume of a prism	Thinking Time (p. 349)			Thinking Time (p. 349) Main Text – 'Can you find a relationship between the volume of a prism and the area of its cross section?' (p. 349)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
24		**14.5 Volume and Surface Area of Cylinders** (pp. 354 – 360)	• Find the volume and surface area of cylinders	Solve problems involving the surface area and volume of a cylinder	Thinking Time (p. 354) Investigation – Comparison between a Cylinder and a Prism (p. 355) Thinking Time (p. 358) Class Discussion – Total Surface Area of Other Types of Cylinders (p. 358)			Thinking Time (p. 354) Just for Fun (p. 356) Thinking Time (p. 358) Class Discussion – Total Surface Area of Other Types of Cylinders (p. 358)
24		**14.6 Volume and Surface Area of Composite Solids** (pp. 361 – 363)	• Solve problems involving the volume and surface area of composite solids	Solve problems involving the areas and volumes of compound shapes				
24		**Miscellaneous**					Solutions for Challenge Yourself	

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
25	**15 Statistical Data Handling**	**15.1 Introduction to Statistics** (p. 369)				Story Time (p. 369)		
25		**15.2 Pictograms and Bar Graphs** (pp. 369 – 374)	• Collect, classify and tabulate data • Construct and interpret data from pictograms and bar graphs	Collect, classify and tabulate statistical data Read, interpret and draw simple inferences from tables and statistical diagrams Construct and interpret pictograms and bar charts	Main Text (p. 370)			Main Text – 'Two levels in the school are selected as the sample group for the survey conducted by the school canteen vendor. Are they representative of the entire school? Explain your answer.' (p. 369) Main Text – 'If the canteen vendor decides to sell three types of fruits to the students, which three should he choose? Explain your answer.' (p. 370) Thinking Time (p. 371) Practise Now Q 2(d)(ii), (e) (p. 372) Ex 15A Q 4(e), 5(iv), 6(iii) (p. 374)
					Thinking Time (p. 371)			

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
25		**15.3 Pie Charts** (pp. 375 – 377)	• Construct and interpret data from pie charts	Construct and interpret pie charts	Main Text (pp. 375 – 376)			Practise Now 1 Q 2(iii) (p. 377)
25		**15.4 Line Graphs** (pp. 377 – 379)	• Construct and interpret data from line graphs • Evaluate the purposes and appropriateness of the use of different statistical diagrams	Read, interpret and draw simple inferences from tables and statistical diagrams	Worked Example 2 Q (ii) (p. 378) Class Discussion – Comparison of Various Statistical Diagrams (p. 379)			Worked Example 2 Q (iv) (p. 378) Practise Now 2 Q (iv) (p. 379) Class Discussion – Comparison of Various Statistical Diagrams (p. 379)
25		**15.5 Statistics in Real-World Contexts** (pp. 380 – 381)			Main Text (pp. 380 – 381) Performance Task (p. 381)	Internet Resources (p. 381) Performance Task (p. 381)		Performance Task (p. 381)
26		**15.6 Evaluation of Statistics** (pp. 382 – 386)	Explain why some statistical information or diagrams can lead to a misinterpretation of data		Class Discussion – Evaluation of Statistics (pp. 382 – 383) Ex 15B Q 10 – 13 (pp. 385 – 386)	Class Discussion – Evaluation of Statistics (pp. 382 – 383)		Class Discussion – Evaluation of Statistics (pp. 382 – 383) Ex 15B Q 8(iv), 10 – 11, 12(iii), 13 (pp. 385 – 386)
26		**Miscellaneous**					Solutions for Challenge Yourself	

Chapter 1 Primes, Highest Common Factor and Lowest Common Multiple

TEACHING NOTES

Suggested Approach

Students have learnt only whole numbers in primary school (they will only learn negative numbers and integers in Chapter 2). They have also learnt how to classify whole numbers into two groups, i.e. odd and even numbers. Teachers can introduce prime numbers as another way in which whole numbers can be classified (see Section 1.1). Traditionally, prime numbers apply to positive integers only, but the syllabus specifies whole numbers, which is not an issue since 0 is not a prime number. Teachers can also arouse students' interest in this topic by bringing in real-life applications (see chapter opener on page 2 of the textbook).

Section 1.1: Prime Numbers

Teachers can build upon prerequisites, namely, factors, to introduce prime numbers by classifying whole numbers according to the number of factors they have (see Investigation: Classification of Whole Numbers). Since the concept of 0 may not be easily understood, it is dealt with separately in the last question of the investigation. Regardless of whether 0 is classified in the same group as 1 or in a new fourth group, 0 and 1 are neither prime nor composite. Teachers are to take note that 1 is not a prime number 'by choice', or else the uniqueness of prime factorisation will fail (see Information on page 8 of the textbook). Also, 0 is not a composite number because it cannot be expressed as a product of prime factors unlike e.g. $40 = 2^3 \times 5$.

To make practice more interesting, a game is designed in Question 2 of Practise Now 1. Teachers can also tell students about the largest known prime number (there is no largest prime number since there are infinitely many primes) and an important real-life application of prime numbers in the encryption of computer data (see chapter opener and Investigation: Interesting Facts about Prime Numbers) in order to arouse their interest in this topic.

Section 1.2: Square Roots and Cube Roots

Teachers can build upon what students have learnt about squares, square roots, cubes and cube roots in primary school. Perfect squares are also called square numbers and perfect cubes are also called cube numbers. Perfect numbers are not the same as perfect squares or perfect cubes. Perfect numbers are numbers which are equal to the sum of its proper factors, where proper factors are factors that are less than the number itself, e.g. $6 = 1 + 2 + 3$ and $28 = 1 + 2 + 4 + 7 + 14$ are the only two perfect numbers less than 100 (perfect numbers are not in the syllabus). After students have learnt negative numbers in Chapter 2, there is a need to revisit square roots and cube roots to discuss negative square roots and negative cube roots (see page 40 of the textbook). Teachers can impress upon students that the square root symbol $\sqrt{}$ refers to the positive square root only.

A common debate among some teachers is whether 0 is a perfect square. There is an argument that 0 is not a perfect square because 0 can multiply by any number (not necessarily itself) to give 0. However, this is not the definition of a perfect square. Since 0 is equal to 0 multiplied by itself, then 0 (the first 0, not the second 0, in this sentence) is a perfect square. Compare this with why 4 is a perfect square (4 is equal to the integer 2 multiplied by itself). Similarly, 0 is a perfect cube.

Section 1.3: Highest Common Factor and Lowest Common Multiple

Teachers can build upon prerequisites, namely, common factors and common multiples, to develop the concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM) respectively (HCF and LCM are no longer in the primary school syllabus although some primary school teachers teach their students HCF and LCM). Since the listing method (see pages 15 and 18 of the textbook) is not an efficient method to find the HCF and the LCM of two or more numbers, there is a need to learn the prime factorisation method and the ladder method (see Methods 1 and 2 in Worked Example 9 and in Worked Example 11). However, when using the ladder method to find the LCM of two or three numbers (see Worked Examples 11 and 12), we stop dividing when there are no common prime factors between any two numbers. The GCE O-level examinations emphasise on the use of the prime factorisation method.

Challenge Yourself

Some of the questions (e.g. Questions 4 and 5) are not easy for average students while others (e.g. Question 2) should be manageable if teachers guide them as follows:

Question 2: The figure consists of 3 identical squares but students are to divide it into 4 identical parts. Teachers can guide students by asking them to find the LCM of 3 and 4, which is 12. Thus students have to divide the figure into 12 equal parts before trying to regroup 3 equal parts to form each of the 4 identical parts.

Questions 4 and 5: Teachers can get students to try different numerical examples before looking for a pattern in order to generalise. In both questions, it is important that students know whether m and n are co-primes, i.e. HCF(m, n) = 1. If m and n are not co-primes, they can be built from the 'basic block' of $\frac{m}{\text{HCF}(m, n)}$ and $\frac{n}{\text{HCF}(m, n)}$, which are co-primes.

WORKED SOLUTIONS

Investigation (Classification of Whole Numbers)

1.

Number	Working	Factors
1	1 is divisible by 1 only.	1
2	$2 = 1 \times 2$	1, 2
3	$3 = 1 \times 3$	1, 3
4	$4 = 1 \times 4 = 2 \times 2$	1, 2, 4
5	$5 = 1 \times 5$	1, 5
6	$6 = 1 \times 6 = 2 \times 3$	1, 2, 3, 6
7	$7 = 1 \times 7$	1, 7
8	$8 = 1 \times 8 = 2 \times 4$	1, 2, 4, 8
9	$9 = 1 \times 9 = 3 \times 3$	1, 3, 9
10	$10 = 1 \times 10 = 2 \times 5$	1, 2, 5, 10
11	$11 = 1 \times 11$	1, 11
12	$12 = 1 \times 12 = 2 \times 6 = 3 \times 4$	1, 2, 3, 4, 6, 12
13	$13 = 1 \times 13$	1, 13
14	$14 = 1 \times 14 = 2 \times 7$	1, 2, 7, 14
15	$15 = 1 \times 15 = 3 \times 5$	1, 3, 5, 15
16	$16 = 1 \times 16 = 2 \times 8 = 4 \times 4$	1, 2, 4, 8, 16
17	$17 = 1 \times 17$	1, 17
18	$18 = 1 \times 18 = 2 \times 9 = 3 \times 6$	1, 2, 3, 6, 9, 18
19	$19 = 1 \times 19$	1, 19
20	$20 = 1 \times 20 = 2 \times 10 = 4 \times 5$	1, 2, 4, 5, 10, 20

Table 1.1

2. **Group A**: 1
Group B: 2, 3, 5, 7, 11, 13, 17, 19
Group C: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

3. 0 is divisible by 1, 2, 3, 4, …
0 has an infinite number of factors.

Thinking Time (Page 4)

1. A prime number is a whole number that has exactly 2 different factors, 1 and itself.
A composite number is a whole number that has more than 2 different factors. A composite number has a finite number of factors.
Since 0 has an infinite number of factors, it is neither a prime nor a composite number.
Since 1 has exactly 1 factor, it is also neither a prime nor a composite number.

2. No, I do not agree with Michael. Consider the numbers 0 and 1. They are neither prime numbers nor composite numbers.

Investigation (Sieve of Eratosthenes)

1.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

2. **(a)** The smallest prime number is 2.
(b) The largest prime number less than or equal to 100 is 97.
(c) There are 25 prime numbers which are less than or equal to 100.
(d) No, not every odd number is a prime number, e.g. the number 9 is an odd number but it is a composite number.
(e) No, not every even number is a composite number, e.g. the number 0 is an even number but it is neither a prime nor a composite number.
(f) For a number greater than 5, if its last digit is 0, 2, 4, 6 or 8, then the number is a multiple of 2, thus it is a composite number; if its last digit is 0 or 5, then the number is a multiple of 5, thus it is a composite number. Hence, for a prime number greater than 5, its last digit can only be 1, 3, 7 or 9.

Journal Writing (Page 5)

1. Yes, the product of two prime numbers can be an odd number, e.g. the product of the two prime numbers 3 and 5 is the odd number 15.

2. Yes, the product of two prime numbers can be an even number, e.g. the product of the two prime numbers 2 and 3 is the even number 6.

3. No, the product of two prime numbers P_1 and P_2 cannot be a prime number since P_1P_2 has at least 3 distinct factors, i.e. 1, P_1 and P_1P_2.

Investigation (Interesting Facts about Prime Numbers)

The 1 000 000$^{\text{th}}$ prime number is 15 485 863.
The last digit of the largest known prime number is 1.

Thinking Time (Page 8)

The index notation is useful for writing complicated and repetitive expressions in a more compact form, e.g. we can write
$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ as 3^8.

Thinking Time (Page 12)

If no brackets are used in pressing the sequence of calculator keys in Worked Example 7, the value obtained would be 60.0416 (to 4 d.p.). The mathematical statement that would have been evaluated is $8^2 + \frac{\sqrt{50}}{7^3} - \sqrt[3]{63}$.

Practise Now 1

1. 537 is an odd number, so it is not divisible by 2.
   Since the sum of the digits of 537 is $5 + 3 + 7 = 15$ which is divisible by 3, therefore 537 is divisible by 3 (divisibility test for 3).
   $\therefore$ 537 is a composite number.

   59 is an odd number, so it is not divisible by 2.
   Since the sum of the digits of 59 is $5 + 9 = 14$ which is not divisible by 3, then 59 is not divisible by 3.
   The last digit of 59 is neither 0 nor 5, so 59 is not divisible by 5.
   A calculator may be used to test whether 59 is divisible by prime numbers more than 5.
   Since 59 is not divisible by any prime numbers less than 59, then 59 is a prime number.

2.

135	49	183	147	93	121	236
201	261	150	11	131	5	89
291	117	153	End	57	0	61
192	231	27	1	111	100	149
17	103	43	7	127	51	53
83	33	32	105	29	71	37

Start

Practise Now 2

1. Since 31 is a prime number, then 1 and 31 are its only two factors. It does not matter whether p or q is 1 or 31 as we only want to find the value of $p + q$.
   $\therefore p + q = 1 + 31 = 32$
2. Since $n \times (n + 28)$ is a prime number, then n and $n + 28$ are its only two factors.
   Since 1 has to be one of its two factors, then $n = 1$.
   $$\therefore n \times (n + 28) = 1 \times (1 + 28) = 1 \times 29 = 29$$

Practise Now 3

1. $126 = 2 \times 3^2 \times 7$
2. $539 = 7^2 \times 11$

Practise Now 4

1. $$\begin{aligned} 784 &= 2 \times 2 \times 2 \times 2 \times 7 \times 7 \\ &= (2 \times 2 \times 7) \times (2 \times 2 \times 7) \\ &= (2 \times 2 \times 7)^2 \end{aligned}$$
   $$\therefore \sqrt{784} = 2 \times 2 \times 7 = 28$$
   Alternatively,
   $$\begin{aligned} 784 &= 2 \times 2 \times 2 \times 2 \times 7 \times 7 \\ &= 2^4 \times 7^2 \end{aligned}$$
   $$\therefore \sqrt{784} = \sqrt{2^4 \times 7^2} = 2^2 \times 7 = 28$$
2. $$\sqrt{7056} = \sqrt{2^4 \times 3^2 \times 7^2} = 2^2 \times 3 \times 7 = 84$$

Practise Now 5

1. $$\begin{aligned} 2744 &= 2 \times 2 \times 2 \times 7 \times 7 \times 7 \\ &= (2 \times 7) \times (2 \times 7) \times (2 \times 7) \\ &= (2 \times 7)^3 \end{aligned}$$
   $$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$$
   Alternatively,
   $$\begin{aligned} 2744 &= 2 \times 2 \times 2 \times 7 \times 7 \times 7 \\ &= 2^3 \times 7^3 \end{aligned}$$
   $$\therefore \sqrt[3]{2744} = \sqrt[3]{2^3 \times 7^3} = 2 \times 7 = 14$$
2. $$\sqrt[3]{9261} = \sqrt[3]{3^3 \times 7^3} = 3 \times 7 = 21$$

Practise Now 6

$$\sqrt{123} \approx \sqrt{121} = 11$$
$$\sqrt[3]{123} \approx \sqrt[3]{125} = 5$$

Practise Now 7

1. **(a)** $23^2 + \sqrt{2025} - 7^3 = 231$

   (b) $\dfrac{3^2 \times \sqrt{20}}{5^3 - \sqrt[3]{2013}} = 0.3582$ (to 4 d.p.)

2. Length of each side of poster = $\sqrt{987}$

$= 31.42$ cm (to 2 d.p.)

Perimeter of poster = 4×31.42

$= 125.7$ cm (to 1 d.p.)

Practise Now 8

$\sqrt{2013} = 44.9$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2013}$ is 43.

2013 is an odd number, so it is not divisible by 2.

Since the sum of the digits of 2013 is $2 + 1 + 3 = 6$ which is divisible by 3, therefore 2013 is divisible by 3 (divisibility test for 3).

$\therefore$ 2013 is a composite number.

$\sqrt{2017} = 44.9$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2017}$ is 43.

Since 2017 is not divisible by any of the prime numbers 2, 3, 5, 7, …, 43, then 2017 is a prime number.

Practise Now 9

1. **Method 1:**

$56 = 2^3 \times 7$

$84 = 2^2 \times 3 \times 7$

HCF of 56 and 84 $= 2^2 \times 7$

$= 28$

Method 2:

$$\begin{array}{r|l} 2 & 56, 84 \\ \hline 2 & 28, 42 \\ \hline 7 & 14, 21 \\ \hline & \;\; 2, 3 \end{array}$$

HCF of 56 and 84 $= 2 \times 2 \times 7$

$= 28$

2. $28 = 2^2 \times 7$

$70 = 2 \times 5 \times 7$

Largest whole number which is a factor of both 28 and 70

= HCF of 28 and 70

$= 2 \times 7$

$= 14$

3. Greatest whole number that will divide both 504 and 588 exactly

= HCF of 504 and 588

$= 2^2 \times 3 \times 7$

$= 84$

Practise Now 10

$90 = 2 \times 3^2 \times 5$

$135 = 3^3 \times 5$

$270 = 2 \times 3^3 \times 5$

HCF of 90, 135 and 270 $= 3^2 \times 5$

$= 45$

Practise Now 11

1. **Method 1:**

$24 = 2^3 \times 3$

$90 = 2 \times 3^2 \times 5$

LCM of 24 and 90 $= 2^3 \times 3^2 \times 5$

$= 360$

Method 2:

$$\begin{array}{r|l} 2 & 24, 90 \\ \hline 3 & 12, 45 \\ \hline & \;\; 4, 15 \end{array}$$

HCF of 24 and 90 $= 2 \times 3 \times 4 \times 15$

$= 360$

2. Smallest whole number that is divisible by both 120 and 126

= LCM of 120 and 126

$= 2^3 \times 3^2 \times 5 \times 7$

$= 2520$

3. $6 = 2 \times 3$

$24 = 2^3 \times 3$

Smallest value of $n = 2^3$

$= 8$

Practise Now 12

$9 = 3^2$

$30 = 2 \times 3 \times 5$

$108 = 2^2 \times 3^3$

LCM of 9, 30 and 108 $= 2^2 \times 3^3 \times 5$

$= 540$

Practise Now 13

1. $15 = 3 \times 5$

$16 = 2^4$

$36 = 2^2 \times 3^2$

LCM of 15, 16 and 36 $= 2^4 \times 3^2 \times 5$

$= 720$

720 minutes = 12 hours

$\therefore$ The three bells will next toll together at 2.00 a.m.

2. **(i)** $140 = 2^2 \times 5 \times 7$

$168 = 2^3 \times 3 \times 7$

$210 = 2 \times 3 \times 5 \times 7$

HCF of 140, 168 and 210 $= 2 \times 7$

$= 14$

Greatest possible length of each of the smaller pieces of rope

= 14 cm

(ii) Number of smallest pieces of rope he can get altogether

$= \dfrac{140}{14} + \dfrac{168}{14} + \dfrac{210}{14}$

$= 10 + 12 + 15$

$= 37$

Exercise 1A

1. **(a)** 87 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 87 is $8 + 7 = 15$ which is divisible by 3, therefore 87 is divisible by 3 (divisibility test for 3).
$\therefore$ 87 is a composite number.

(b) 67 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 67 is $6 + 7 = 13$ which is not divisible by 3, then 67 is not divisible by 3.
The last digit of 67 is neither 0 nor 5, so 67 is not divisible by 5.
A calculator may be used to test whether 67 is divisible by prime numbers more than 5.
Since 67 is not divisible by any prime numbers less than 67, then 67 is a prime number.

(c) 73 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 73 is $7 + 3 = 10$ which is not divisible by 3, then 73 is not divisible by 3.
The last digit of 73 is neither 0 nor 5, so 73 is not divisible by 5.
A calculator may be used to test whether 73 is divisible by prime numbers more than 5.
Since 73 is not divisible by any prime numbers less than 73, then 73 is a prime number.

(d) 91 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 91 is $9 + 1 = 10$ which is not divisible by 3, then 91 is not divisible by 3.
The last digit of 91 is neither 0 nor 5, so 91 is not divisible by 5.
A calculator may be used to test whether 91 is divisible by prime numbers more than 5.
Since 91 is divisible by 7, therefore 91 is a composite number.

2. **(a)** $72 = 2^3 \times 3^2$ **(b)** $187 = 11 \times 17$
(c) $336 = 2^4 \times 3 \times 7$ **(d)** $630 = 2 \times 3^2 \times 5 \times 7$

3. **(a)**
$$\begin{aligned}1764 &= 2 \times 2 \times 3 \times 3 \times 7 \times 7\\ &= (2 \times 3 \times 7) \times (2 \times 3 \times 7)\\ &= (2 \times 3 \times 7)^2\end{aligned}$$
$$\begin{aligned}\therefore \sqrt{1764} &= 2 \times 3 \times 7\\ &= 42\end{aligned}$$
Alternatively,
$$\begin{aligned}1764 &= 2 \times 2 \times 3 \times 3 \times 7 \times 7\\ &= 2^2 \times 3^2 \times 7^2\end{aligned}$$
$$\begin{aligned}\therefore \sqrt{1764} &= \sqrt{2^2 \times 3^2 \times 7^2}\\ &= 2 \times 3 \times 7\\ &= 42\end{aligned}$$

(b)
$$\begin{aligned}576 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3\\ &= (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3)\\ &= (2 \times 2 \times 2 \times 3)^2\end{aligned}$$
$$\begin{aligned}\therefore \sqrt{576} &= 2 \times 2 \times 2 \times 3\\ &= 24\end{aligned}$$
Alternatively,
$$\begin{aligned}576 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3\\ &= 2^6 \times 3^2\end{aligned}$$
$$\begin{aligned}\therefore \sqrt{576} &= \sqrt{2^6 \times 3^2}\\ &= 2^3 \times 3\\ &= 24\end{aligned}$$

(c)
$$\begin{aligned}3375 &= 3 \times 3 \times 3 \times 5 \times 5 \times 5\\ &= (3 \times 5) \times (3 \times 5) \times (3 \times 5)\\ &= (3 \times 5)^3\end{aligned}$$
$$\begin{aligned}\therefore \sqrt[3]{3375} &= 3 \times 5\\ &= 15\end{aligned}$$
Alternatively,
$$\begin{aligned}3375 &= 3 \times 3 \times 3 \times 5 \times 5 \times 5\\ &= 3^3 \times 5^3\end{aligned}$$
$$\begin{aligned}\therefore \sqrt[3]{3375} &= \sqrt[3]{3^3 \times 5^3}\\ &= 3 \times 5\\ &= 15\end{aligned}$$

(d)
$$\begin{aligned}1728 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3\\ &= (2 \times 2 \times 3) \times (2 \times 2 \times 3) \times (2 \times 2 \times 3)\\ &= (2 \times 2 \times 3)^3\end{aligned}$$
$$\begin{aligned}\therefore \sqrt[3]{1728} &= 2 \times 2 \times 3\\ &= 12\end{aligned}$$
Alternatively,
$$\begin{aligned}1728 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3\\ &= 2^6 \times 3^3\end{aligned}$$
$$\begin{aligned}\therefore \sqrt[3]{1728} &= \sqrt[3]{2^6 \times 3^3}\\ &= 2^2 \times 3\\ &= 12\end{aligned}$$

4.
$$\begin{aligned}\sqrt{9801} &= \sqrt{3^4 \times 11^2}\\ &= 3^2 \times 11\\ &= 99\end{aligned}$$

5.
$$\begin{aligned}\sqrt[3]{21\ 952} &= \sqrt[3]{2^6 \times 7^3}\\ &= 2^2 \times 7\\ &= 28\end{aligned}$$

6. **(a)**
$$\begin{aligned}\sqrt{66} &\approx \sqrt{64}\\ &= 8\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{80} &\approx \sqrt{81}\\ &= 9\end{aligned}$$
(c)
$$\begin{aligned}\sqrt[3]{218} &\approx \sqrt[3]{216}\\ &= 6\end{aligned}$$
(d)
$$\begin{aligned}\sqrt[3]{730} &\approx \sqrt[3]{729}\\ &= 9\end{aligned}$$

7. **(a)** $7^2 - \sqrt{361} + 21^3 = 9291$

(b) $\dfrac{\sqrt{555} + 5^2}{2^3 \times \sqrt[3]{222}} = 1.0024$ (to 4 d.p.)

(c) $\sqrt{4^3 + \sqrt[3]{4913}} = 9$

8. Length of each side of photo frame = $\sqrt{250}$

$= 15.81$ cm (to 2 d.p.)

Perimeter of photo frame $= 4 \times 15.81$

$= 63.2$ cm (to 1 d.p.)

9. Length of each side of box = $\sqrt[3]{2197}$

$= 13$ cm

Area of one side of box $= 13^2$

$= 169\text{ cm}^2$

10. **(a)** $\sqrt{667} = 25.8$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{667}$ is 23.

667 is an odd number, so it is not divisible by 2.

Since the sum of the digits of 667 is $6 + 6 + 7 = 19$ which is not divisible by 3, then 667 is not divisible by 3.

The last digit of 667 is neither 0 nor 5, so 667 is not divisible by 5.

A calculator may be used to test whether 667 is divisible by prime numbers more than 5.

Since 667 is divisible by 23, therefore 667 is a composite number.

(b) $\sqrt{677} = 26.0$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{677}$ is 23.

Since 677 is not divisible by any of the prime numbers 2, 3, 5, 7, …, 23, then 677 is a prime number.

(c) $\sqrt{2021} = 45.0$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2021}$ is 43.

2021 is an odd number, so it is not divisible by 2.

Since the sum of the digits of 2021 is $2 + 0 + 2 + 1 = 5$ which is not divisible by 3, then 2021 is not divisible by 3.

The last digit of 2021 is neither 0 nor 5, so 2021 is not divisible by 5.

A calculator may be used to test whether 2021 is divisible by prime numbers more than 5.

Since 2021 is divisible by 43, therefore 2021 is a composite number.

(d) $\sqrt{2027} = 45.0$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2027}$ is 43.

Since 2027 is not divisible by any of the prime numbers 2, 3, 5, 7, …, 43, then 2027 is a prime number.

11. Since 37 is a prime number, then 1 and 37 are its only two factors.

It does not matter whether p or q is 1 or 37 as we only want to find the value of $p + q$.

$\therefore p + q = 1 + 37 = 38$

12. Since $n \times (n + 42)$ is a prime number, then n and $n + 42$ are its only two factors.

Since 1 has to be one of its two factors, then $n = 1$.

$\therefore n \times (n + 42) = 1 \times (1 + 42)$

$= 1 \times 43$

$= 43$

Exercise 1B

1. **(a)** $12 = 2^2 \times 3$

$30 = 2 \times 3 \times 5$

HCF of 12 and 30 $= 2 \times 3$

$= 6$

(b) $84 = 2^2 \times 3 \times 7$

$156 = 2^2 \times 3 \times 13$

HCF of 84 and 156 $= 2^2 \times 3$

$= 12$

(c) $15 = 3 \times 5$

$60 = 2^2 \times 3 \times 5$

$75 = 3 \times 5^2$

HCF of 15, 60 and 75 $= 3 \times 5$

$= 15$

(d) $77 = 7 \times 11$

$91 = 7 \times 13$

$143 = 11 \times 13$

HCF of 77, 91 and 143 = 1

2. **(a)** $24 = 2^3 \times 3$

$30 = 2 \times 3 \times 5$

LCM of 24 and 30 $= 2^3 \times 3 \times 5$

$= 120$

(b) $42 = 2 \times 3 \times 7$

$462 = 2 \times 3 \times 7 \times 11$

LCM of 42 and 462 $= 2 \times 3 \times 7 \times 11$

$= 462$

(c) $12 = 2^2 \times 3$

$18 = 2 \times 3^2$

$81 = 3^4$

LCM of 12, 18 and 81 $= 2^2 \times 3^4$

$= 324$

(d) $63 = 3^2 \times 7$

$80 = 2^4 \times 5$

$102 = 2 \times 3 \times 17$

LCM of 63, 80 and 102 $= 2^4 \times 3^2 \times 5 \times 7 \times 17$

$= 85\,680$

3. $42 = 2 \times 3 \times 7$

$98 = 2 \times 7^2$

Largest whole number which is a factor of both 42 and 98

= HCF of 42 and 98

$= 2 \times 7$

$= 14$

4. Greatest whole number that will divide both 792 and 990 exactly

= HCF of 792 and 990

$= 2 \times 3^2 \times 11$

$= 198$

5. Smallest whole number that is divisble by both 176 and 342

= LCM of 176 and 342

$= 2^4 \times 3^2 \times 11 \times 19$

$= 30\,096$

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6. $15 = 3 \times 5$

$45 = 3^2 \times 5$

Smallest value of $n = 3^2$

$= 9$

7. **(i)** $171 = 3^2 \times 19$

$63 = 3^2 \times 7$

$27 = 3^3$

HCF of 171, 63 and $27 = 3^2$

$= 9$

Largest number of gift bags that can be packed = 9

(ii) Number of pens in a gift bag $= 171 \div 9$

$= 19$

Number of pencils in a gift bag $= 63 \div 9$

$= 7$

Number of erasers in a gift bag $= 27 \div 9$

$= 3$

8. **(i)** $60 = 2^2 \times 3 \times 5$

$80 = 2^4 \times 5$

LCM of 60 and $80 = 2^4 \times 3 \times 5$

$= 240$

It will take 240 s for both cars to be back at the starting point at the same time.

(ii) 5×240 s = 1200 s

= 20 minutes

It will take 20 minutes for the faster car to be 5 laps ahead of the slower car.

9. **(a)** True.

If 6 is a factor of a whole number n, then $n = 6k$ for some whole number k.

We have $n = 6k = 2(3k)$. Since $3k$ is a whole number, then 2 is a factor of n.

We also have $n = 6k = 3(2k)$. Since $2k$ is a whole number, then 3 is a factor of n.

(b) True. Since 2 and 3 are distinct prime factors of the whole number, then the prime factorisation of the whole number will contain both of these prime factors.

(c) False, e.g. 2 and 4 are factors of 4, but 8 is not a factor of 4.

(d) True. If f is a factor of n, then $n = fk$ for some whole number k. Thus $\frac{n}{f} = k$ is a whole number. Since n can be written as a product of the whole numbers $\frac{n}{f}$ and f, then $\frac{n}{f}$ is a factor of n.

(e) True. Since h is a factor of both p and q, then both p and q are divisible by h.

10. $9 = 3^2$

$12 = 2^2 \times 3$

$252 = 2^2 \times 3^2 \times 7$

Possible values of $n = 7, 3 \times 7$ or $3^2 \times 7$

$= 7, 21$ or 63

11. **(a)** True. If 6 is a multiple of a whole number n, then $6 = nk$ for some whole number k. We have $12 = 2nk = n(2k)$. Since $2k$ is a whole number, then 12 is a multiple of n.

(b) False, e.g. 12 is a multiple of 4, but 6 is not a multiple of 4.

(c) True. If 18 is a multiple of a whole number n, then $18 = nk$ for some whole number k. Thus $\frac{18}{n} = k$ is a whole number, i.e. 18 is divisible by n.

(d) True. Since m is a multiple of p, by the same reasoning as in **(c)**, then m is divisible by p. Similarly, m is divisible by q.

12. **(i)** $64 = 2^6$

$48 = 2^4 \times 3$

HCF of 64 and $48 = 2^4$

$= 16$

Length of each square = 16 cm

(ii) Number of squares that can be cut altogether $= \frac{64}{16} \times \frac{48}{16}$

$= 4 \times 3$

$= 12$

13. **(i)** Let the number of boys in the class be n.

Then $15 \times n = 3 \times 5 \times n$ is divisible by $21 = 3 \times 7$.

Thus the possible values of n are multiples of 7.

Hence, $n = 14$ since $14 + 20 = 34$ students is the only possibility where the number of students in the class is between 30 and 40.

$\therefore$ Number of students in the class = 34

(ii) Number of chocolate bars their form teacher receive

$= \frac{15 \times 14}{21}$

$= 10$

14. **(i)** $126 = 2 \times 3^2 \times 7$

$108 = 2^2 \times 3^3$

HCF of 126 and $108 = 2 \times 3^3$

$= 18$

Length of each square = 18 cm

Least number of square patterns that could be formed on the sheet of paper

$= \frac{126}{18} \times \frac{108}{18}$

$= 7 \times 6$

$= 42$

(ii) To fit the sheet of paper perfectly, the patterns can be rectangular, triangular or trapeziums with two right angles, etc.

15. **(i)** $45 = 3^2 \times 5$

$42 = 2 \times 3 \times 7$

LCM of 45 and $42 = 2 \times 3^2 \times 5 \times 7$

$= 630$

Number of patterns needed to form the smallest square

$= \frac{630}{45} \times \frac{630}{42}$

$= 14 \times 15$

$= 210$

(ii) 630 mm = 0.63 m

Area of smallest square that can be formed $= 0.63^2$

$= 0.3969 \text{ m}^2$

By trial and error,

Area of largest square that can be formed

$= 0.3969 \times 2^2$

$= 1.5876 \text{ m}^2 < 1.6 \text{ m}^2$

$\therefore$ Length of largest square that can be formed $= \sqrt{1.5876}$

$= 1.26$ m

Review Exercise 1

1. **(a)** $1225 = 5 \times 5 \times 7 \times 7$
$= (5 \times 7) \times (5 \times 7)$
$= (5 \times 7)^2$
$\therefore \sqrt{1225} = 5 \times 7$
$= 35$

Alternatively,
$1225 = 5 \times 5 \times 7 \times 7$
$= 5^2 \times 7^2$
$\therefore \sqrt{1225} = \sqrt{5^2 \times 7^2}$
$= 5 \times 7$
$= 35$

(b) $13\ 824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3)$
$= (2 \times 2 \times 2 \times 3)^3$
$\therefore \sqrt[3]{13\ 824} = 2 \times 2 \times 2 \times 3$
$= 24$

Alternatively,
$13\ 824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^9 \times 3^3$
$\therefore \sqrt[3]{13\ 824} = \sqrt[3]{2^9 \times 3^3}$
$= 2^3 \times 3$
$= 24$

2. **(a)** $\sqrt{63} \approx \sqrt{64}$
$= 8$

(b) $\sqrt[3]{345} \approx \sqrt[3]{343}$
$= 7$

3. **(a)** $\sqrt{753} = 27.4$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{753}$ is 23.
753 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 753 is $7 + 5 + 3 = 15$ which is divisible by 3, therefore 753 is divisible by 3 (divisibility test for 3).
$\therefore$ 753 is a composite number.

(b) $\sqrt{757} = 27.5$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{757}$ is 23.
Since 757 is not divisible by any of the prime numbers 2, 3, 5, 7, …, 23, then 757 is a prime number.

4. **(i)** Greatest whole number that will divide both 840 and 8316 exactly
= HCF of 840 and 8316
$= 2^2 \times 3 \times 7$
$= 84$

(ii) Smallest whole number that is divisible by both 840 and 8316
= LCM of 840 and 8316
$= 2^3 \times 3^3 \times 5 \times 7 \times 11$
$= 83\ 160$

5. $6 = 2 \times 3$
$12 = 2^2 \times 3$
$660 = 2^2 \times 3 \times 5 \times 11$
Possible values of $n = 5 \times 11, 2 \times 5 \times 11, 3 \times 5 \times 11,$
$2^2 \times 5 \times 11, 2 \times 3 \times 5 \times 11$
or $2^2 \times 3 \times 5 \times 11$
$= 55, 110, 165, 220, 330$ or 660

6. **(i)** $108 = 2^2 \times 3^3$
$81 = 3^4$
$54 = 2 \times 3^3$
HCF of 108, 81 and $54 = 3^3$
$= 27$
Largest number of baskets that can be packed = 27

(ii) Number of stalks of roses in a basket $= 108 \div 27$
$= 4$
Number of stalks of lilies in a basket $= 81 \div 27$
$= 3$
Number of stalks of orchids in a basket $= 54 \div 27$
$= 2$

7. Time taken for Khairul to run 1 round = 360 s
= 6 minutes
Time taken for Devi to cycle 1 round $= 4 \div 2$
= 2 minutes
$18 = 2 \times 3^2$
$6 = 2 \times 3$
$2 = 2$
LCM of 18, 6 and $2 = 2 \times 3^2$
$= 18$
All three of them will next meet at 6.03 p.m.

8. **(i)** By counting, they will next have the same day off on 7 May.

(ii) $4 = 2^2$
$6 = 2 \times 3$
LCM of 4 and $6 = 2^2 \times 3$
$= 12$
Subsequently, they will have the same day off every 12 days.

Challenge Yourself

1. **(i)** The six adjacent numbers are 11, 12, 1, 2, 3, 4.

(ii) The other six numbers are 5, 6, 7, 8, 9, 10.
Make a list where the sum of each of the pairs of numbers is a prime number:

- $4 + 7 = 11; 4 + 9 = 13$
- $5 + 6 = 11; 5 + 8 = 13$
- $6 + 5 = 11; 6 + 7 = 13; 6 + 11 = 17$
- $7 + 4 = 11; 7 + 6 = 13; 7 + 10 = 17$
- $8 + 5 = 13; 8 + 9 = 17; 8 + 11 = 19$
- $9 + 4 = 13; 9 + 8 = 17; 9 + 10 = 19$
- $10 + 7 = 17; 10 + 9 = 19$
- $11 + 6 = 17; 11 + 8 = 19$

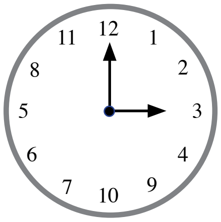
Notice that 5 and 10 are the only numbers that can be adjacent to two numbers only:

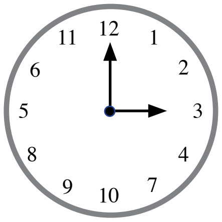
- 5 can be adjacent to 6 and 8 only, i.e. 6 – 5 – 8 or 8 – 5 – 6;
- 10 can be adjacent to 7 and 9 only, i.e. 7 – 10 – 9 or 9 – 10 – 7.

Since 4 and 11 are adjacent to another number on one side, then

- the only two possibilities for the other side of 4 are 7 and 9;
- the only two possibilities for the other side of 11 are 6 and 8.

With the above information, we can narrow down the possible arrangements to only two ways:





2. LCM of 3 and 4 = 12

$\therefore$ We divide the 3 identical squares into 12 equal parts.

Thus we have:

3. **(i)** $120 = 2^3 \times 3 \times 5$

$126 = 2 \times 3^2 \times 7$

HCF of 120 and 126 $= 2 \times 3$

$= 6$

LCM of 120 and 126 $= 2^3 \times 3^2 \times 5 \times 7$

$= 2520$

(ii) HCF × LCM $= 6 \times 2520$

$= 15\ 120$

$= 120 \times 126$ (Shown)

$120 = \mathbf{2^3} \times \mathbf{3} \times \mathbf{5}$

$126 = \mathbf{2} \times \mathbf{3^2} \times \mathbf{7}$

To obtain the **HCF** of 120 and 126, we choose the power of each of the common prime factors with the smaller index, i.e. **2** and **3**.

On the other hand, to obtain the **LCM** of 120 and 126, we choose the power of each of the common prime factors with the higher index, i.e. $\mathbf{2^3}$ and $\mathbf{3^2}$, and the remaining factors, i.e. **5** and **7**.

Since each term in the prime factorisation of 120 and 126 is used to find either their **HCF** or their **LCM**, the product of the **HCF** and **LCM** of 120 and 126 is equal to the product of 120 and 126.

(iii) Yes, the result in **(ii)** can be generalised for any two numbers.

Proof:

Consider two numbers x and y.

Then $x = \text{HCF} \times p$, — (1)

$y = \text{HCF} \times q$, — (2)

where the HCF of p and q is 1.

$(1) \times q$: $x \times q = \text{HCF} \times p \times q$ — (3)

$(2) \times p$: $y \times p = \text{HCF} \times p \times q$ — (4)

$(3) \times (4)$: $x \times y \times p \times q = \text{HCF} \times p \times q \times \text{HCF} \times p \times q$

$x \times y = \text{HCF} \times \text{HCF} \times p \times q$

Since the HCF of p and q is 1, we cannot take out a factor greater than 1 in the product $p \times q$, thus $\text{HCF} \times p \times q = \text{LCM}$.

$\therefore x \times y = \text{HCF} \times \text{LCM}$

(iv) No, the result in **(ii)** cannot be generalised for any three numbers.

For example, consider the numbers 10, 20 and 25.

$10 = 2 \times 5$

$20 = 2^2 \times 5$

$25 = 5^2$

HCF of 10, 20 and 25 = 5

LCM of 10, 20 and 25 $= 2^2 \times 5^2$

$= 100$

HCF × LCM $= 5 \times 100$

$= 500$

$\neq 10 \times 20 \times 25$

4. Number of squares passed through by a diagonal of a m-by-n rectangle

$= m + n - \text{HCF}(m, n)$

5. **(i)** Fraction of a sausage each person gets $= \dfrac{12}{18}$

$= \dfrac{2}{3}$

$\therefore$ Least number of cuts required = 12

(ii) Least number of cuts required $= n - \text{HCF}(m, n)$

Chapter 2 Integers, Rational Numbers and Real Numbers

TEACHING NOTES

Suggested Approach

Although the concept of negative numbers is new to most students as they have not learnt this in primary school, they do encounter negative numbers in their daily lives, e.g. in weather forecasts. Therefore, teachers can get students to discuss examples of the use of negative numbers in the real world to bring across the idea of negative numbers (see Class Discussion: Uses of Negative Numbers in the Real World). The learning experiences in the new syllabus specify the use of algebra discs. In this chapter, only number discs (or counters) showing the numbers 1 and –1 are needed. Since many Secondary 1 students are still in the concrete operational stage (according to Piaget), the use of algebra discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use algebra discs in examinations, and partly because they cannot use algebra discs to add or subtract large negative integers, and decimals (see Section 2.2).

Section 2.1: Negative Numbers

Teachers should teach students to read the negative number –2 as negative 2, not minus 2 ('negative' is a state while 'minus' is an operation). For example, if you have \$5 and you owe your friend \$2, how much do you have left? Since nothing is mentioned about you returning money to your friend, you have \$5 left. Thus \$2 is a state of owing money. However, if you return \$2 to your friend, you have \$5 + (–\$2) = \$5 – \$2 = \$3 left, i.e. 5 minus 2 is an operation of returning money.

Students should also learn about the absolute value of a negative number (see page 29 of the textbook) because they will need it in Section 2.2.

In primary school, students have only learnt the terms 'less than' and 'more than', so there is a need to teach them how to use the symbols '<' and '>' when comparing numbers. It is not necessary to teach them about 'less than or equal to' and 'more than or equal to' now.

Section 2.2: Addition and Subtraction involving Negative Numbers

Algebra discs cannot be used to add or subtract large negative integers, and decimals, so there is a need to help students consolidate what they have learnt in the class discussions on pages 33 and 35 of the textbook by moving away from the 'concrete' to the following two key 'abstract' concepts:

Key Concept 1: Adding a negative number is the same as subtracting the absolute value of the number, e.g. $5 + (-2) = 5 - 2$.

Key Concept 2: Subtracting a negative number is the same as adding the absolute value of the number, e.g. $5 - (-2) = 5 + 2$.

To make the key concepts less abstract, numerical examples are used. Do not use algebra now because students are still unfamiliar with algebra even though they have learnt some basic algebra in primary school. Avoid teaching students '$- \times - = +$' now because the idea behind $5 - (-2)$ is subtraction, not multiplication. To make practice more interesting, a puzzle is designed on page 36 of the textbook.

Section 2.3: Multiplication and Division involving Negative Numbers

The idea of flipping over a disc to obtain the negative of a number, e.g. $-(-3) = 3$, is important in teaching multiplication involving negative numbers. Since algebra discs cannot be used to teach division involving negative numbers, another method is adopted (see page 40 of the textbook).

There is a need to revisit square roots and cube roots in this section to discuss negative square roots and negative cube roots (see page 40 of the textbook). Teachers can impress upon students that the square root symbol $\sqrt{}$ refers to the positive square root only.

Section 2.4: Rational Numbers and Real Numbers

Traditionally, real numbers are classified as either rational or irrational numbers. Another way to classify real numbers is according to whether their decimal forms are terminating, recurring, or non-recurring (see page 50 of the textbook). If teachers show students the first million digits of π (see page 51 of the textbook), many students may be surprised that π has so many digits! This suggests that students do not know that π has an infinite number of decimal places. Teachers may wish to celebrate Pi Day with students on March 14 by talking about π or singing the Pi song.

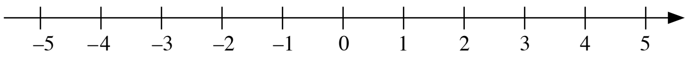
WORKED SOLUTIONS

Class Discussion (Uses of Negative Numbers in the Real World)

- One of the most common uses of negative numbers is in the measurement of temperature, where negative numbers are used to show temperatures below the freezing point of water, i.e. 0 °C. Absolute zero, defined as 0 Kelvin, is the theoretical lowest possible temperature. 0 Kelvin is equivalent to a temperature of –273.15 °C, therefore the theoretical lowest possible temperature is 273.15 °C below 0 °C.
- The elevation of a location commonly refers to its height with reference to Earth's sea level and can be represented by a positive or a negative number. Given a point with an elevation of –200 m, we can deduce that the point is 200 m below sea level. The lowest elevation on Earth that is on dry land is the Dead Sea shore in Asia with an elevation of –423 m, i.e. the shore of the Dead Sea is 423 m below sea level.
- Negative numbers are also used to tell time zones, which are based on Greenwich Mean Time (GMT). A country which is in the time zone of GMT –2 means that the time in that country is 2 hours behind the GMT. For example, Honolulu, Hawaii is in the time zone of GMT –10, while Liverpool, United Kingdom is in the time zone of GMT 0, therefore when it is 10 a.m. in Liverpool, it is 12 midnight in Honolulu.
- Latitude and longitude are a set of coordinates that allow for the specification of a geographical location on the Earth's surface and can be represented by positive and/or negative numbers. The latitude of a point is determined with reference to the equatorial plane; the North Pole has a latitude of +90°, which means that it is 90° north of the equator while the South Pole has a latitude of –90°, which means that it is 90° south of the equator. The longitude of a point gives its east-west position relative to the Prime Meridian (longitude 0°); a location with a longitude of +50° means that it is 50° east of the Prime Meridian while a location with a longitude of –50° means that it is 50° west of the Prime Meridian. The latitude and longitude of Rio Grande, Mexico are approximately –32° and –52° respectively, which means that it is 32° south of the equator and 52° west of the Prime Meridian.
- The use of negative numbers can also be seen in scoring systems, such as in golf. Each hole has a par score, which indicates the number of strokes required and a golfer's score for that hole is calculated based on the number of strokes played. A score of +3 on a hole shows that the golfer played three strokes above par, while a score of –3 on a hole shows that the golfer played three strokes under par.

Teachers may wish to note that the list is not exhaustive.

Thinking Time (Page 28)



(a) Since –3 is on the left of 2, we say '–3 is less than 2' and we write '$-3 < 2$'.

(b) Since –3 is on the right of –5, we say '–3 is more than –5' and we write '$-3 > -5$'.

Class Discussion (Addition involving Negative Numbers)

Part I

1. **(a)** $7 + (-3) = 4$
(b) $6 + (-4) = 2$

2. **(a)** $(-7) + 3 = -4$
(b) $(-6) + 4 = -2$

3. **(a)** $(-7) + (-3) = -10$
(b) $(-6) + (-4) = -10$

Note:

- If we add a positive number and a negative number,
  (i) we take the *difference* between the absolute values of the two numbers, and
  (ii) the sign of the answer follows the sign of the number with the greater absolute value,
  e.g. $5 + (-2) = 3$ and $(-5) + 2 = -3$.
- If we add two negative numbers,
  (i) we take the *sum* of the absolute values of the two numbers, and
  (ii) the answer is negative,
  e.g. $(-5) + (-2) = -7$.

Class Discussion (Subtraction involving Negative Numbers)

Part I

1. **(a)** $7 - (-3) = 7 + 3$
$= 10$
(b) $6 - (-4) = 6 + 4$
$= 10$

2. **(a)** $(-7) - 3 = (-7) + (-3)$
$= -10$
(b) $(-6) - 4 = (-6) + (-4)$
$= -10$

3. **(a)** $(-7) - (-3) = (-7) + 3$
$= -4$
(b) $(-4) - (-6) = (-4) + 6$
$= 2$

4. **(a)** $3 - 7 = 3 + (-7)$
$= -4$
(b) $4 - 6 = 4 + (-6)$
$= -2$

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Note:

- If we take the difference of a positive number and a negative number,
  (i) we *add* the absolute values of the two numbers, and
  (ii) the sign of the answer follows the sign of the number with the greater absolute value,
  e.g. $5 - (-2) = 7$ and $(-5) - 2 = -7$.
- If we take the difference of two negative numbers or two positive numbers,
  (i) we take the *difference* between the absolute values of the two numbers, and
  (ii) the sign of the answer depends on whether the first number is greater than or smaller than the second number,
  e.g. $(-5) - (-2) = -3$ but $(-2) - (-5) = 3$;
  $2 - 5 = -3$ but $5 - 2 = 3$.

Class Discussion (Multiplication involving Negative Numbers)

Part I

1. **(a)** $1 \times (-4) = -4$
(b) $2 \times (-4) = -8$
(c) $3 \times (-4) = -12$

2. **(a)** $(-1) \times 4 = -4$
(b) $(-2) \times 4 = -8$
(c) $(-3) \times 4 = -12$

3. **(a)** $(-1) \times (-4) = 4$
(b) $(-2) \times (-4) = 8$
(c) $(-3) \times (-4) = 12$

Note: In general,

> *positive* number × *negative* number = *negative* number,
> *negative* number × *positive* number = *negative* number,
> *negative* number × *negative* number = *positive* number.

Thinking Time (Page 41)

It is not possible to obtain the square roots of a negative number, e.g. $\pm\sqrt{-16}$, because the square of any number is more than or equal to 0.

Teachers may wish to take this opportunity to highlight to higher-ability students that even though $\pm\sqrt{-16}$ is not defined in the set of real numbers, it is defined in the set of complex numbers.

Thinking Time (Page 49)

(a) Any integer m can be expressed in the form $\frac{m}{1}$, e.g. $2 = \frac{2}{1}$ and $-3 = \frac{-3}{1}$.

In particular, the integer 0 can be expressed in the form $\frac{0}{n}$, where n is any integer except 0.

(b) There is more than one way to express a decimal in the form $\frac{a}{b}$,

e.g. $0.5 = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} \ldots$ and $0.333\ldots = \frac{1}{3} = \frac{2}{6} = \frac{3}{9} \ldots$

Investigation (Terminating, Recurring and Non-Recurring Decimals)

Group 1	Group 2	Group 3
$\frac{9}{4} = 2.25$	$\frac{1}{3} = 0.333\ 333\ 333\ 3$	$\frac{1}{\sqrt{2}} = 0.707\ 106\ 781\ 2$
$-3\frac{1}{8} = -3.125$	$-\frac{123}{99} = -1.242\ 424\ 242$	$-\sqrt[3]{5} = -1.709\ 975\ 947$
$\frac{63}{64} = 0.984\ 375$	$\frac{22}{7} = 3.142\ 857\ 143$	$\pi = 3.141\ 592\ 654$

Table 2.1

1. Based on the calculator values, π is not equal to $\frac{22}{7}$.

2. For each of the numbers in Group 2, some digits after the decimal point repeat themselves indefinitely. The numbers in Group 2 are rational numbers.

3. For each of the numbers in Group 1, the digits after the decimal point terminate. The numbers in Group 1 are rational numbers.
For each of the numbers in Group 3, the digits after the decimal point do not repeat but they continue indefinitely. The numbers in Group 3 are irrational numbers.

4.

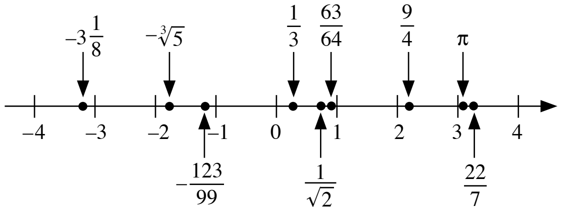


Fig. 2.7

5. $\frac{22}{7}, \pi, \frac{9}{4}, \frac{63}{64}, \frac{1}{\sqrt{2}}, \frac{1}{3}, -\frac{123}{99}, -\sqrt[3]{5}, -3\frac{1}{8}$

Investigation (Some Interesting Facts about the Irrational Number π)

1. The 1 000 000$^{\text{th}}$ digit of π is 1.

2. The 5 000 000 000 000$^{\text{th}}$ digit of π is 2.

3. Lu Chao, a graduate student from China, took 24 hours and 4 minutes to recite π to 67 890 decimal places in 2005.

Practise Now (Page 27)

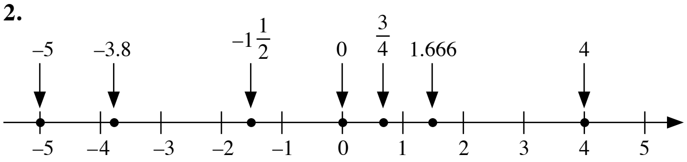
1. **(i)** 2013, 6
(ii) –5, –17
(iii) 2013, 1.666, $\frac{3}{4}$, 6
(iv) –5, $-\frac{1}{2}$, –3.8, –17, $-\frac{2}{3}$

2. **(a)** –43.6 °C
(b) –423 m
(c) –1
(d) –$10 000

Practise Now (Page 29)

1. **(b)** $<$ **(c)** $>$ **(d)** $>$

2.



$\therefore -5, -3.8, -1\frac{1}{2}, 0, \frac{3}{4}, 1.666, 4$

Practise Now (Page 33)

(a) $9 + (-2) = 7$
(b) $-7 + 4 = -3$
(c) $3 + (-5) = -2$
(d) $-6 + (-8) = -14$
(e) $27 + (-13) = 14$
(f) $-25 + 11 = -14$
(g) $14 + (-16) = -2$
(h) $-12 + (-15) = -27$

Practise Now 1

Temperature in the morning $= -8\text{ °C} + 2\text{ °C}$
$= -6\text{ °C}$

Practise Now (Page 35)

(a) $9 - (-2) = 9 + 2$
$= 11$
(b) $-7 - 4 = -11$
(c) $-3 - (-5) = -3 + 5$
$= 2$
(d) $-8 - (-6) = -8 + 6$
$= -2$
(e) $4 - 8 = -4$
(f) $27 - (-13) = 27 + 13$
$= 40$
(g) $-25 - 11 = -36$
(h) $-14 - (-16) = -14 + 16$
$= 2$
(i) $-15 - (-12) = -15 + 12$
$= -3$
(j) $10 - 28 = -18$

Practise Now 2

1. Point *A* shows –5 °C.
Point *B* shows 23 °C.
Difference in temperature $= 23\text{ °C} - (-5\text{ °C})$
$= 23\text{ °C} + 5\text{ °C}$
$= 28\text{ °C}$

2. Altitude at $D = -165$ m
Difference in altitude $= 314\text{ m} - (-165\text{ m})$
$= 314\text{ m} + 165\text{ m}$
$= 479\text{ m}$

Practise Now (Page 39)

(a) $2 \times (-6) = -12$
(b) $-5 \times 4 = -20$
(c) $-1 \times (-8) = 8$
(d) $-3 \times (-7) = 21$
(e) $-(-10) = 10$
(f) $-9(-2) = 18$
(g) $15 \times (-2) = -30$
(h) $-3 \times 12 = -36$
(i) $-4 \times (-10) = 40$
(j) $-2(-100) = 200$

Practise Now (Page 40)

(a) $-8 \div 2 = -4$
(b) $15 \div (-3) = -5$
(c) $-8 \div (-4) = 2$
(d) $\frac{-6}{3} = -2$
(e) $\frac{20}{-5} = -4$
(f) $\frac{-12}{-3} = 4$

Practise Now 3a

(a) Square roots of 64 $= \pm\sqrt{64}$
$= \pm 8$
(b) Negative square root of 9 $= -\sqrt{9}$
$= -3$
(c) $\sqrt{36} = 6$

Practise Now 3b

(a) $(-3)^3 = -27$ **(b)** $(-4)^3 = -64$
(c) $\sqrt[3]{216} = 6$ **(d)** $\sqrt[3]{-8} = -2$

Practise Now 4a

(a) $-3 \times (15 - 7 + 2) = -3 \times (8 + 2)$
$= -3 \times 10$
$= -30$
(b) $4^3 - 7 \times [16 - (\sqrt[3]{64} - 5)] = 64 - 7 \times [16 - (4 - 5)]$
$= 64 - 7 \times [16 - (-1)]$
$= 64 - 7 \times (16 + 1)$
$= 64 - 7 \times 17$
$= 64 - 119$
$= -55$

Practise Now 4b

(a) $-3 \times (15 - 7 + 2) = -30$
(b) $4^3 - 7 \times [16 - (\sqrt[3]{64} - 5)] = -55$

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Practise Now 5

(a) $7\frac{1}{2} + \left(-3\frac{3}{5}\right) = 7\frac{1}{2} - 3\frac{3}{5}$

$= 7\frac{5}{10} - 3\frac{6}{10}$

$= \left(6 + \frac{10}{10}\right) + \frac{5}{10} - 3\frac{6}{10}$

$= 3\frac{9}{10}$

(b) $-2\frac{3}{4} + \left(-\frac{5}{6}\right) - \left(-\frac{2}{3}\right) = -2\frac{3}{4} - \frac{5}{6} + \frac{2}{3}$

$= -\frac{11}{4} - \frac{5}{6} + \frac{2}{3}$

$= -\frac{33}{12} - \frac{10}{12} + \frac{8}{12}$

$= \frac{-33 - 10 + 8}{12}$

$= \frac{-43 + 8}{12}$

$= \frac{-35}{12}$

$= -2\frac{11}{12}$

Practise Now 6

(a) $2\frac{2}{3} \times \frac{9}{4} = \frac{{}^{2}\cancel{8}}{{}_{1}\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{4}_{1}}$

$= 2 \times 3$

$= 6$

(b) $4\frac{1}{6} \div \frac{5}{2} = \frac{25}{6} \div \frac{5}{2}$

$= \frac{{}^{5}\cancel{25}}{{}_{3}\cancel{6}} \times \frac{\cancel{2}^{1}}{\cancel{5}_{1}}$

$= \frac{5}{3}$

$= 1\frac{2}{3}$

Practise Now 7a

(a) $5\frac{1}{4} \div \left(-2\frac{4}{5}\right) = \frac{21}{4} \div \left(-\frac{14}{5}\right)$

$= \frac{{}^{3}\cancel{21}}{4} \times \left(-\frac{5}{\cancel{14}_{2}}\right)$

$= -\frac{15}{8}$

$= -1\frac{7}{8}$

(b) $1\frac{3}{4} \times \left[\frac{6}{5} + \left(-\frac{1}{2}\right)\right] = \frac{7}{4} \times \left(\frac{6}{5} - \frac{1}{2}\right)$

$= \frac{7}{4} \times \left(\frac{12}{10} - \frac{5}{10}\right)$

$= \frac{7}{4} \times \frac{7}{10}$

$= \frac{49}{40}$

$= 1\frac{9}{40}$

Practise Now 7b

Practise Now 5

(a) $7\frac{1}{2} + \left(-3\frac{3}{5}\right) = 3\frac{9}{10}$

(b) $-2\frac{3}{4} + \left(-\frac{5}{6}\right) - \left(-\frac{2}{3}\right) = -2\frac{11}{12}$

Practise Now 6

(a) $2\frac{2}{3} \times \frac{9}{4} = 6$

(b) $4\frac{1}{6} \div \frac{5}{2} = 1\frac{2}{3}$

Practise Now 7a

(a) $5\frac{1}{4} \div \left(-2\frac{4}{5}\right) = -1\frac{7}{8}$

(b) $1\frac{3}{4} \times \left[\frac{6}{5} + \left(-\frac{1}{2}\right)\right] = 1\frac{9}{40}$

Practise Now 8

(a)

$$\begin{array}{r} 13\ .\ 56 \\ \times \quad 2\ .\ 4 \\ \hline 5\ 424 \\ +\ 27\ 12 \\ \hline 32.544 \\ \hline \end{array}$$

$\therefore 13.56 \times 2.4 = 32.544$

(b)

$$\begin{array}{r} 13\ 7\ .\ 8 \\ \times \quad 0\ .\ 35 \\ \hline 6\ 890 \\ +\ 41\ 34 \\ \hline 48.230 \\ \hline \end{array}$$

$\therefore 137.8 \times 0.35 = 48.23$

Practise Now 9

(a) $0.92 \div 0.4 = \dfrac{0.92}{0.4}$

$= \dfrac{9.2}{4}$

$$
\begin{array}{r}
2.3 \\
4\overline{)\;9.2} \\
\underline{-\;8} \\
1\;2 \\
\underline{-\;1\;2} \\
0
\end{array}
$$

$\therefore 0.92 \div 0.4 = 2.3$

(b) $1.845 \div 0.15 = \dfrac{1.845}{0.15}$

$= \dfrac{184.5}{15}$

$$
\begin{array}{r}
1\;2\;.\;3 \\
15\overline{)\;1\;8\;4\;.\;5} \\
\underline{-\;1\;5} \\
3\;4 \\
\underline{-\;3\;0} \\
4\;5 \\
\underline{-\;4\;5} \\
0
\end{array}
$$

$\therefore 1.845 \div 0.15 = 12.3$

Practise Now 10

(a) $32 - (-1.6) = 32 + 1.6$

$= 33.6$

(b) $1.3 + (-3.5) = -2.2$

(c) $\dfrac{0.12}{0.4} \times \left(\dfrac{-0.23}{0.6}\right) = \dfrac{1.2}{4} \times \left(\dfrac{-0.23}{0.6}\right)$

$= 0.3 \times \left(\dfrac{-0.23}{0.6}\right)$

$= {}^{1}\cancel{3} \times \left(\dfrac{-0.23}{\cancel{6}_2}\right)$

$= -0.115$

(d) $-0.3^2 \times \left(\dfrac{4.5}{-2.7}\right) - 0.65 = -0.3^2 \times \dfrac{45}{-27} - 0.65$

$= {}^{0.03}\cancel{-0.09} \times \dfrac{\cancel{45}^{\,5}}{-\cancel{27}\;\cancel{3}_1} - 0.65$

$= 0.15 - 0.65$

$= -0.5$

Practise Now 11

$\dfrac{\pi \times 0.7^2}{\sqrt[3]{2.4} + 1\frac{3}{10}} = 0.583$ (to 3 d.p.)

Exercise 2A

1. **(i)** 10 001, 4 **(ii)** –12, –2017

(iii) $\dfrac{1}{5}$, 4.33, 10 001, 4 **(iv)** $-0.3, -\dfrac{5}{7}, -12, -1\dfrac{1}{2}, -2017$

2. **(a)** 30 m above sea level

(b) –35

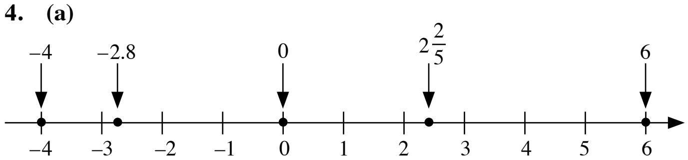
(c) An anticlockwise rotation of 30°

(d) A speed of 45 km/h of a car travelling West

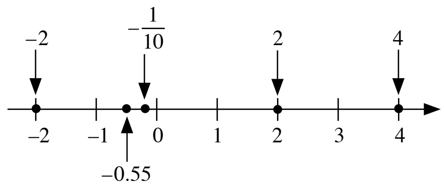
3. **(a)** $<$ **(b)** $<$ **(c)** $<$

(d) $>$ **(e)** $<$ **(f)** $>$

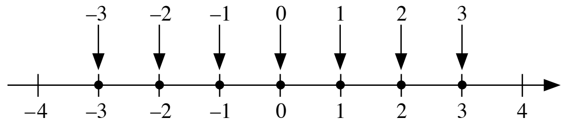
4. **(a)**



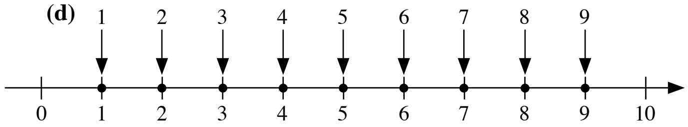
(b)



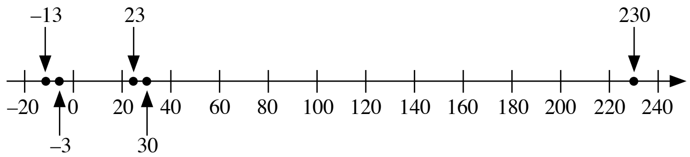
(c)



(d)

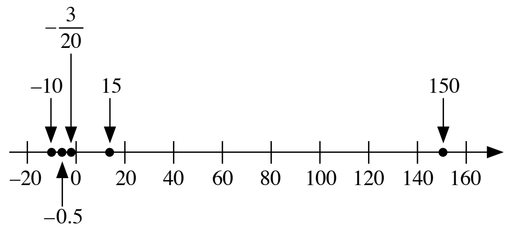


5. **(a)**



$\therefore -13, -3, 23, 30, 230$

(b)



$\therefore -10, -0.5, -\dfrac{3}{20}, 15, 150$

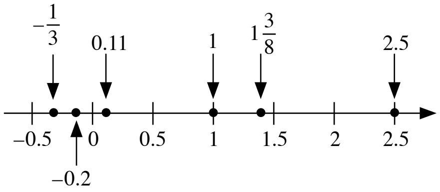
6. **(a)** –273.15 °C

(b) –86 m

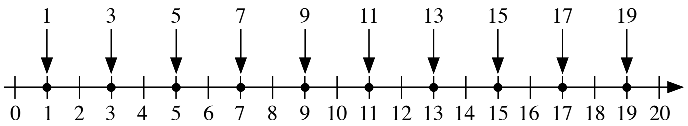
7. **(a)** $>$ **(b)** $>$

(c) $<$ **(d)** $>$

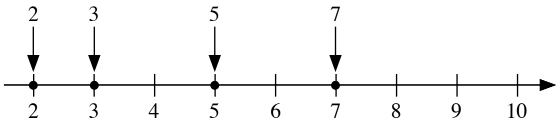
8. **(a)**



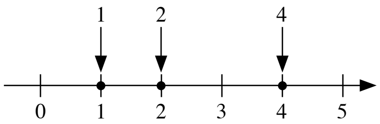
(b)



(c)



(d)



Exercise 2B

1. **(a)** $6 + (-2) = 4$
(b) $-5 + 8 = 3$
(c) $4 + (-10) = -6$
(d) $-1 + (-7) = -8$
(e) $9 + (-3) = 6$
(f) $-11 + (-5) = -16$
(g) $-10 + 2 = -8$
(h) $1 + (-8) = -7$

2. **(a)** $-(-7) = 7$
(b) $5 - (-3) = 5 + 3$
$= 8$
(c) $-4 - 7 = -11$
(d) $-8 - (-2) = -8 + 2$
$= -6$
(e) $-1 - (-10) = -1 + 10$
$= 9$
(f) $6 - 9 = -3$
(g) $-8 - 3 = -11$
(h) $2 - (-7) = 2 + 7$
$= 9$

3. **(a)** $4 + (-7) - (-3) = 4 + (-7) + 3$
$= 0$
(b) $-3 - 5 + (-9) = -17$
(c) $1 - 8 - (-8) = 1 - 8 + 8$
$= 1$
(d) $-2 + (-1) - 6 = -9$
(e) $8 - (-9) + 1 = 8 + 9 + 1$
$= 18$
(f) $-5 + (-3) + (-2) = -10$
(g) $6 + (-5) - (-8) = 6 + (-5) + 8$
$= 9$
(h) $2 - (-7) - 8 = 2 + 7 - 8$
$= 1$

4. **(a)** $23 + (-11) = 12$
(b) $-19 + 12 = -7$
(c) $17 + (-29) = -12$
(d) $-21 + (-25) = -46$
(e) $-13 + 18 = 5$
(f) $-24 + (-13) = -37$
(g) $16 + (-27) = -11$
(h) $-26 + 14 = -12$

5. **(a)** $22 - (-13) = 22 + 13$
$= 35$
(b) $-14 - 16 = -30$
(c) $-19 - (-11) = -19 + 11$
$= -8$
(d) $-18 - (-22) = -18 + 22$
$= 4$
(e) $17 - 23 = -6$
(f) $-20 - 15 = -35$
(g) $12 - (-17) = 12 + 17$
$= 29$
(h) $-21 - 17 = -38$

6. Temperature in the morning = –11 °C + 7 °C
= –4 °C

7. Point A shows –7 °C.
Point B shows 16 °C.
Difference in temperature = 16 °C – (–7 °C)
= 16 °C + 7 °C
= 23 °C

8. Altitude of town = –51 m
Difference in altitude = 138 m – (–51 m)
= 138 m + 51 m
= 189 m

9. **(i)** Difference between –2 and 3 = 3 – (–2)
= 3 + 2
= 5

(ii) The timeline for BC and AD does not have a zero while the number line has a zero.

(iii) There are 4 years between 2 BC and 3 AD.
Note: As there is no zero on the timeline, we cannot use 3 – (–2) to find the difference between 2 BC and 3 AD. In fact, the calculation should be 3 – (–2) – 1, provided one year is in BC and the other year is in AD. If both are in BC, or both are in AD, the calculation is the same as that in **(i)**.

(iv) A real-life example is the floors in a building, i.e. we can consider B1 (Basement 1) as –1 but there is no floor with the number 0.

Exercise 2C

1. **(a)** $3 \times (-9) = -27$
(b) $-8 \times 4 = -32$
(c) $-7 \times (-5) = 35$
(d) $-1 \times (-6) = 6$

(e) $-2(-7) = 14$

(f) $-6 \times 0 = 0$

2. (a) $-21 \div 7 = -3$

(b) $16 \div (-2) = -8$

(c) $-8 \div (-2) = 4$

(d) $\frac{-14}{2} = -7$

(e) $\frac{15}{-5} = -3$

(f) $\frac{-18}{-3} = 6$

3. (a) Square roots of $81 = \pm\sqrt{81}$
$= \pm 9$

(b) Square roots of $16 = \pm\sqrt{16}$
$= \pm 4$

(c) Square roots of $25 = \pm\sqrt{25}$
$= \pm 5$

(d) Square roots of $100 = \pm\sqrt{100}$
$= \pm 10$

4. (a) $\sqrt{81} = 9$

(b) $\sqrt{4} = 2$

(c) $-\sqrt{9} = -3$

(d) Not possible

5. (a) $(-2)^3 = -8$

(b) $(-5)^3 = -125$

(c) $(-10)^3 = -1000$

(d) $(-6)^3 = -216$

6. (a) $\sqrt[3]{27} = 3$

(b) $-\sqrt[3]{64} = -4$

(c) $\sqrt[3]{8} = 2$

(d) $\sqrt[3]{-216} = -6$

7. (a) $-55 + (-10) - 10 = -65 - 10$
$= -75$

(b) $-12 - [(-8) - (-2)] + 3 = -12 - [(-8) + 2] + 3$
$= -12 - (-6) + 3$
$= -12 + 6 + 3$
$= -6 + 3$
$= -3$

(c) $-100 + (-45) + (-5) + 20 = -145 + (-5) + 20$
$= -150 + 20$
$= -130$

(d) $-2 + 3 \times 15 = -2 + 45$
$= 43$

(e) $(-5 - 2) \times (-3) = (-7) \times (-3)$
$= 21$

(f) $-25 \times (-4) \div (-12 + 32) = -25 \times (-4) \div 20$
$= 100 \div 20$
$= 5$

(g) $3 \times (-3)^2 - (7 - 2)^2 = 3 \times (-3)^2 - 5^2$
$= 3 \times 9 - 25$
$= 27 - 25$
$= 2$

(h) $5 \times [3 \times (-2) - 10] = 5 \times (-6 - 10)$
$= 5 \times (-16)$
$= -80$

(i) $-12 \div [2^2 - (-2)] = -12 \div [4 - (-2)]$
$= -12 \div (4 + 2)$
$= -12 \div 6$
$= -2$

(j) $\sqrt{10 - 3 \times (-2)} = \sqrt{10 - (-6)}$
$= \sqrt{10 + 6}$
$= \sqrt{16}$
$= 4$

8. (a) $-55 + (-10) - 10 = -75$

(b) $-12 - [(-8) - (-2)] + 3 = -3$

(c) $-100 + (-45) + (-5) + 20 = -130$

(d) $-2 + 3 \times 15 = 43$

(e) $(-5 - 2) \times (-3) = 21$

(f) $-25 \times (-4) \div (-12 + 32) = 5$

(g) $3 \times (-3)^2 - (7 - 2)^2 = 2$

(h) $5 \times [3 \times (-2) - 10] = -80$

(i) $-12 \div [2^2 - (-2)] = -2$

(j) $\sqrt{10 - 3 \times (-2)} = 4$

9. (a) $24 \times (-2) \times 5 \div (-6) = -48 \times 5 \div (-6)$
$= -240 \div (-6)$
$= 40$

(b) $4 \times 10 - 13 \times (-5) = 40 - (-65)$
$= 40 + 65$
$= 105$

(c) $(16 - 24) - (57 - 77) \div (-2) = (-8) - (-20) \div (-2)$
$= (-8) - 10$
$= -18$

(d) $160 \div (-40) - 20 \div (-5) = -4 - (-4)$
$= -4 + 4$
$= 0$

(e) $[(12 - 18) \div 3 - 5] \times (-4) = (-6 \div 3 - 5) \times (-4)$
$= (-2 - 5) \times (-4)$
$= (-7) \times (-4)$
$= 28$

(f) $\{[(-15 + 5) \times 2 + 8] - 32 \div 8\} - (-7)$
$= \{[(-10) \times 2 + 8] - 32 \div 8\} - (-7)$
$= [(-20 + 8) - 32 \div 8] - (-7)$
$= [(-12) - 32 \div 8] - (-7)$
$= [(-12) - 4] - (-7)$
$= (-16) - (-7)$
$= (-16) + 7$
$= -9$

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(g) $(5-2)^3 \times 2 + [-4 + (-7)] \div (-2+4)^2 = 3^3 \times 2 + (-11) \div 2^2$
$= 27 \times 2 + (-11) \div 4$
$= 54 + \left(-2\frac{3}{4}\right)$
$= 51\frac{1}{4}$

(h) $\{-10 - [12 + (-3)^2] + 3^3\} \div (-3)$
$= [-10 - (12 + 9) + 3^3] \div (-3)$
$= (-10 - 21 + 3^3) \div (-3)$
$= (-10 - 21 + 27) \div (-3)$
$= (-31 + 27) \div (-3)$
$= (-4) \div (-3)$
$= 1\frac{1}{3}$

10. **(a)** $24 \times (-2) \times 5 \div (-6) = 40$
(b) $4 \times 10 - 13 \times (-5) = 105$
(c) $(16 - 24) - (57 - 77) \div (-2) = -18$
(d) $160 \div (-40) - 20 \div (-5) = 0$
(e) $[(12 - 18) \div 3 - 5] \times (-4) = 28$
(f) $\{[(-15 + 5) \times 2 + 8] - 32 \div 8\} - (-7) = -9$
(g) $(5-2)^3 \times 2 + [-4 + (-7)] \div (-2+4)^2 = 51\frac{1}{4}$
(h) $\{-10 - [12 + (-3)^2] + 3^3\} \div (-3) = 1\frac{1}{3}$

11. $\sqrt[3]{-2 \times (-6.5) - [-2 \times (-3) + 8 \times (-2) - 8 \times 2] + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - [6 + (-16) - 16] + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - (-10 - 16) + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - (-26) + 5^2}$
$= \sqrt[3]{-2 \times (-6.5) - (-26) + 25}$
$= \sqrt[3]{13 - (-26) + 25}$
$= \sqrt[3]{13 + 26 + 25}$
$= \sqrt[3]{39 + 25}$
$= \sqrt[3]{64}$
$= 4$

Exercise 2D

1. **(a)** $-\frac{1}{2} + \left(-\frac{3}{4}\right) = -\frac{1}{2} - \frac{3}{4}$
$= -\frac{2}{4} - \frac{3}{4}$
$= \frac{-2-3}{4}$
$= \frac{-5}{4}$
$= -1\frac{1}{4}$

(b) $3\frac{1}{8} + \left(-\frac{1}{4}\right) = 3\frac{1}{8} - \frac{1}{4}$
$= 3\frac{1}{8} - \frac{2}{8}$
$= \left(2 + \frac{8}{8}\right) + \frac{1}{8} - \frac{2}{8}$
$= 2\frac{7}{8}$

(c) $5\frac{1}{5} - 4\frac{1}{2} = 5\frac{2}{10} - 4\frac{5}{10}$
$= \left(4 + \frac{10}{10}\right) + \frac{2}{10} - 4\frac{5}{10}$
$= \frac{7}{10}$

(d) $-3\frac{1}{6} + \left(-4\frac{2}{3}\right) = -3\frac{1}{6} - 4\frac{2}{3}$
$= -3\frac{1}{6} - 4\frac{4}{6}$
$= -7\frac{5}{6}$

2. **(a)** $-\frac{1}{2} + \left(-\frac{3}{4}\right) = -1\frac{1}{4}$
(b) $3\frac{1}{8} + \left(-\frac{1}{4}\right) = 2\frac{7}{8}$
(c) $5\frac{1}{5} - 4\frac{1}{2} = \frac{7}{10}$
(d) $-3\frac{1}{6} + \left(-4\frac{2}{3}\right) = -7\frac{5}{6}$

3. **(a)** $\frac{{}^{5}\cancel{15}}{{}_{2}\cancel{8}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}} = \frac{5}{2}$
$= 2\frac{1}{2}$

(b) $2\frac{3}{5} \times \frac{15}{26} = \frac{{}^{1}\cancel{13}}{{}_{1}\cancel{5}} \times \frac{\cancel{15}^{3}}{\cancel{26}_{2}}$
$= \frac{3}{2}$
$= 1\frac{1}{2}$

(c) $\frac{15}{4} \div \frac{5}{2} = \frac{{}^{3}\cancel{15}}{{}_{2}\cancel{4}} \times \frac{\cancel{2}^{1}}{\cancel{5}_{1}}$
$= \frac{3}{2}$
$= 1\frac{1}{2}$

(d) $1\frac{7}{9} \div \frac{4}{3} = \frac{16}{9} \div \frac{4}{3}$
$= \frac{{}^{4}\cancel{16}}{{}_{3}\cancel{9}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{1}}$
$= \frac{4}{3}$
$= 1\frac{1}{3}$

4. **(a)** $\frac{15}{8} \times \frac{4}{3} = 2\frac{1}{2}$

(b) $2\frac{3}{5} \times \frac{15}{26} = 1\frac{1}{2}$

(c) $\frac{15}{4} \div \frac{5}{2} = 1\frac{1}{2}$

(d) $1\frac{7}{9} \div \frac{4}{3} = 1\frac{1}{3}$

5. **(a)** $\frac{{}^{8}\cancel{64}}{{}_{5}\cancel{15}} \times \left(-\frac{\cancel{3}^{1}}{\cancel{8}_{1}}\right) = -\frac{8}{5}$

$= -1\frac{3}{5}$

(b) $\frac{4}{15} \div \left(-\frac{10}{3}\right) = \frac{{}^{2}\cancel{4}}{{}_{5}\cancel{15}} \times \left(-\frac{\cancel{3}^{1}}{\cancel{10}_{5}}\right)$

$= -\frac{2}{25}$

(c) $-6\frac{1}{8} \times \frac{3}{14} = -\frac{{}^{7}\cancel{49}}{8} \times \frac{3}{\cancel{14}_{2}}$

$= -\frac{21}{16}$

$= -1\frac{5}{16}$

(d) $-2\frac{1}{2} \times 4\frac{2}{5} = -\frac{{}^{1}\cancel{5}}{{}_{1}\cancel{2}} \times \frac{\cancel{22}^{11}}{\cancel{5}_{1}}$

$= -11$

(e) $-1\frac{1}{4} \div \frac{3}{8} = -\frac{5}{4} \div \frac{3}{8}$

$= -\frac{5}{{}_{1}\cancel{4}} \times \frac{\cancel{8}^{2}}{3}$

$= -\frac{10}{3}$

$= -3\frac{1}{3}$

(f) $-\frac{8}{9} \div \left(-1\frac{2}{3}\right) = -\frac{8}{9} \div \left(-\frac{5}{3}\right)$

$= -\frac{8}{{}_{3}\cancel{9}} \times \left(-\frac{\cancel{3}^{1}}{5}\right)$

$= \frac{8}{15}$

6. **(a)** $\frac{64}{15} \times \left(-\frac{3}{8}\right) = -1\frac{3}{5}$

(b) $\frac{4}{15} \div \left(-\frac{10}{3}\right) = -\frac{2}{25}$

(c) $-6\frac{1}{8} \times \frac{3}{14} = -1\frac{5}{16}$

(d) $-2\frac{1}{2} \times 4\frac{2}{5} = -11$

(e) $-1\frac{1}{4} \div \frac{3}{8} = -3\frac{1}{3}$

(f) $-\frac{8}{9} \div \left(-1\frac{2}{3}\right) = \frac{8}{15}$

7. **(a)**

$$\begin{array}{r} 14.72 \\ \times \quad 1.2 \\ \hline 2\,944 \\ +14\,72 \\ \hline 17.664 \\ \hline \end{array}$$

$\therefore 14.72 \times 1.2 = 17.664$

(b)

$$\begin{array}{r} 130.4 \\ \times \quad 0.15 \\ \hline 6\,520 \\ +13\,04 \\ \hline 19.560 \\ \hline \end{array}$$

$\therefore 130.4 \times 0.15 = 19.56$

(c)

$$\begin{array}{r} 0.27 \\ \times \quad 0.08 \\ \hline 0.0216 \\ \hline \end{array}$$

$\therefore 0.27 \times 0.08 = 0.0216$

(d)

$$\begin{array}{r} 0.25 \\ \times \quad 1.96 \\ \hline 150 \\ 225 \\ +\quad 25 \\ \hline 0.4900 \\ \hline \end{array}$$

$\therefore 0.25 \times 1.96 = 0.49$

8. **(a)** $0.81 \div 0.3 = \frac{0.81}{0.3}$

$= \frac{8.1}{3}$

$$\begin{array}{r} 2.7 \\ 3\,\overline{)\,8.1} \\ -6 \\ \hline 2\,1 \\ -2\,1 \\ \hline 0 \end{array}$$

$\therefore 0.81 \div 0.3 = 2.7$

(b) $1.32 \div 0.12 = \frac{1.32}{0.12}$

$= \frac{132}{12}$

$= 11$

(c) $3.426 \div 0.06 = \frac{3.426}{0.06}$

$= \frac{342.6}{6}$

$$\begin{array}{r} 5\,7.1 \\ 6\,\overline{)\,3\,4\,2.6} \\ -3\,0 \\ \hline 4\,2 \\ -4\,2 \\ \hline 6 \\ -\,6 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 3.426 \div 0.06 = 57.1$

(d) $4.35 \div 1.5 = \dfrac{4.35}{1.5}$

$= \dfrac{43.5}{15}$

$$\begin{array}{r} 2.9 \\ 15\overline{)\ 4\,3.5} \\ -\,3\,0 \\ \hline 1\,3\,5 \\ -\,1\,3\,5 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 4.35 \div 1.5 = 2.9$

9. **(a)** $4.3 - (-3.9) = 4.3 + 3.9$

$= 8.2$

(b) $2.8 + (-1.5) = 1.3$

(c) $-5.9 + 2.7 = -3.2$

(d) $-6.7 - 5.4 = -12.1$

10. **(a)** $-\dfrac{8}{5} - \left(-2\dfrac{1}{4}\right) - \dfrac{1}{2} = -\dfrac{8}{5} + 2\dfrac{1}{4} - \dfrac{1}{2}$

$= -\dfrac{8}{5} + \dfrac{9}{4} - \dfrac{1}{2}$

$= -\dfrac{32}{20} + \dfrac{45}{20} - \dfrac{10}{20}$

$= \dfrac{-32 + 45 - 10}{20}$

$= \dfrac{3}{20}$

(b) $6\dfrac{1}{5} - \left(-\dfrac{3}{4}\right) + \left(-4\dfrac{1}{10}\right) = 6\dfrac{1}{5} + \dfrac{3}{4} - 4\dfrac{1}{10}$

$= \dfrac{31}{5} + \dfrac{3}{4} - \dfrac{41}{10}$

$= \dfrac{124}{20} + \dfrac{15}{20} - \dfrac{82}{20}$

$= \dfrac{124 + 15 - 82}{20}$

$= \dfrac{57}{20}$

$= 2\dfrac{17}{20}$

(c) $4\dfrac{2}{7} + \left(-6\dfrac{1}{3}\right) - \left(-\dfrac{4}{21}\right) = 4\dfrac{2}{7} - 6\dfrac{1}{3} + \dfrac{4}{21}$

$= \dfrac{30}{7} - \dfrac{19}{3} + \dfrac{4}{21}$

$= \dfrac{90}{21} - \dfrac{133}{21} + \dfrac{4}{21}$

$= \dfrac{90 - 133 + 4}{21}$

$= \dfrac{-39}{21}$

$= \dfrac{-13}{7}$

$= -1\dfrac{6}{7}$

(d) $-4 + \left(-3\dfrac{1}{8}\right) + \left(-\dfrac{4}{3}\right) = -4 - 3\dfrac{1}{8} - \dfrac{4}{3}$

$= -4 - 3\dfrac{1}{8} - 1\dfrac{1}{3}$

$= -4 - 3\dfrac{3}{24} - 1\dfrac{8}{24}$

$= -8\dfrac{11}{24}$

(e) $-\dfrac{1}{5} + 2\dfrac{1}{4} + \left(-\dfrac{7}{2}\right) = -\dfrac{1}{5} + 2\dfrac{1}{4} - \dfrac{7}{2}$

$= -\dfrac{1}{5} + \dfrac{9}{4} - \dfrac{7}{2}$

$= -\dfrac{4}{20} + \dfrac{45}{20} - \dfrac{70}{20}$

$= \dfrac{-4 + 45 - 70}{20}$

$= \dfrac{-29}{20}$

$= -1\dfrac{9}{20}$

11. **(a)** $-\dfrac{8}{5} - \left(-2\dfrac{1}{4}\right) - \dfrac{1}{2} = \dfrac{3}{20}$

(b) $6\dfrac{1}{5} - \left(-\dfrac{3}{4}\right) + \left(-4\dfrac{1}{10}\right) = 2\dfrac{17}{20}$

(c) $4\dfrac{2}{7} + \left(-6\dfrac{1}{3}\right) - \left(-\dfrac{4}{21}\right) = -1\dfrac{6}{7}$

(d) $-4 + \left(-3\dfrac{1}{8}\right) + \left(-\dfrac{4}{3}\right) = -8\dfrac{11}{24}$

(e) $-\dfrac{1}{5} + 2\dfrac{1}{4} + \left(-\dfrac{7}{2}\right) = -1\dfrac{9}{20}$

12. **(a)** $-\dfrac{5}{7} \times \left(-\dfrac{28}{15} + 1\dfrac{2}{3}\right) = -\dfrac{5}{7} \times \left(-\dfrac{28}{15} + \dfrac{5}{3}\right)$

$= -\dfrac{5}{7} \times \left(-\dfrac{28}{15} + \dfrac{25}{15}\right)$

$= -\dfrac{5}{7} \times \left(-\dfrac{3}{15}\right)$

$= -\dfrac{{}^{1}\cancel{5}}{7} \times \left(-\dfrac{1}{\cancel{5}_{1}}\right)$

$= \dfrac{1}{7}$

(b) $\left[-\dfrac{1}{4} - \left(-\dfrac{1}{3}\right)\right] \div \left(\dfrac{1}{4} - \dfrac{1}{3}\right) = \left(-\dfrac{1}{4} + \dfrac{1}{3}\right) \div \left(\dfrac{1}{4} - \dfrac{1}{3}\right)$

$= \left(-\dfrac{3}{12} + \dfrac{4}{12}\right) \div \left(\dfrac{3}{12} - \dfrac{4}{12}\right)$

$= \dfrac{1}{12} \div \left(-\dfrac{1}{12}\right)$

$= \dfrac{1}{{}_{1}\cancel{12}} \times (-\cancel{12}^{1})$

$= -1$

(c) $10 - \frac{15}{8} \times \left(\frac{3}{2} \div 4\frac{1}{2}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{15}{8} \times \left(\frac{3}{2} \div \frac{9}{2}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{15}{8} \times \left(\frac{{}^{1}\cancel{3}}{{}_{1}\cancel{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{{}^{5}\cancel{15}}{8} \times \frac{1}{\cancel{3}_{1}} + \left(-\frac{1}{4}\right)$

$= 10 - \frac{5}{8} + \left(-\frac{1}{4}\right)$

$= 10 - \frac{5}{8} - \frac{1}{4}$

$= 10 - \frac{5}{8} - \frac{2}{8}$

$= \left(9 + \frac{8}{8}\right) - \frac{5}{8} - \frac{2}{8}$

$= 9\frac{1}{8}$

(d) $\left(\frac{1}{2}\right)^3 - \left(\frac{3}{4}\right)^2 + \left(-\frac{3}{4}\right) = \frac{1}{8} - \frac{9}{16} + \left(-\frac{3}{4}\right)$

$= \frac{1}{8} - \frac{9}{16} - \frac{3}{4}$

$= \frac{2}{16} - \frac{9}{16} - \frac{12}{16}$

$= \frac{2 - 9 - 12}{16}$

$= \frac{-19}{16}$

$= -1\frac{3}{16}$

(e) $\frac{1}{3} + \frac{4}{9} \times \left(-\frac{1}{2}\right)^2 = \frac{1}{3} + \frac{{}^{1}\cancel{4}}{9} \times \frac{1}{\cancel{4}_{1}}$

$= \frac{1}{3} + \frac{1}{9}$

$= \frac{3}{9} + \frac{1}{9}$

$= \frac{4}{9}$

(f) $\left(\frac{3}{2}\right)^2 \times \left(\frac{1}{15} - 2\frac{1}{3}\right) = \left(\frac{3}{2}\right)^2 \times \left(\frac{1}{15} - \frac{7}{3}\right)$

$= \left(\frac{3}{2}\right)^2 \times \left(\frac{1}{15} - \frac{35}{15}\right)$

$= \left(\frac{3}{2}\right)^2 \times \left(-\frac{34}{15}\right)$

$= \frac{{}^{3}\cancel{9}}{{}_{2}\cancel{4}} \times \left(-\frac{\cancel{34}^{17}}{\cancel{15}_{5}}\right)$

$= -\frac{51}{10}$

$= -5\frac{1}{10}$

13. (a) $-\frac{5}{7} \times \left(-\frac{28}{15} + 1\frac{2}{3}\right) = \frac{1}{7}$

(b) $\left[-\frac{1}{4} - \left(-\frac{1}{3}\right)\right] \div \left(\frac{1}{4} - \frac{1}{3}\right) = -1$

(c) $10 - \frac{15}{8} \times \left(\frac{3}{2} \div 4\frac{1}{2}\right) + \left(-\frac{1}{4}\right) = 9\frac{1}{8}$

(d) $\left(\frac{1}{2}\right)^3 - \left(\frac{3}{4}\right)^2 + \left(-\frac{3}{4}\right) = -1\frac{3}{16}$

(e) $\frac{1}{3} + \frac{4}{9} \times \left(-\frac{1}{2}\right)^2 = \frac{4}{9}$

(f) $\left(\frac{3}{2}\right)^2 \times \left(\frac{1}{15} - 2\frac{1}{3}\right) = -5\frac{1}{10}$

14. (a) $\frac{0.15}{0.5} \times \left(\frac{-0.16}{1.2}\right) = \frac{1.5}{5} \times \left(\frac{-0.16}{1.2}\right)$

$= 0.3 \times \left(\frac{-0.16}{1.2}\right)$

$= {}^{1}\cancel{3} \times \left(\frac{-0.16}{\cancel{12}_{4}}\right)$

$= -0.04$

(b) $\frac{0.027}{0.03} \times \left(\frac{1.4}{-0.18}\right) = \frac{2.7}{3} \times \left(\frac{1.4}{-0.18}\right)$

$= 0.9 \times \left(\frac{1.4}{-0.18}\right)$

$= {}^{1}\cancel{9} \times \left(\frac{14}{-\cancel{18}_{2}}\right)$

$= -7$

(c) $-0.4^2 \times \left(\frac{-1.3}{0.8}\right) - 0.62 = -0.4^2 \times \left(\frac{-13}{8}\right) - 0.62$

$= {}^{0.02}\cancel{-0.16} \times \left(\frac{-13}{\cancel{8}_{1}}\right) - 0.62$

$= 0.26 - 0.62$

$= -0.36$

(d) $(-0.2)^3 \times \frac{27}{1.6} + 0.105 = (-0.2)^3 \times \frac{270}{16} + 0.105$

$= {}^{0.001}\cancel{-0.008} \times \frac{\cancel{270}^{135}}{\cancel{16}_{\cancel{8}\,1}} + 0.105$

$= -0.135 + 0.105$

$= -0.03$

15. (a) $\left(\frac{\pi + 5\frac{1}{2}}{-2.1}\right)^2 = 16.934$ (to 3 d.p.)

(b) $-\frac{\pi^2 + \sqrt{2}}{7 - \sqrt[3]{4}} = -2.085$ (to 3 d.p.)

(c) $\frac{\sqrt[3]{14^2 + 19^2}}{\pi - 4.55} = -5.842$ (to 3 d.p.)

(d) $\sqrt{\frac{4.6^2 + 8.3^2 - \left(6\frac{1}{2}\right)^2}{2 \times 4.6 - 8.3}} = 7.288$ (to 3 d.p.)

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16. Amount of time Nora spent on visiting old folks' homes

$= \frac{4}{7} \times 8\frac{1}{16}$

$= \frac{\cancel{4}^{1}}{7} \times \frac{129}{\cancel{16}_{4}}$

$= \frac{129}{28}$

$= 4\frac{17}{28}$ hours

17. $5\frac{3}{4} - 2\frac{5}{6} + \left(-\frac{23}{15}\right) - \left(-4\frac{7}{10}\right) = 5\frac{3}{4} - 2\frac{5}{6} - \frac{23}{15} + 4\frac{7}{10}$

$= \frac{23}{4} - \frac{17}{6} - \frac{23}{15} + \frac{47}{10}$

$= \frac{345}{60} - \frac{170}{60} - \frac{92}{60} + \frac{282}{60}$

$= \frac{345 - 170 - 92 + 282}{60}$

$= \frac{365}{60}$

$= \frac{73}{12}$

$= 6\frac{1}{12}$

18. Fraction of sum of money left after Farhan has taken his share

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

Fraction of sum of money left after Khairul has taken his share

$= \left(1 - \frac{1}{3}\right) \times \frac{4}{5}$

$= \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

Fraction of sum of money left after Huixian has taken her share

$= \left(1 - \frac{1}{4}\right) \times \frac{8}{15}$

$= \frac{{}^{1}\cancel{3}}{{}_{1}\cancel{4}} \times \frac{\cancel{8}^{2}}{\cancel{15}_{5}}$

$= \frac{2}{5}$

Fraction of sum of money taken by Jun Wei $= \left(1 - \frac{1}{7}\right) \times \frac{2}{5}$

$= \frac{6}{7} \times \frac{2}{5}$

$= \frac{12}{35}$

Review Exercise 2

1. **(a)** $-7 - 38 = -45$

$8 + (-55) = -47$

$\therefore -7 - 38 > 8 + (-55)$

(b) $2.36 - 10.58 = -8.22$

$-11.97 - (-2.69) = -11.97 + 2.69$

$= -9.28$

$\therefore 2.36 - 10.58 > -11.97 - (-2.69)$

(c) $-5 \times 1.5 = -7.5$

$50 \div (-8) = -6.25$

$\therefore -5 \times 1.5 < 50 \div (-8)$

(d) $7\frac{1}{5} - \left(-3\frac{3}{10}\right) = 7\frac{1}{5} + 3\frac{3}{10}$

$= 7\frac{2}{10} + 3\frac{3}{10}$

$= 10\frac{5}{10}$

$= 10\frac{1}{2}$

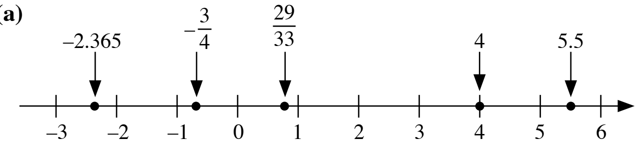
$19\frac{2}{5} + \left(-8\frac{1}{10}\right) = 19\frac{2}{5} - 8\frac{1}{10}$

$= 19\frac{4}{10} - 8\frac{1}{10}$

$= 11\frac{3}{10}$

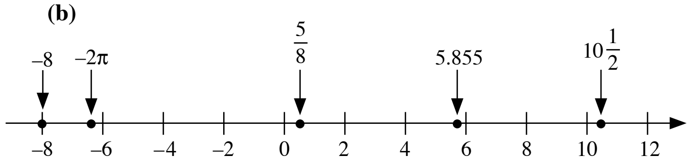
$\therefore 7\frac{1}{5} - \left(-3\frac{3}{10}\right) < 19\frac{2}{5} + \left(-8\frac{1}{10}\right)$

2. **(a)**



$\therefore 5.5, 4, \frac{29}{33}, -\frac{3}{4}, -2.365$

(b)



$\therefore 10\frac{1}{2}, 5.855, \frac{5}{8}, -2\pi, -8$

3. **(a)** $13 - (-54) = 13 + 54$

$= 67$

(b) $(-74) - (-46) = -74 + 46$

$= -28$

(c) $11 + (-33) - (-7) = -22 - (-7)$

$= -22 + 7$

$= -15$

(d) $-13 + (-15) + (-8) = -28 + (-8)$

$= -36$

4. **(a)** $-12 \times 7 = -84$

(b) $4 \times (-5) \times (-6) = -20 \times (-6)$

$= 120$

(c) $-600 \div 15 = -40$

(d) $50 \div (-8) \div (-5) = -\frac{50}{8} \div (-5)$

$= -\frac{25}{4} \div (-5)$

$= -\frac{^{5}\cancel{25}}{4} \times \left(-\frac{1}{\cancel{5}_{1}}\right)$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

5. (a) $(-3-5) \times (-3-4) = (-8) \times (-7)$

$= 56$

(b) $4 \times (-5) \div (-2) = -20 \div (-2)$

$= 10$

(c) $-5 \times 6 - 18 \div (-3) = -30 - (-6)$

$= -30 + 6$

$= -24$

(d) $2 \times (-3)^2 - 3 \times 4 = 2 \times 9 - 3 \times 4$

$= 18 - 12$

$= 6$

(e) $-3 \times (-2) \times (2-5)^2 = -3 \times (-2) \times (-3)^2$

$= -3 \times (-2) \times 9$

$= 6 \times 9$

$= 54$

(f) $(-2)^2 - (-2) \times 3 + 2 \times 3^2 = 4 - (-2) \times 3 + 2 \times 9$

$= 4 - (-6) + 18$

$= 4 + 6 + 18$

$= 10 + 18$

$= 28$

(g) $(-4)^2 \div (-8) + 3 \times (-2)^3 = 16 \div (-8) + 3 \times (-8)$

$= (-2) + (-24)$

$= -26$

(h) $4 \times 3^2 \div (-6) - (-1)^3 \times (-3)^2 = 4 \times 9 \div (-6) - (-1) \times 9$

$= 36 \div (-6) - (-1) \times 9$

$= -6 - (-9)$

$= -6 + 9$

$= 3$

(i) $-2 \times (-2)^3 \times (-2) \times 3 + (-2) \times 3 \times (-1)^2$

$= -2 \times (-8) \times (-2) \times 3 + (-2) \times 3 \times 1$

$= 16 \times (-2) \times 3 + (-6)$

$= -32 \times 3 + (-6)$

$= -96 + (-6)$

$= -102$

(j) $5 - \{12 \times [(-5)^2 - 7] \div 3\} = 5 - [12 \times (25 - 7) \div 3]$

$= 5 - (12 \times 18 \div 3)$

$= 5 - (216 \div 3)$

$= 5 - 72$

$= -67$

6. $\frac{-18 - \left[\sqrt[3]{-3375} - (-6)^2\right]}{\sqrt{4} + 9} = \frac{-18 - \left[\sqrt[3]{-(3^3 \times 5^3)} - (-6)^2\right]}{\sqrt{4} + 9}$

$= \frac{-18 - [-(3 \times 5) - 36]}{2 + 9}$

$= \frac{-18 - (-15 - 36)}{11}$

$= \frac{-18 - (-51)}{11}$

$= \frac{-18 + 51}{11}$

$= \frac{33}{11}$

$= 3$

7. (a) $3\frac{4}{7} + 1\frac{2}{5} - \left(-\frac{3}{7}\right) = 3\frac{4}{7} + 1\frac{2}{5} + \frac{3}{7}$

$= \frac{25}{7} + \frac{7}{5} + \frac{3}{7}$

$= \frac{125}{35} + \frac{49}{35} + \frac{15}{35}$

$= \frac{125 + 49 + 15}{35}$

$= \frac{189}{35}$

$= \frac{27}{5}$

$= 5\frac{2}{5}$

(b) $\frac{2}{3} - \left(-3\frac{3}{20}\right) + \left(-\frac{2}{5}\right) = \frac{2}{3} + 3\frac{3}{20} - \frac{2}{5}$

$= \frac{2}{3} + \frac{63}{20} - \frac{2}{5}$

$= \frac{40}{60} + \frac{189}{60} - \frac{24}{60}$

$= \frac{40 + 189 - 24}{60}$

$= \frac{205}{60}$

$= \frac{41}{12}$

$= 3\frac{5}{12}$

(c) $-6\frac{4}{9} - 3\frac{3}{4} - 3\frac{5}{9} = -6\frac{16}{36} - 3\frac{27}{36} - 3\frac{20}{36}$

$= -12\frac{63}{36}$

$= -12\frac{7}{4}$

$= -13\frac{3}{4}$

(d) $\left(-\frac{1}{2}+\frac{1}{3}\right)+\left[\frac{1}{4}+\left(-\frac{1}{3}\right)\right]+\left(-\frac{1}{20}\right)$

$=-\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{3}-\frac{1}{20}$

$=-\frac{1}{2}+\frac{1}{3}-\frac{1}{3}+\frac{1}{4}-\frac{1}{20}$

$=-\frac{1}{2}+\frac{1}{4}-\frac{1}{20}$

$=-\frac{10}{20}+\frac{5}{20}-\frac{1}{20}$

$=\frac{-10+5-1}{20}$

$=\frac{-6}{20}$

$=-\frac{3}{10}$

(e) $-3\frac{1}{4}\times 1\frac{3}{5}\times\left(-1\frac{2}{13}\right)=-\frac{{}^{1}\cancel{13}}{{}_{1}\cancel{4}}\times\frac{\cancel{8}^{2}}{\cancel{5}_{1}}\times\left(-\frac{\cancel{15}^{3}}{\cancel{13}_{1}}\right)$

$=6$

(f) $\frac{3}{5}\times\left(-\frac{1}{4}-\frac{1}{6}\right)\div\left(-2\frac{1}{3}+1\frac{1}{4}\right)$

$=\frac{3}{5}\times\left(-\frac{3}{12}-\frac{2}{12}\right)\div\left(-2\frac{4}{12}+1\frac{3}{12}\right)$

$=\frac{3}{5}\times\left(-\frac{5}{12}\right)\div\left(-1\frac{1}{12}\right)$

$=\frac{3}{5}\times\left(-\frac{5}{12}\right)\div\left(-\frac{13}{12}\right)$

$=\frac{3}{{}_{1}\cancel{5}}\times\left(-\frac{\cancel{5}^{1}}{\cancel{12}_{1}}\right)\times\left(-\frac{\cancel{12}^{1}}{13}\right)$

$=\frac{3}{13}$

(g) $-3\frac{9}{16}\div 1\frac{3}{16}-\frac{1}{3}\times\left(-1\frac{3}{4}\right)=-\frac{57}{16}\div\frac{19}{16}-\frac{1}{3}\times\left(-\frac{7}{4}\right)$

$=-\frac{{}^{3}\cancel{57}}{{}_{1}\cancel{16}}\times\frac{\cancel{16}^{1}}{\cancel{19}_{1}}-\frac{1}{3}\times\left(-\frac{7}{4}\right)$

$=-3-\left(-\frac{7}{12}\right)$

$=-3+\frac{7}{12}$

$=-2\frac{5}{12}$

(h) $-12\frac{1}{2}+1\frac{2}{3}\div(-4)-\frac{5}{7}\times\left(-2\frac{4}{5}\right)$

$=-12\frac{1}{2}+\frac{5}{3}\div(-4)-\frac{5}{7}\times\left(-\frac{14}{5}\right)$

$=-12\frac{1}{2}+\frac{5}{3}\times\left(-\frac{1}{4}\right)-\frac{{}^{1}\cancel{5}}{{}_{1}\cancel{7}}\times\left(-\frac{\cancel{14}^{2}}{\cancel{5}_{1}}\right)$

$=-12\frac{1}{2}+\left(-\frac{5}{12}\right)-(-2)$

$=-12\frac{1}{2}-\frac{5}{12}+2$

$=-12\frac{6}{12}-\frac{5}{12}+2$

$=-10\frac{11}{12}$

8. $\dfrac{\left(-\frac{4}{7}\right)^2-\left(-\frac{2}{5}\right)^3}{-\sqrt{\frac{64}{625}}\div\sqrt[3]{-\frac{8}{125}}}=\frac{598}{1225}$

9. **(a)** $-12.8-88.2=-101$

(b) $500.3-(-200.2)-210.1=500.3+200.2-210.1$

$=700.5-210.1$

$=490.4$

(c) $1.44\div 1.2\times(-0.4)=\frac{1.44}{1.2}\times(-0.4)$

$=\frac{14.4}{12}\times(-0.4)$

$=1.2\times(-0.4)$

$=-0.48$

(d) $(-0.3)^2\div(-0.2)+(-2.56)=0.09\div(-0.2)+(-2.56)$

$=\frac{0.09}{-0.2}+(-2.56)$

$=\frac{0.9}{-2}+(-2.56)$

$=-0.45+(-2.56)$

$=-0.45-2.56$

$=-3.01$

Challenge Yourself

1. Since $\sqrt{x-3}$ and $(y+2)^2$ cannot be negative,

$\sqrt{x-3}=0$ and $(y+2)^2=0$

$x-3=0$ and $y+2=0$

$\therefore x=3$ and $y=-2$

2. **(a)**

$$\begin{array}{r} 3\,2\,4 \\ \times \quad 5\,7 \\ \hline 2\,2\,6\,8 \\ +\,1\,6\,2\,0 \\ \hline 1\,8\,4\,6\,8 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} 4\,8 \\ 28\overline{)\,1\,3\,4\,4} \\ -\,1\,1\,2 \\ \hline 2\,2\,4 \\ -\,2\,2\,4 \\ \hline 0 \\ \hline \end{array}$$

3. **(b)** $(3+3)\div 3+3-3=2$

(c) $3+3-3-3+3=3$

(d) $(3+3+3+3)\div 3=4$

(e) $3+3\div 3+3\div 3=5$

(f) $3+3+(3-3)\times 3=6$

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Chapter 3 Approximation and Estimation

TEACHING NOTES

Suggested Approach

Teachers can give students a real-life example when an approximated or estimated value is used before getting them to discuss occasions when they use approximation and estimation in their daily lives. In this chapter, they will first learn the five rules to identify the digits which are significant in a number before learning how to round off numbers to a specified number of significant figures. Students will also learn how to carry out estimation through worked examples that involve situations in real-world contexts.

Section 3.1: Approximation

To make learning of mathematics relevant, students should know some reasons why they need to use approximations in their daily lives (see Class Discussion: Actual and Approximated Values).

Teachers should do a recap with students on what they have learnt in primary school, i.e. how to round off numbers to the nearest tenth, whole number and 10 etc.

Section 3.2: Significant Figures

Through the example on measuring cylinders on page 63 of the textbook, students will learn that a number is more accurate when it is given to a greater number of significant figures.

After learning how to round off numbers to a specified number of significant figures, teachers can arouse students' interest in this topic by bringing in real-life situations where they cannot just round off a number using the rules they have learnt (see Investigation: Rounding in Real Life). The journal writing on page 67 of the textbook requires students to cite examples of such situations.

Section 3.3: Rounding and Truncation Errors

Teachers should tell students that the general instructions for O-level Mathematics examinations state, 'If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.' The investigation on page 68 of the textbook highlights the importance of giving intermediate values correct to four significant figures if we want the final answer to be accurate to three significant figures. Otherwise, a rounding error may occur.

Students should also learn that there is a difference between 'approximately 2.5 million' and 'equal to 2.5 million (to 2 s.f.)' (see the thinking time on page 69 of the textbook).

Teachers should tell students the difference between rounding off a number to, say, 3 significant figures and truncating the same number to 3 significant figures. The investigation on page 70 of the textbook enables students to find out more about rounding and truncation errors in calculators.

Section 3.4: Estimation

Teachers can impress upon students that there are differences between approximation and estimation. Since students need to be aware when an answer is obviously wrong, estimation allows them to check the reasonableness of an answer obtained from a calculator (see Worked Example 6).

Students will also learn an important estimation strategy: use a smaller quantity to estimate a larger quantity (see Investigation: Use of a Smaller Quantity to Estimate a Larger Quantity).

Teachers should get students to work in groups to estimate quantities in a variety of contexts, compare their estimates and share their estimation strategies with one another. (see the performance task on page 76 of the textbook).

WORKED SOLUTIONS

Class Discussion (Actual and Approximated Values)

1. The actual values indicated in the article include '42 038 777 passengers', '13.0%', '24 awards' and 'four terminals' while approximated values include 'over 360 awards' and '73 million passengers'. Actual values are exact numbers while approximated values are values which are usually rounded off.
2. **(a)** It is not necessary to specify the actual number of awards won, as an approximation is sufficient to show that Changi Airport has won many awards.
   (b) A headline serves as a brief summary of the article to draw readers' attentions, thus it is more appropriate to use an approximated value instead of the actual value.

Investigation (Rounding in Real Life)

Scenario 1

Total number of passengers = 215 + 5
= 220

Number of buses required = 220 ÷ 30

$= 7\frac{1}{3}$

The nearest whole number to $7\frac{1}{3}$ is 7. However, 7 buses are not enough to carry 220 passengers, thus we round up to find the number of buses required to carry all the passengers.
∴ The number of buses required is 8.

Scenario 2

Maximum mass of lift = 897 kg
= 900 kg (to the nearest 100 kg)

If the maximum mass of the lift is given as 900 kg, it means that the lift is able to carry a mass of ⩽ 900 kg. However, the maximum mass allowed is only 897 kg.
∴ The maximum mass of the lift should be given as 800 kg.

Scenario 3

In Singapore, the issue of 1-cent coins has ceased since 2002; while the coins are legal tender and are still in circulation, most shops have stopped accepting 1-cent coins. As such, when people wish to pay for their purchases in cash, the prices of their purchases have to be rounded off to the nearest 5 cents which is now considered to be the smallest denomination of currency in Singapore.

Teachers may wish to ask students to explain why when other methods of payment are used, it is not necessary to round off the prices of their purchases to the nearest 5 cents.

Journal Writing (Page 67)

- A developer wants to build a house on a plot of land that has a height restriction of 10 m. The height from the floor to the ceiling of each level is about 2.6 m.
  Number of levels the developer can build = 10 m ÷ 2.6 m
  = 3.85 (to 3 s.f.)
  The nearest whole number to 3.85 is 4. However, a house with 4 levels will be taller than 10 m, and thus will go against the height restrictions. Hence, the maximum number of levels that the developer can build is 3.
- The boiling point of oxygen, i.e. the temperature at which liquid oxygen boils to form gaseous oxygen, is –183 °C. The maximum temperature, correct to the nearest 10 °C, at which liquid oxygen can be stored is –190 °C as oxygen will be in its gaseous state at a temperature of –180 °C.

Teachers may wish to note that the list is not exhaustive.

Investigation (The Missing 0.1% Votes)

1. The percentage of votes for each candidate given is correct to 3 significant figures. Due to rounding errors in the intermediate steps, there is a follow-through error, resulting in the missing 0.1% of the votes. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.
2. Percentage of votes for Vishal $= \frac{188}{301} \times 100\%$
   = 62.5% (to 3 s.f.)

   Percentage of votes for Rui Feng $= \frac{52}{301} \times 100\%$
   = 17.3% (to 3 s.f.)

   Percentage of votes for Huixian $= \frac{61}{301} \times 100\%$
   = 20.3% (to 3 s.f.)

   Total percentage of votes = 62.5% + 17.3% + 20.3%
   = 100.1%

   The percentage of votes for each candidate given is correct to 3 significant figures. Due to rounding errors in the intermediate steps, which results in a follow through error, the total percentage of votes is 100.1%. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.

Thinking Time (Page 69)

1. **(i)** When the population of City *A* is approximately 2.5 million, it is possible for the exact population size to be 2.47 million.
   (ii) When the population of City *A* is approximately 2.5 million, it is possible for the exact population size to be 2.6 million.

2. **(i)** When the population of City B is equal to 2.5 million (to 2 s.f.), it is possible for the exact population size to be 2.47 million as it is equal to 2.5 million when rounded off to 2 significant figures.
(ii) When the population of City B is equal to 2.5 million (to 2 s.f.), it is not possible for the exact population size to be 2.6 million as it is still equal to 2.6 million when rounded off to 2 significant figures.

Note: There is a difference between 'approximately 2.5 million' and 'equal to 2.5 million (to 2 s.f.)'.

Investigation (Rounding and Truncation Errors in Calculators)

For this activity, the calculator model used is SHARP EL-509VM.

(a) **1.** 0.727 922 061
2. 7.27 922 061 3
3. 2.7 922 061 3
The calculator stores 12 digits.
The calculator truncates the value of $\sqrt{162}$ at the 12th digit to give 12.727 922 061 3, instead of rounding $\sqrt{162}$ to 12.727 922 061 4.

(b) **5.** 6.999 999 999

Investigation (Use of a Smaller Quantity to Estimate a Larger Quantity)

For this investigation, the smaller box used is of length 9.2 cm, width 5.6 cm and height 2.7 cm.

Three trials are carried out to find the average number of 10¢ coins that can fill the box. The result of each trial is shown in the table.

Trial	Number of 10¢ coins
1	294
2	280
3	284

Average number of 10¢ coins that can fill the smaller box

$= \dfrac{294 + 280 + 284}{3}$

$= \dfrac{858}{3}$

$= 286$

Volume of smaller box $= 9.2 \times 5.6 \times 2.7$
$= 139.104 \text{ cm}^3$

Volume of tank $= 50 \times 23 \times 13$
$= 14\,950 \text{ cm}^3$

Number of 10¢ coins that can fill the tank $= \dfrac{286}{139.104} \times 14\,950$
$= 30\,737$ (to the nearest whole number)

$\therefore$ Amount of money in the tank $= 30\,737 \times 10¢$
$= \$3073.70$

Performance Task (Estimation in Our Daily Lives)

1. Use surveys, questionnaires or verbal questioning to find out the number of hours spent surfing the Internet by each student in the class on *a weekday* and on *a Saturday or Sunday*. Ensure that students have a common understanding of the phrase 'surfing the Internet'.

Calculate the total number of hours spent surfing the Internet by *all* the students in the class on *a weekday* and on *a Saturday or Sunday*.

Total amount of time spent surfing the Internet by all the students in the class on a weekday = x hours

Total amount of time spent surfing the Internet by all the students in the class on a Saturday or Sunday = y hours

Estimate the total number of hours spent surfing the Internet by *all* the students in the class in *a month*. Assume that the average number of weekdays and the average number of Saturdays and Sundays in a month are 22 and 8 respectively.

Total amount of time spent surfing the Internet by all the students in the class in a month $\approx (22x + 8y)$ hours

2. Assume that there are 8 slices in a large pizza. Use verbal questioning to find out the number of slices needed to feed *one class* (e.g. about 40 students) in the school when they go for an excursion.

Number of slices needed to feed one class in the school = x

Number of pizzas needed to feed one class in the school = $\dfrac{x}{8}$

Find out the number of classes in the school. Ensure that there is approximately the same number of students in each class, e.g. 40 students.

Number of classes in the school = y

Estimate the amount of pizza needed to feed all the students in the school during an excursion.

Total number of pizzas needed to feed all the students in the school $\approx \dfrac{xy}{8}$

3. Find out the opening hours of the drinks stall on a weekday and determine the durations of the peak (e.g. recess and lunchtime) and non-peak periods respectively.

Duration of peak period = x hours

Duration of non-peak period = y hours

Find out the amount of money collected by the drinks stall in *half an hour* during the peak period and *half an hour* during the non-peak period.

Amount of money collected by drinks stall in half an hour during peak period = \$$p$

Amount of money collected by drinks stall in half an hour during non-peak period = \$$q$

Estimate the total amount of money collected for both the peak and non-peak periods.

Total amount of money collected by drinks stall during peak period $\approx \$2px$

Total amount of money collected by drinks stall during non-peak period $\approx \$2qy$

Hence,

Total amount of money collected by drinks stall on a weekday $\approx \$(2px + 2qy)$

Practise Now 1

1. (a) 3 409 725 = 3 409 730 (to the nearest 10)
   (b) 3 409 725 = 3 409 700 (to the nearest 100)
   (c) 3 409 725 = 3 410 000 (to the nearest 1000)
   (d) 3 409 725 = 3 410 000 (to the nearest 10 000)
2. Largest possible number of overseas visitors = 11 649 999
   Smallest possible number of overseas visitors = 11 550 000

Practise Now 2

1. (a) 78.4695 = 78.5 (to 1 d.p.)
   (b) 78.4695 = 78 (to the nearest whole number)
   (c) 78.4695 = 78.47 (to the nearest hundredth)
   (d) 78.4695 = 78.470 (to the nearest 0.001)
2. No, I do not agree with Jun Wei. 8.40 is rounded off to 2 decimal places which is more accurate than 8.4 which is rounded off to 1 decimal place.

Practise Now 3

Cost of 450 kWh of electricity $= 450 \times \$0.29$
$= \$130.50$

Cost of 38 m^3 of water $= 38 \times \$1.17$
$= \$44.46$

Total amount of money the household has to pay $= \$130.50 + \$44.46$
$= \$174.96$
$= \$175$ (to the nearest dollar)

Practise Now (Page 64)

(a) The number 192 has 3 significant figures.
(b) The number 83.76 has 4 significant figures.
(c) The number 3 has 1 significant figure.
(d) The number 4.5 has 2 significant figures.

Practise Now (Page 64)

(a) The number 506 has 3 significant figures.
(b) The number 1.099 has 4 significant figures.
(c) The number 3.0021 has 5 significant figures.
(d) The number 70.8001 has 6 significant figures.

Practise Now (Page 64)

1. (a) The number 0.10 has 2 significant figures.
   (b) The number 0.500 has 3 significant figures.
   (c) The number 41.0320 has 6 significant figures.
   (d) The number 6.090 has 4 significant figures.
2. 4.10 cm is more accurate because 4.10 cm is measured to 3 significant figures, while 4.1 cm is measured to 2 significant figures.

Practise Now (Page 65)

(a) The number 0.021 has 2 significant figures.
(b) The number 0.603 has 3 significant figures.
(c) The number 0.001 73 has 3 significant figures.
(d) The number 0.1090 has 4 significant figures.

Practise Now (Page 65)

(a) 3800 m, which is corrected to the nearest 10 m, has 3 significant figures.
(b) 25 000 km, which is corrected to the nearest km, has 5 significant figures.
(c) 100 000 g, which is corrected to the nearest 10 000 g, has 2 significant figures.

Practise Now 4

1. (a) 3748 = 3750 (to 3 s.f.)
   (b) 0.004 709 89 = 0.004 710 (to 4 s.f.)
   (c) 4971 = 5000 (to 2 s.f.)
   (d) 0.099 99 = 0.10 (to 2 s.f.)
   0.099 99 = 0.100 (to 3 s.f.)
2. Since 67 0X1 (to 3 s.f.), then the possible values of X are 5, 6, 7, 8 or 9.
   If 67 0X1 is a perfect square, then by trial and error, $X = 8$.

Practise Now 5

(i) Length of square $= \sqrt{105}$
$= 10.2$ m (to 3 s.f.)
(ii) Perimeter of square $= 10.25 \times 4$
$= 41.0$ m (to 3 s.f.)

Practise Now 6

1. $798 \times 195 \approx 800 \times 200$
   $= 160\ 000$
   $\therefore$ Nora's answer is not reasonable.
2. (a) $5712 \div 297 \approx 5700 \div 300$
   $= 19$
   Using a calculator, $5712 \div 297 = 19.2$ (to 3 s.f.).
   $\therefore$ The estimated value is close to the actual value.

(b) $\sqrt{63} \times \sqrt[3]{129} \approx \sqrt{64} \times \sqrt[3]{125}$
$= 8 \times 5$
$= 40$

Using a calculator, $\sqrt{63} \times \sqrt[3]{129} = 40.1$ (to 3 s.f.).
$\therefore$ The estimated value is close to the actual value.

3. Time taken to drive from Singapore to Malacca $= \frac{250}{80}$
$\approx \frac{240}{80}$ hours

Practise Now 7

Rp 10 000 ≈ S\$1.50, so Rp 20 000 ≈ S\$3, Rp 5000 ≈ S\$0.75
$\therefore$ The price of the pair of earrings is Rp 25 000 ≈ S\$3.75.

Practise Now 8

For option A, 300 ml costs about \$9.
Thus 100 ml will cost about \$3, and 50 ml will cost about \$1.50.
$\therefore$ For option A, 350 ml will cost about \$9 + \$1.50 = \$10.50.
For option B, 350 ml costs \$10.40 which is \$0.10 cheaper than option A.
However, for option A, 300 ml actually costs \$8.80 which is less than \$9.
Thus for option A, 350 ml will cost at least \$0.20 less than the estimated \$10.50.
$\therefore$ Option A is better value for money.

Practise Now 9

Percentage of shaded region $= \frac{2}{3} \times 100\%$
$= 66\frac{2}{3}\%$

Exercise 3A

1. (a) 698 352 = 698 400 (to the nearest 100)
   (b) 698 352 = 698 000 (to the nearest 1000)
   (c) 698 352 = 700 000 (to the nearest 10 000)
2. (a) 45.7395 = 45.7 (to 1 d.p.)
   (b) 45.7395 = 46 (to the nearest whole number)
   (c) 45.7395 = 45.740 (to 3 d.p.)
3. (i) Perimeter of land = 2(28.3 + 53.7)
   = 2(82)
   = 160 m (to the nearest 10 m)
   (ii) Area of grass needed to fill up the entire plot of land
   $= 28.3 \times 53.7$
   $= 1519.71$ m^2
   $= 1500$ m^2 (to the nearest 100 m^2)
4. (a) 4.918 m = 4.9 m (to the nearest 0.1 m)
   (b) 9.71 cm = 10 cm (to the nearest cm)
   (c) \$10.982 = \$11.00 (to the nearest ten cents)
   (d) 6.489 kg = 6.49 kg (to the nearest $\frac{1}{100}$ kg)
5. No, I do not agree with Kate. She needs to put a '0' in the ones place as a place holder after dropping the digit '2', i.e. 5192.3 = 5190 (to the nearest 10).
6. Largest possible value of Singapore's population = 5 077 499
   Smallest possible value of Singapore's population = 5 076 500
7. No, I do not agree with Farhan. 27.0 is rounded off to 1 decimal place which is more accurate than 27 which is rounded off to the nearest whole number.

Exercise 3B

1. (a) The number 39 018 has 5 significant figures.
   (b) The number 0.028 030 has 5 significant figures.
   (c) 2900, which is corrected to the nearest 10, has 3 significant figures.
2. (a) 728 = 730 (to 2 s.f.)
   (b) 503.88 = 503.9 (to 4 s.f.)
   (c) 0.003 018 5 = 0.003 019 (to 4 s.f.)
   (d) 6396 = 6400 (to 2 s.f.)
   6396 = 6400 (to 3 s.f.)
   (e) 9.9999 = 10.0 (to 3 s.f.)
   (f) 8.076 = 8.08 (to 3 s.f.)
3. Possible values of x = 4, 5 or 6
4. (a) $\frac{1}{99} = 0.010\ 10$ (to 4 s.f.)
   (b) $871 \times 234 = 203\ 814$
   $= 200\ 000$ (to 2 s.f.)
   (c) $\frac{21^2}{0.219} = 2013.698\ 63$
   $= 2013.7$ (to 5 s.f.)
   (d) $\frac{3.91^3 - 2.1}{6.41} = 9.0$ (to 2 s.f.)
5. Greatest number of sweets that can be bought
   $= \frac{\$2}{\$0.30}$
   = 6 (to the nearest whole number)
6. (i) Length of square $= \sqrt{264}$
   $= 16.2$ cm (to 3 s.f.)
   (ii) Perimeter of square $= 16.25 \times 4$
   $= 65.0$ cm (to 3 s.f.)
7. (i) Radius of circle $= \frac{136}{2\pi}$
   $= 21.6$ m (to 3 s.f.)
   (ii) Area of circle $= \pi(21.65)^2$
   $= 1470$ m^2 (to 3 s.f.)
8. Since 21 X09 = 22 000 (to 2 s.f.), then the possible values of X are 5, 6, 7, 8 or 9.
   If 21 X09 is a perfect square, then by trial and error, X = 6.
9. Largest possible number of people at the concert = 21 249
   Smallest possible number of people at the concert = 21 150
10. (i) 987 654 321 + 0.000 007 – 987 654 321 = 0.000 007
    (ii) 987 654 321 + 0.000 007 – 987 654 321 = 0

(iii) No, the answers for **(i)** and **(ii)** are different. This is because the calculator truncates the value of 987 654 321 + 0.000 007 to give 987 654 321. Hence, the answer for **(ii)** is 0.

Exercise 3C

1. $218 \div 31 \approx 210 \div 30$
$= 7$
$\therefore$ Priya's answer is not reasonable.
Using a calculator, $218 \div 31 = 7.03$ (to 3 s.f.).
$\therefore$ The estimated value is close to the actual value.

2. (a) $2013 \times 39 \approx 2000 \times 40$
$= 80\ 000$
Using a calculator, $2013 \times 39 = 78\ 507$.
$\therefore$ The estimated value is close to the actual value.

(b) $\sqrt{145.6} \div \sqrt[3]{65.4} \approx \sqrt{144} \div \sqrt[3]{64}$
$= 12 \div 4$
$= 3$
Using a calculator, $\sqrt{145.6} \div \sqrt[3]{65.4} = 2.99$ (to 3 s.f.).
$\therefore$ The estimated value is close to the actual value.

3. (i) $3.612 = 3.6$ (to 2 s.f.)
$29.87 = 30$ (to 2 s.f.)
(ii) $3.612 \div 29.87 \approx 3.6 \div 30$
$= 0.12$ (to 2 s.f.)

4. Amount of petrol used $= \dfrac{274}{9.1}$
$\approx \dfrac{270}{9}\ l$

5. Ratio of area of shaded region to that of unshaded region = 1 : 2

6. Total amount of money that the shopkeeper has to pay
$= 32 \times \$18 + 18 \times \$8 + 47 \times \$26 + 63 \times \$23 + 52 \times \$9$
$\approx 30 \times \$20 + 20 \times \$10 + 50 \times \$30 + 60 \times \$20 + 50 \times \$10$
$= \$600 + \$200 + \$1500 + \$1200 + \$500$
$= \$4000$ (to the nearest hundred dollars)

7. RM10 ≈ S\$4, so RM20 ≈ S\$8, RM5 ≈ S\$2.
$\therefore$ The price of the bag is RM25 ≈ S\$10.

8. For option *A*, 300 g costs about \$6.
Thus 100 g will cost about \$2.
$\therefore$ For option *A*, 500 g will cost about $5 \times \$2 = \$10$.
For option *B*, 500 g costs \$9.90 which is \$0.10 cheaper than option *A*.
However, for option *A*, 300 g actually costs \$5.80 which is \$0.20 less than \$6.
Thus for option *A*, 500 g will cost at least \$0.20 less than the estimated \$10.
$\therefore$ Option *A* is better value for money.

9. Price of dress in Shop *A* after a 20% discount $= \dfrac{80}{100} \times \$79.50$
$\approx \dfrac{80}{100} \times \$80$
Price of dress in Shop *B* after a 10% discount $= \dfrac{90}{100} \times \$69.50$
$\approx \dfrac{90}{100} \times \$70$

10. KRW 900 ≈ S\$1
$\therefore$ Price of handbag = KRW 26 700
≈ KRW 27 000
= 30 × KRW 900
≈ 30 × S\$1
= S\$30

Review Exercise 3

1. (a) $6479.952 = 6500$ (to the nearest 100)
(b) $6479.952 = 6000$ (to the nearest 1000)
(c) $6479.952 = 6480.0$ (to the nearest tenth)

2. (i) $4.793 = 4.8$ (to 2 s.f.)
$39.51 = 40$ (to 2 s.f.)
(ii) $4.793 \div 39.51 \approx 4.8 \div 40$
$= 0.12$ (to 2 s.f.)

3. Smallest possible mass of chocolate truffle = 0.0245 kg

4. Rp 10 000 ≈ S\$1.50, so Rp 30 000 ≈ S\$4.50, Rp 5000 ≈ S\$0.75.
$\therefore$ The price of the toy is Rp 35 000 ≈ S\$5.25.

5. Total mass $= 3 \times 109 + 2 \times 148 + 5 \times 84$
$\approx (3 \times 110 + 2 \times 150 + 5 \times 80)$ g

6. Number of batteries required $= \dfrac{28.2}{4.03}$
$\approx \dfrac{28}{4}$

7. Price of hard disk in Store *A* after a 20% discount $= \dfrac{80}{100} \times \$85.05$
$\approx \dfrac{80}{100} \times \$85$
Price of hard disk in Store *B* after a 10% discount $= \dfrac{90}{100} \times \$76.05$
$\approx \dfrac{90}{100} \times \$76$

8. For option *A*, 250 m*l* costs about \$15.
Thus 50 m*l* will cost about \$3, and 100 m*l* will cost about \$6.
$\therefore$ For option *A*, 300 m*l* will cost about $3 \times \$6 = \$18$.
Furthermore, for option *A*, 250 m*l* actually costs \$15.20 which is \$0.20 more than \$15.
Thus for option *A*, 300 m*l* will cost at least \$0.20 more than the estimated \$18.
$\therefore$ Option *B* is better value for money.

Challenge Yourself

1. 987×123 is more than 988×122 because 987×123 $= 987 \times (122 + 1)$, i.e. there is an additional 987×1; but $988 \times 122 = (987 + 1) \times 122$, i.e. there is only an additional 1×122. In fact, $987 \times 123 - 988 \times 122 = 987 - 122 = 865$.

2. This question tests students' sense of mass. The mass of an ordinary car is likely to be 2000 kg.
Teachers may wish to get students to give examples of objects with masses of 20 kg, 200 kg and 20 000 kg, e.g. 2 10-kg bags of rice have a total mass of 20 kg, 5 Secondary 1 students have a total mass of about 200 kg and a rocket has a mass of about 20 000 kg.

Chapter 4 Basic Algebra and Algebraic Manipulation

TEACHING NOTES

Suggested Approach

Some students are still unfamiliar with algebra even though they have learnt some basic algebra in primary school. Thus for the lower ability students, teachers should teach this chapter as though they do not know algebra at all. The learning experiences in the new syllabus specify the use of algebra discs. In addition to the algebra discs showing the numbers 1 and –1 which students have encountered in Chapter 2, algebra discs showing x, $-x$, y and $-y$ are needed. Since many Secondary 1 students are still in the concrete operational stage (according to Piaget), the use of algebra discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use algebra discs in examinations, and partly because they cannot use algebra discs to manipulate algebraic expressions which consist of algebraic terms that have large or fractional coefficients (see Section 4.1, 4.2 and 4.3).

Section 4.1: Fundamental Algebra

Teachers should teach students how to use letters to represent numbers and interpret basic algebraic notations such as $ab = a \times b$. Teachers should illustrate the definitions of mathematical terms such as 'algebraic term', 'coefficient', 'algebraic expression' and 'linear expression' using appropriate examples.

In the class discussion on page 83 of the textbook, students are required to use algebraic expressions to express mathematical relationships.

To make learning more interactive, students are given the opportunity to use a spreadsheet to explore the concept of variables (see Investigation: Comparison between Pairs of Expressions). Through this investigation, students should be able to observe that evaluating an algebraic expression means finding the value of the expression when the variables take on certain values. This investigation also provides students with an intuitive sense of the difference between pairs of expressions such as $2n$ and $2 + n$, n^2 and $2n$, and $2n^2$ and $(2n)^2$. Students are expected to give a more rigorous mathematical explanation for the difference between such a pair of expressions in the journal writing on page 85 of the textbook.

Algebra discs cannot be used to add or subtract algebraic terms with large coefficients, so there is a need to help students consolidate what they have learnt in Worked Example 2. For the lower ability students, before going through Worked Example 2(d) and (e), teachers should revisit the procedure for simplifying ordinary numerical fractions, e.g. $\frac{1}{2} + \frac{1}{3}$.

Section 4.2: Expansion and Simplification of Linear Expressions

The idea of flipping over a disc to obtain the negative of a number or variable, e.g. $-(-x) = x$, is needed to teach students how to obtain the negative of a linear expression. Algebra discs cannot be used to manipulate algebraic expressions which consist of algebraic terms that have large coefficients, so there is a need to help students consolidate what they have learnt in the class discussion on page 94 of the textbook by moving away from the 'concrete' to the following 'abstract' concept:

Distributive Law: $a(b + c) = ab + ac$

Teachers should emphasise the importance of the rules by which operations are performed when an algebraic expression involves brackets by using the thinking time on page 96 of the textbook.

Section 4.3: Simplification of Linear Expressions with Fractional Coefficients

After going through Worked Example 5 and 6, students should observe that the procedure for simplifying linear expressions with fractional coefficients is similar to that of simplifying ordinary numerical fractions.

Section 4.4: Factorisation

Students should learn how to appreciate the factorisation process, i.e. it is the reverse of expansion. Teachers should tell students the difference between 'complete' and 'incomplete' factorisation. In Secondary 1, students only need to know how to factorise algebraic expressions by extracting the common factors.

The class discussion on page 101 of the textbook requires students to work in pairs to select and justify pairs of equivalent expressions. Teachers should make use ofthis opportunity to highlight some common errors made by students when manipulating algebraic expressions.

WORKED SOLUTIONS

Class Discussion (Expressing Mathematical Relationships using Algebra)

1.

	In words	Algebraic expression
(a)	Sum of $2x$ and $3z$	$2x + 3z$
(b)	Product of x and $7y$	$7xy$
(c)	Divide $3ab$ by $2c$	$\frac{3ab}{2c}$
(d)	Subtract $6q$ from $10z$	$10z - 6q$
(e)	Subtract the product of x and y from the sum of p and q	$(p + q) - xy$
(f)	Divide the sum of 3 and y by 5	$\frac{3 + y}{5}$
(g)	Subtract the product of 2 and c from the positive square root of b	$\sqrt{b} - 2c$
(h)	There are three times as many girls as boys in a school. Find an expression, in terms of x, for the total number of students in the school, where x represents the number of boys in the school.	It is given that x represents the number of boys. $\therefore$ $3x$ represents the number of girls. Total number of students $= x + 3x$ $= 4x$
(i)	The age of Nora's father is thrice hers. The age of Nora's brother is 5 years more than hers. Find an expression, in terms of y, for the sum of their ages, where y represents Nora's age.	It is given that y represents Nora's age. $\therefore$ Nora's father is $3y$ years old. Nora's brother is $(y + 5)$ years old. Sum of their ages $= y + 3y + y + 5$ $= (5y + 5)$ years
(j)	The length is three times as long as the breadth of the rectangle. Find an expression, in terms of b, for the perimeter and the area of the rectangle, where b represents the breadth of the rectangle.	It is given that b represents the breadth of the rectangle in m. $3b$ represents the length of the rectangle in m. Perimeter of rectangle $= 2(3b + b)$ $= 2(4b)$ $= 8b$ m Area of rectangle $= 3b \times b$ $= 3b^2$ m^2

Table 4.3

Investigation (Comparison between Pairs of Expressions)

2.

	A	B	C	D	E	F
1						
2	n	$2n$	$2 + n$	n^2	$2n^2$	$(2n)^2$
3	1	2				
4	2	4				
5	3	6				
6	4	8				
7	5	10				

(i) The value of $2n$ changes as n changes.

(ii) We multiply the given value of n by 2 to obtain the corresponding value of $2n$.

(iii) When $n = 8$,
$2n = 2 \times 8$
$= 16$
When $n = 9$,
$2n = 2 \times 9$
$= 18$
When $n = 10$,
$2n = 2 \times 10$
$= 20$

3.

	A	B	C	D	E	F
1						
2	n	$2n$	$2 + n$	n^2	$2n^2$	$(2n)^2$
3	1	2	3	1	2	4
4	2	4	4	4	8	16
5	3	6	5	9	18	36
6	4	8	6	16	32	64
7	5	10	7	25	50	100

4.
- $2n$ and $2 + n$
  Referring to columns B and C on the spreadsheet, the expressions $2n$ and $2 + n$ are equal only when $n = 2$. When $n < 2$, $2n < 2 + n$. When $n > 2$, $2n > 2 + n$.
- n^2 and $2n$
  Referring to columns B and D on the spreadsheet, the expressions n^2 and $2n$ are equal when $n = 2$. By observation, they are also equal when $n = 0$. When $n < 0$ or $n > 2$, $n^2 > 2n$. When $0 < x < 2$, $n^2 < 2n$.
- $2n^2$ and $(2n)^2$
  By observation, the expressions $2n^2$ and $(2n)^2$ are equal when $n = 0$. For any value of $n \neq 0$, $(2n)^2 > 2n^2$.

Journal Writing (Page 85)

By observation, the expressions $5 + n$ and $5n$ are equal only when $n = 1\frac{1}{4}$. When $n < 1\frac{1}{4}$, $5n < 5 + n$. When $n > 1\frac{1}{4}$, $5n > 5 + n$.

Class Discussion (The Distributive Law)

1. **(a)** $2(-x - 4) = -2x - 8$
(b) $-2(-x - 4) = 2x + 8$
(c) $3(y - 2x) = 3y - 6x$
(d) $-3(y - 2x) = -3y + 6x$

2. $a(b + c) = ab + ac$

Thinking Time (Page 96)

$$\begin{aligned}-(x-5)+6x-(7x-2)+12 &= -x+5+6x-7x+2+12\\ &= -x+6x-7x+5+2+12\\ &= -2x+19\end{aligned}$$

Possible ways:

- $$\begin{aligned}-(x-5)+6x-7x-(2+12) &= -(x-5)+6x-7x-14\\ &= -x+5+6x-7x-14\\ &= -x+6x-7x+5-14\\ &= -2x-9\end{aligned}$$
- $$\begin{aligned}-x-(5+6x)-(7x-2)+12 &= -x-5-6x-7x+2+12\\ &= -x-6x-7x-5+2+12\\ &= -14x+9\end{aligned}$$
- $$\begin{aligned}-x-(5+6x)-7x-(2+12) &= -x-(5+6x)-7x-14\\ &= -x-5-6x-7x-14\\ &= -x-6x-7x-5-14\\ &= -14x-19\end{aligned}$$

Class Discussion (Equivalent Expressions)

The five pairs of equivalent expressions are as follows:

1. **D** and **F**

$$\begin{aligned}3(x-2y)-2(3x-y) &= 3x-6y-6x+2y\\ &= 3x-6x-6y+2y\\ &= -3x-4y\end{aligned}$$

*Students may mistakenly match **D** and **O** due to an error in their working as shown:*

$$\begin{aligned}3(x-2y)-2(3x-y) &= 3x-6y-6x - 2y\\ &= 3x-6x-6y-2y\\ &= -3x-8y\end{aligned}$$

2. **A** and **E**

$$\begin{aligned}\frac{x-3}{2}-\frac{2x-5}{3} &= \frac{3(x-3)-2(2x-5)}{6}\\ &= \frac{3x-9-4x+10}{6}\\ &= \frac{3x-4x-9+10}{6}\\ &= \frac{-x+1}{6}\\ &= \frac{1-x}{6}\end{aligned}$$

*Students may mistakenly match **E** and **H** due to an error in their working as shown:*

$$\begin{aligned}\frac{x-3}{2}-\frac{2x-5}{3} &= \frac{3(x-3)-2(2x-5)}{6}\\ &= \frac{3x-9-4x-10}{6}\\ &= \frac{3x-4x-9-10}{6}\\ &= \frac{-x-19}{6}\end{aligned}$$

3. **G** and **N**

$$\begin{aligned}\frac{3(x+3)}{4}-\frac{4(2x+3)}{3} &= \frac{9(x+3)-16(2x+3)}{12}\\ &= \frac{9x+27-32x-48}{12}\\ &= \frac{9x-32x+27-48}{12}\\ &= \frac{-23x-21}{12}\end{aligned}$$

*Students may mistakenly match **B** and **G** due to an error in their working as shown:*

$$\begin{aligned}\frac{3(x+3)}{4}-\frac{4(2x+3)}{3} &= \frac{9(x+3)-16(2x+3)}{12}\\ &= \frac{9x+27-32x+48}{12}\\ &= \frac{9x-32x+27+48}{12}\\ &= \frac{-23x+75}{12}\end{aligned}$$

4. **I** and **M**

$$\begin{aligned}2x-3[5x-y-2(7x-y)] &= 2x-3(5x-y-14x+2y)\\ &= 2x-3(5x-14x-y+2y)\\ &= 2x-3(-9x+y)\\ &= 2x+27x-3y\\ &= 29x-3y\end{aligned}$$

*Students may mistakenly match **L** and **M** due to errors in their working as shown:*

$$\begin{aligned}2x-3[5x-y-2(7x-y)] &= 2x-3(5x-y-14x - 2y)\\ &= 2x-3(5x-14x-y-2y)\\ &= 2x-3(-9x-3y)\\ &= 2x - 27x - 9y\\ &= -25x-9y\end{aligned}$$

5. **C** and **J, C** and **K** or **J** and **K**

$7ay-49y = 7(ay-7y) = 7y(a-7)$

Teachers may wish to get students to indicate the expression which is obtained when the expression 7ay – 49y is factorised completely.

Practise Now 1

1. **(a)** $$\begin{aligned}5y-4x &= 5(4)-4(-2)\\ &= 20+8\\ &= 28\end{aligned}$$

(b) $$\begin{aligned}\frac{1}{x}-y+3 &= \frac{1}{-2}-4+3\\ &= -\frac{1}{2}-4+3\\ &= -4\frac{1}{2}+3\\ &= -1\frac{1}{2}\end{aligned}$$

2. $p^2 + 3q^2 = \left(-\frac{1}{2}\right)^2 + 3(-2)^2$

$= \frac{1}{4} + 3(4)$

$= \frac{1}{4} + 12$

$= 12\frac{1}{4}$

Practise Now (Page 87)

(a) $3x + 4x = 7x$
(b) $3x + (-4x) = -x$
(c) $-3x + 4x = x$
(d) $-3x + (-4x) = -7x$

Practise Now (Page 88)

(a) $4x - 3x = x$
(b) $4x - (-3x) = 4x + 3x$
$= 7x$
(c) $-4x - 3x = -7x$
(d) $-4x - (-3x) = -4x + 3x$
$= -x$

Practise Now (Page 89)

(a) $x + 2 + 5x - 4 = x + 5x + 2 - 4$
$= 6x - 2$
(b) $2x + (-3) - 3x + 5 = 2x - 3x + (-3) + 5$
$= -x + 2$
(c) $-x - y - (-2x) + 4y = -x - y + 2x + 4y$
$= -x + 2x - y + 4y$
$= x + 3y$
(d) $-3x - 7y + (-2y) - (-4x) = -3x - 7y + (-2y) + 4x$
$= -3x + 4x - 7y + (-2y)$
$= x - 9y$

Practise Now 2

1. **(a)** $2x - 5y + 4y + 8x = 2x + 8x - 5y + 4y$
$= 10x - y$
(b) $11x - (-5y) - 14x - 2y = 11x + 5y - 14x - 2y$
$= 11x - 14x + 5y - 2y$
$= -3x + 3y$
(c) $-9x - (-y) + (-3x) - 7y = -9x + y - 3x - 7y$
$= -9x - 3x + y - 7y$
$= -12x - 6y$
(d) $\frac{1}{2}x - \frac{1}{3}x = \frac{3}{6}x - \frac{2}{6}x$
$= \frac{1}{6}x$
(e) $\frac{7}{4}y - \frac{5}{8}y = \frac{14}{8}y - \frac{5}{8}y$
$= \frac{9}{8}y$

2. **(i)** $2p - 5q + 7r - 4p + 2q - 3r = 2p - 4p - 5q + 2q + 7r - 3r$
$= -2p - 3q + 4r$
(ii) When $p = \frac{1}{2}$, $q = -\frac{1}{3}$, $r = 4$,
$-2p - 3q + 4r = -2\left(\frac{1}{2}\right) - 3\left(-\frac{1}{3}\right) + 4(4)$
$= -1 + 1 + 16$
$= 0 + 16$
$= 16$

Practise Now (Page 92)

(a) $-(3x + 2) = -3x - 2$
(b) $-(3x - 2) = -3x + 2$
(c) $-(-3x - 2) = 3x + 2$
(d) $-(2x + y - 4) = -2x - y + 4$

Practise Now (Page 92)

(a) $x + 1 + [-(3x - 1)] = x + 1 - 3x + 1$
$= x - 3x + 1 + 1$
$= -2x + 2$
(b) $5x - 3 + [-(4x + 1)] = 5x - 3 - 4x - 1$
$= 5x - 4x - 3 - 1$
$= x - 4$
(c) $3x + 2y + [-(-y + 2x)] = 3x + 2y + y - 2x$
$= 3x - 2x + 2y + y$
$= x + 3y$
(d) $-4x + 2y + [-(-x - 5y)] = -4x + 2y + x + 5y$
$= -4x + x + 2y + 5y$
$= -3x + 7y$

Practise Now (Page 93)

(a) $3(5x) = 15x$
(b) $3(-5x) = -15x$
(c) $-3(5x) = -15x$
(d) $-3(-5x) = 15x$

Practise Now 3

(a) $3(x + 2) = 3x + 6$
(b) $-5(x - 4y) = -5x + 20y$
(c) $-a(x + 2y) = -ax - a(2y)$
$= -ax - 2ay$

Practise Now (Page 95)

(a) $x + 7 + 3(x - 2) = x + 7 + 3x - 6$
$= x + 3x + 7 - 6$
$= 4x + 1$
(b) $3(x + 2) + 2(-2x + 1) = 3x + 6 - 4x + 2$
$= 3x - 4x + 6 + 2$
$= -x + 8$
(c) $2(-x - y) - (2x - y) = -2x - 2y - 2x + y$
$= -2x - 2x - 2y + y$
$= -4x - y$

(d) $-(x + 4y) - 2(3x - y) = -x - 4y - 6x + 2y$
$= -x - 6x - 4y + 2y$
$= -7x - 2y$

Practise Now 4

1. (a) $6(4x + y) + 2(x - y) = 24x + 6y + 2x - 2y$
$= 24x + 2x + 6y - 2y$
$= 26x + 4y$

(b) $x - [y - 3(2x - y)] = x - (y - 6x + 3y)$
$= x - (-6x + y + 3y)$
$= x - (-6x + 4y)$
$= x + 6x - 4y$
$= 7x - 4y$

(c) $7x - 2[3(x - 2) - 2(x - 5)] = 7x - 2(3x - 6 - 2x + 10)$
$= 7x - 2(3x - 2x - 6 + 10)$
$= 7x - 2(x + 4)$
$= 7x - 2x - 8$
$= 5x - 8$

2. (i) Michael's present age = $(p + 5)$ years

(ii) Vishal's present age = $3(p + 5)$
$= (3p + 15)$ years

(iii) Sum of their ages in 6 years' time
$= p + p + 5 + 3p + 15 + 3 \times 6$
$= p + p + 5 + 3p + 15 + 18$
$= p + p + 3p + 5 + 15 + 18$
$= (5p + 38)$ years

(iv) Sum of their ages 3 years ago $= p + p + 5 + 3p + 15 - 3 \times 3$
$= p + p + 5 + 3p + 15 - 9$
$= p + p + 3p + 5 + 15 - 9$
$= (5p + 11)$ years

Alternatively,
Sum of their ages 3 years ago $= 5p + 38 - 3 \times 9$
$= 5p + 38 - 27$
$= (5p + 11)$ years

Practise Now 5

(a) $\frac{1}{2}x + \frac{1}{4}y - \frac{2}{5}y - \frac{1}{3}x = \frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}y - \frac{2}{5}y$
$= \frac{3}{6}x - \frac{2}{6}x + \frac{5}{20}y - \frac{8}{20}y$
$= \frac{1}{6}x - \frac{3}{20}y$

(b) $\frac{1}{8}[-y - 3(16x - 3y)] = \frac{1}{8}(-y - 48x + 9y)$
$= \frac{1}{8}(-y + 9y - 48x)$
$= \frac{1}{8}(8y - 48x)$
$= y - 6x$

Practise Now 6

1. (a) $\frac{x-3}{2} + \frac{2x-5}{3} = \frac{3(x-3)}{6} + \frac{2(2x-5)}{6}$
$= \frac{3(x-3) + 2(2x-5)}{6}$
$= \frac{3x - 9 + 4x - 10}{6}$
$= \frac{3x + 4x - 9 - 10}{6}$
$= \frac{7x - 19}{6}$

(b) $\frac{x-2}{4} - \frac{2x-7}{3} = \frac{3(x-2)}{12} - \frac{4(2x-7)}{12}$
$= \frac{3(x-2) - 4(2x-7)}{12}$
$= \frac{3x - 6 - 8x + 28}{12}$
$= \frac{3x - 8x - 6 + 28}{12}$
$= \frac{-5x + 22}{12}$

2. (a) $\frac{x-1}{3} + \frac{1}{2} - \frac{2x-3}{4} = \frac{4(x-1)}{12} + \frac{6}{12} - \frac{3(2x-3)}{12}$
$= \frac{4(x-1) + 6 - 3(2x-3)}{12}$
$= \frac{4x - 4 + 6 - 6x + 9}{12}$
$= \frac{4x - 6x - 4 + 6 + 9}{12}$
$= \frac{-2x + 11}{12}$

(b) $2x + \frac{x-4}{9} - \frac{2x-5}{3} = \frac{9(2x)}{9} + \frac{x-4}{9} - \frac{3(2x-5)}{9}$
$= \frac{9(2x) + x - 4 - 3(2x-5)}{9}$
$= \frac{18x + x - 4 - 6x + 15}{9}$
$= \frac{18x + x - 6x - 4 + 15}{9}$
$= \frac{13x + 11}{9}$

Practise Now 7

(a) $-10x + 25 = -5(2x - 5)$
(b) $18a - 54ay + 36az = 9a(2 - 6y + 4z)$

Exercise 4A

1. (a) $ab + 5y$ (b) $f^3 - 3$
(c) $6kq$ (d) $\frac{2w}{3xy}$
(e) $3x - 4\sqrt{z}$ (f) $\frac{2p}{5q}$

2. **(a)** $4x - 7y = 4(6) - 7(-4)$
$= 24 + 28$
$= 52$

(b) $\frac{5x}{3y} + x = \frac{5(6)}{3(-4)} + 6$
$= \frac{30}{-12} + 6$
$= -2\frac{1}{2} + 6$
$= 3\frac{1}{2}$

(c) $2x^2 - y^3 = 2(6)^2 - (-4)^3$
$= 72 - (-64)$
$= 72 + 64$
$= 136$

(d) $3x + \frac{x}{y} - y^2 = 3(6) + \frac{6}{-4} - (-4)^2$
$= 18 - 1\frac{1}{2} - 16$
$= 16\frac{1}{2} - 16$
$= \frac{1}{2}$

3. **(a)** $a(3c - b) = 3[3(6) - (-5)]$
$= 3(18 + 5)$
$= 3(23)$
$= 69$

(b) $ab^2 - ac = 3(-5)^2 - 3(6)$
$= 3(25) - 18$
$= 75 - 18$
$= 57$

(c) $\frac{b}{a} - \frac{c}{b} = \frac{-5}{3} - \frac{6}{-5}$
$= -1\frac{2}{3} + 1\frac{1}{5}$
$= -\frac{7}{15}$

(d) $\frac{b + c}{a} + \frac{a + c}{b} = \frac{-5 + 6}{3} + \frac{3 + 6}{-5}$
$= \frac{1}{3} + \frac{9}{-5}$
$= \frac{1}{3} - 1\frac{4}{5}$
$= -1\frac{7}{15}$

4. **(a)** $5x + 22 - 6x - 23 = 5x - 6x + 22 - 23$
$= -x - 1$

(b) $x + 3y + 6x + 4y = x + 6x + 3y + 4y$
$= 7x + 7y$

(c) $6xy + 13x - 2yx - 5x = 6xy - 2yx + 13x - 5x$
$= 4xy + 8x$

(d) $6x - 20y + 7z - 8x + 25y - 11z$
$= 6x - 8x - 20y + 25y + 7z - 11z$
$= -2x + 5y - 4z$

5. **(a)** Required answer $= 2x + 4y + (-5y)$
$= 2x + 4y - 5y$
$= 2x - y$

(b) Required answer $= -b - 4a + 7b - 6a$
$= -4a - 6a - b + 7b$
$= -10a + 6b$

(c) Required answer $= 6d - 4c + (-7c + 6d)$
$= 6d - 4c - 7c + 6d$
$= -4c - 7c + 6d + 6d$
$= -11c + 12d$

(d) Required answer $= 3pq - 6hk + (-3qp + 14kh)$
$= 3pq - 6hk - 3qp + 14kh$
$= 3pq - 3qp - 6hk + 14kh$
$= 8hk$

6. **(a)** $(a + b)^2 - \sqrt[3]{3xy}$

(b) Total value = $(20x + 500y)$ cents

7. **(a)** $\frac{3a - b}{2c} + \frac{3a - c}{c - b} = \frac{3(3) - (-4)}{2(-2)} + \frac{3(3) - (-2)}{-2 - (-4)}$
$= \frac{9 + 4}{-4} + \frac{9 + 2}{-2 + 4}$
$= \frac{13}{-4} + \frac{11}{2}$
$= -3\frac{1}{4} + 5\frac{1}{2}$
$= 2\frac{1}{4}$

(b) $\frac{2c - a}{3c + b} - \frac{5a + 4c}{c - a} = \frac{2(-2) - 3}{3(-2) + (-4)} - \frac{5(3) + 4(-2)}{-2 - 3}$
$= \frac{-4 - 3}{-6 - 4} - \frac{15 - 8}{-5}$
$= \frac{-7}{-10} - \frac{7}{-5}$
$= \frac{7}{10} + 1\frac{2}{5}$
$= 2\frac{1}{10}$

(c) $\frac{a + b + 2c}{3c - a - b} - \frac{5c}{4b} = \frac{3 + (-4) + 2(-2)}{3(-2) - 3 - (-4)} - \frac{5(-2)}{4(-4)}$
$= \frac{3 - 4 - 4}{-6 - 3 + 4} - \frac{-10}{-16}$
$= \frac{-5}{-5} - \frac{5}{8}$
$= 1 - \frac{5}{8}$
$= \frac{3}{8}$

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(d) $\dfrac{b-c}{3c+4b} \div \left(\dfrac{bc}{a}+\dfrac{ac}{b}\right)$

$= \dfrac{-4-(-2)}{3(-2)+4(-4)} \div \left[\dfrac{(-4)(-2)}{3}+\dfrac{3(-2)}{-4}\right]$

$= \dfrac{-4+2}{-6-16} \div \left(\dfrac{8}{3}+\dfrac{-6}{-4}\right)$

$= \dfrac{-2}{-22} \div \left(2\dfrac{2}{3}+1\dfrac{1}{2}\right)$

$= \dfrac{1}{11} \div 4\dfrac{1}{6}$

$= \dfrac{6}{275}$

8. **(a)** $15x + (-7y) + (-18x) + 4y = 15x - 7y - 18x + 4y$
$= 15x - 18x - 7y + 4y$
$= -3x - 3y$

(b) $-3x + (-5y) - (-10y) - 7x = -3x - 5y + 10y - 7x$
$= -3x - 7x - 5y + 10y$
$= -10x + 5y$

(c) $9x - (-2y) - 8x - (-12y) = 9x + 2y - 8x + 12y$
$= 9x - 8x + 2y + 12y$
$= x + 14y$

(d) $-7x - (-15y) - (-2x) + (-6y) = -7x + 15y + 2x - 6y$
$= -7x + 2x + 15y - 6y$
$= -5x + 9y$

9. **(a)** $\dfrac{1}{4}x + \dfrac{1}{3}x = \dfrac{3}{12}x + \dfrac{4}{12}x$
$= \dfrac{7}{12}x$

(b) $\dfrac{2}{5}y - \dfrac{1}{3}y = \dfrac{6}{15}y - \dfrac{5}{15}y$
$= \dfrac{1}{15}y$

(c) $-\dfrac{3}{7}a + \dfrac{3}{5}a = -\dfrac{15}{35}a + \dfrac{21}{35}a$
$= \dfrac{6}{35}a$

(d) $\dfrac{9}{4}b - \dfrac{4}{3}b = \dfrac{27}{12}b - \dfrac{16}{12}b$
$= \dfrac{11}{12}b$

10. **(i)** $3p + (-q) - 7r - (-8p) - q + 2r = 3p - q - 7r + 8p - q + 2r$
$= 3p + 8p - q - q - 7r + 2r$
$= 11p - 2q - 5r$

(ii) When $p = 2$, $q = -1\dfrac{1}{2}$, $r = -5$,

$11p - 2q - 5r = 11(2) - 2\left(-1\dfrac{1}{2}\right) - 5(-5)$
$= 22 + 3 + 25$
$= 25 + 25$
$= 50$

11. **(i)** Raj's age 5 years later = $(12m + 5)$ years

(ii) Present age of Raj's son = $12m - 9m$
$= 3m$ years
Age of Raj's son 5 years later = $(3m + 5)$ years
Sum of their ages in 5 years' time = $12m + 5 + 3m + 5$
$= 12m + 3m + 5 + 5$
$= (15m + 10)$ years

12. Amount of money Huixian had at first
$= 8 \times \$w + 7 \times \$m + \$(3w + 5m)$
$= \$8w + \$7m + \$(3w + 5m)$
$= \$(8w + 3w + 7m + 5m)$
$= \$(11w + 12m)$

13. **(a)** Number of people who order plain prata = $\dfrac{5}{2}a$

(b) Number of people who order egg prata = $\dfrac{2}{5}b$

(c) Number of people who order egg prata = $\dfrac{2}{7}c$

Exercise 4B

1. **(a)** $-(x + 5) = -x - 5$
(b) $-(4 - x) = -4 + x$
(c) $2(3y + 7) = 6y + 14$
(d) $8(2y - 5) = 16y - 40$
(e) $8(3a - 4b) = 24a - 32b$
(f) $-3(c + 6) = -3c - 18$
(g) $-4(d - 6) = -4d + 24$
(h) $2a(x - y) = 2ax - 2ay$

2. **(a)** $5(a + 2b) - 3b = 5a + 10b - 3b$
$= 5a + 7b$

(b) $7(p + 10q) + 2(6p + 7q) = 7p + 70q + 12p + 14q$
$= 7p + 12p + 70q + 14q$
$= 19p + 84q$

(c) $a + 3b - (5a - 4b) = a + 3b - 5a + 4b$
$= a - 5a + 3b + 4b$
$= -4a + 7b$

(d) $x + 3(2x - 3y + z) + 7z = x + 6x - 9y + 3z + 7z$
$= 7x - 9y + 10z$

3. Present age of Khairul's uncle = $4(x + 5)$
$= (4x + 20)$ years

4. Total cost = $4x + 6(x - y)$
$= 4x + 6x - 6y$
$= (10x - 6y)$ cents

5. Total cost of skirts Devi bought
$= 7 \times \$x + n \times \$12 + (2n + 1) \times \$15 + 4 \times \$3x$
$= \$7x + \$12n + \$15(2n + 1) + \$12x$
$= \$7x + \$12n + \$(30n + 15) + \$12x$
$= \$(7x + 12n + 30n + 15 + 12x)$
$= \$(7x + 12x + 12n + 30n + 15)$
$= \$(19x + 42n + 15)$

6. **(a)** $4u - 3(2u - 5v) = 4u - 6u + 15v$
$= -2u + 15v$

(b) $-2a - 3(a - b) = -2a - 3a + 3b$
$= -5a + 3b$

(c) $7m - 2n - 2(3n - 2m) = 7m - 2n - 6n + 4m$
$= 7m + 4m - 2n - 6n$
$= 11m - 8n$

(d) $5(2x + 4) - 3(-6 - x) = 10x + 20 + 18 + 3x$
$= 10x + 3x + 20 + 18$
$= 13x + 38$

(e) $-4(a - 3b) - 5(a - 3b) = -4a + 12b - 5a + 15b$
$= -4a - 5a + 12b + 15b$
$= -9a + 27b$

(f) $5(3p - 2q) - 2(3p + 2q) = 15p - 10q - 6p - 4q$
$= 15p - 6p - 10q - 4q$
$= 9p - 14q$

(g) $x + y - 2(3x - 4y + 3) = x + y - 6x + 8y - 6$
$= x - 6x + y + 8y - 6$
$= -5x + 9y - 6$

(h) $3(p - 2q) - 4(2p - 3q - 5) = 3p - 6q - 8p + 12q + 20$
$= 3p - 8p - 6q + 12q + 20$
$= -5p + 6q + 20$

(i) $9(2a + 4b - 7c) - 4(b - c) - 7(-c - 4b)$
$= 18a + 36b - 63c - 4b + 4c + 7c + 28b$
$= 18a + 36b - 4b + 28b - 63c + 4c + 7c$
$= 18a + 60b - 52c$

(j) $-4[5(2x + 3y) - 4(x + 2y)] = -4(10x + 15y - 4x - 8y)$
$= -4(10x - 4x + 15y - 8y)$
$= -4(6x + 7y)$
$= -24x - 28y$

7. **(a)** Required answer $= 2x - 5 - (-6x - 3)$
$= 2x - 5 + 6x + 3$
$= 2x + 6x - 5 + 3$
$= 8x - 2$

(b) Required answer $= 10x - 2y + z - (6x - y + 5z)$
$= 10x - 2y + z - 6x + y - 5z$
$= 10x - 6x - 2y + y + z - 5z$
$= 4x - y - 4z$

(c) Required answer $= -4p - 4q + 15sr - (8p + 9q - 5rs)$
$= -4p - 4q + 15sr - 8p - 9q + 5rs$
$= -4p - 8p - 4q - 9q + 15sr + 5rs$
$= -12p - 13q + 20rs$

(d) Required answer $= 10a - b - 4c - 8d - (8a - 3b + 5c - 4d)$
$= 10a - b - 4c - 8d - 8a + 3b - 5c + 4d$
$= 10a - 8a - b + 3b - 4c - 5c - 8d + 4d$
$= 2a + 2b - 9c - 4d$

8. **(a)** $-2\{3a - 4[a - (2 + a)]\} = -2[3a - 4(a - 2 - a)]$
$= -2[3a - 4(-2)]$
$= -2(3a + 8)$
$= -6a - 16$

(b) $5\{3c - [d - 2(c + d)]\} = 5[3c - (d - 2c - 2d)]$
$= 5[3c - (-2c - d)]$
$= 5(3c + 2c + d)$
$= 5(5c + d)$
$= 25c + 5d$

9. Average monthly salary of the female employees

$$= \$\left[\frac{2000(m + f) - m(b + 200)}{f}\right]$$
$$= \$\left(\frac{2000m + 2000f - mb - 200m}{f}\right)$$
$$= \$\left(\frac{2000m - 200m - mb + 2000f}{f}\right)$$
$$= \$\left(\frac{1800m - mb + 2000f}{f}\right)$$

Exercise 4C

1. **(a)** $\frac{1}{4}x + \frac{1}{5}y - \frac{1}{6}x - \frac{1}{10}y = \frac{1}{4}x - \frac{1}{6}x + \frac{1}{5}y - \frac{1}{10}y$
$= \frac{3}{12}x - \frac{2}{12}x + \frac{2}{10}y - \frac{1}{10}y$
$= \frac{1}{12}x + \frac{1}{10}y$

(b) $\frac{2}{3}a - \frac{1}{7}b + 2a - \frac{3}{5}b = \frac{2}{3}a + 2a - \frac{1}{7}b - \frac{3}{5}b$
$= \frac{2}{3}a + \frac{6}{3}a - \frac{5}{35}b - \frac{21}{35}b$
$= \frac{8}{3}a - \frac{26}{35}b$

(c) $\frac{5}{9}c + \frac{3}{4}d - \frac{7}{8}c - \frac{4}{3}d = \frac{5}{9}c - \frac{7}{8}c + \frac{3}{4}d - \frac{4}{3}d$
$= \frac{40}{72}c - \frac{63}{72}c + \frac{9}{12}d - \frac{16}{12}d$
$= -\frac{23}{72}c - \frac{7}{12}d$

(d) $2f - \frac{5}{3}h + \frac{9}{4}k - \frac{1}{2}f - \frac{28}{5}k + \frac{5}{4}h$
$= 2f - \frac{1}{2}f - \frac{5}{3}h + \frac{5}{4}h + \frac{9}{4}k - \frac{28}{5}k$
$= \frac{4}{2}f - \frac{1}{2}f - \frac{20}{12}h + \frac{15}{12}h + \frac{45}{20}k - \frac{112}{20}k$
$= \frac{3}{2}f - \frac{5}{12}h - \frac{67}{20}k$

2. **(a)** $5a + 4b - 3c - \left(2a - \frac{3}{2}b + \frac{3}{2}c\right)$
$= 5a + 4b - 3c - 2a + \frac{3}{2}b - \frac{3}{2}c$
$= 5a - 2a + 4b + \frac{3}{2}b - 3c - \frac{3}{2}c$
$= 3a + \frac{8}{2}b + \frac{3}{2}b - \frac{6}{2}c - \frac{3}{2}c$
$= 3a + \frac{11}{2}b - \frac{9}{2}c$

(b) $\frac{1}{2}[2x + 2(x - 3)] = \frac{1}{2}(2x + 2x - 6)$
$= \frac{1}{2}(4x - 6)$
$= 2x - 3$

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(c) $\frac{2}{5}[12p-(5+2p)] = \frac{2}{5}(12p-5-2p)$

$= \frac{2}{5}(12p-2p-5)$

$= \frac{2}{5}(10p-5)$

$= 4p-2$

(d) $\frac{1}{2}[8x+10-6(1-4x)] = \frac{1}{2}(8x+10-6+24x)$

$= \frac{1}{2}(8x+24x+10-6)$

$= \frac{1}{2}(32x+4)$

$= 16x+2$

3. **(a)** $\frac{x}{2}+\frac{2x}{5} = \frac{5x}{10}+\frac{4x}{10}$

$= \frac{9}{10}x$

(b) $\frac{a}{3}-\frac{a}{4} = \frac{4a}{12}-\frac{3a}{12}$

$= \frac{1}{12}a$

(c) $\frac{2h}{7}+\frac{h+1}{5} = \frac{10h}{35}+\frac{7(h+1)}{35}$

$= \frac{10h+7(h+1)}{35}$

$= \frac{10h+7h+7}{35}$

$= \frac{17h+7}{35}$

(d) $\frac{3x}{8}-\frac{x+2}{4} = \frac{3x}{8}-\frac{2(x+2)}{8}$

$= \frac{3x-2(x+2)}{8}$

$= \frac{3x-2x-4}{8}$

$= \frac{x-4}{8}$

(e) $\frac{4x+1}{5}+\frac{3x-1}{2} = \frac{2(4x+1)}{10}+\frac{5(3x-1)}{10}$

$= \frac{2(4x+1)+5(3x-1)}{10}$

$= \frac{8x+2+15x-5}{10}$

$= \frac{8x+15x+2-5}{10}$

$= \frac{23x-3}{10}$

(f) $\frac{3y-1}{4}-\frac{2y-3}{6} = \frac{3(3y-1)}{12}-\frac{2(2y-3)}{12}$

$= \frac{3(3y-1)-2(2y-3)}{12}$

$= \frac{9y-3-4y+6}{12}$

$= \frac{9y-4y-3+6}{12}$

$= \frac{5y+3}{12}$

(g) $\frac{a-2}{4}-\frac{a+7}{8} = \frac{2(a-2)}{8}-\frac{a+7}{8}$

$= \frac{2(a-2)-(a+7)}{8}$

$= \frac{2a-4-a-7}{8}$

$= \frac{2a-a-4-7}{8}$

$= \frac{a-11}{8}$

(h) $\frac{3p-2q}{3}-\frac{4p-5q}{4} = \frac{4(3p-2q)}{12}-\frac{3(4p-5q)}{12}$

$= \frac{4(3p-2q)-3(4p-5q)}{12}$

$= \frac{12p-8q-12p+15q}{12}$

$= \frac{12p-12p-8q+15q}{12}$

$= \frac{7}{12}q$

4. **(a)** $12x-9 = 3(4x-3)$

(b) $-25y-35 = -5(5y+7)$

(c) $27b-36by = 9b(3-4y)$

(d) $8ax+12a-4az = 4a(2x+3-z)$

(e) $4m-6my-18mz = 2m(2-3y-9z)$

5. **(a)** $y-\frac{2}{3}(9x-3y) = y-2(3x-y)$

$= y-6x+2y$

$= -6x+y+2y$

$= -6x+3y$

(b) $-\frac{1}{3}\{6(p+q)-3[p-2(p-3q)]\}$

$= -\frac{1}{3}[6(p+q)-3(p-2p+6q)]$

$= -\frac{1}{3}[6(p+q)-3(-p+6q)]$

$= -\frac{1}{3}(6p+6q+3p-18q)$

$= -\frac{1}{3}(6p+3p+6q-18q)$

$= -\frac{1}{3}(9p-12q)$

$= -3p+4q$

6. **(a)** $\dfrac{7(x+3)}{2} + \dfrac{5(2x-5)}{3} = \dfrac{21(x+3)}{6} + \dfrac{10(2x-5)}{6}$

$= \dfrac{21(x+3)+10(2x-5)}{6}$

$= \dfrac{21x+63+20x-50}{6}$

$= \dfrac{21x+20x+63-50}{6}$

$= \dfrac{41x+13}{6}$

(b) $\dfrac{3x-4}{5} - \dfrac{3(x-1)}{2} = \dfrac{2(3x-4)}{10} - \dfrac{15(x-1)}{10}$

$= \dfrac{2(3x-4)-15(x-1)}{10}$

$= \dfrac{6x-8-15x+15}{10}$

$= \dfrac{6x-15x-8+15}{10}$

$= \dfrac{-9x+7}{10}$

(c) $\dfrac{3(z-2)}{4} - \dfrac{4(2z-3)}{5} = \dfrac{15(z-2)}{20} - \dfrac{16(2z-3)}{20}$

$= \dfrac{15(z-2)-16(2z-3)}{20}$

$= \dfrac{15z-30-32z+48}{20}$

$= \dfrac{15z-32z-30+48}{20}$

$= \dfrac{-17z+18}{20}$

(d) $\dfrac{2(p-4q)}{3} - \dfrac{3(2p+q)}{2} = \dfrac{4(p-4q)}{6} - \dfrac{9(2p+q)}{6}$

$= \dfrac{4(p-4q)-9(2p+q)}{6}$

$= \dfrac{4p-16q-18p-9q}{6}$

$= \dfrac{4p-18p-16q-9q}{6}$

$= \dfrac{-14p-25q}{6}$

(e) $-\dfrac{2b}{3} - \dfrac{3(a-2b)}{5} = -\dfrac{10b}{15} - \dfrac{9(a-2b)}{15}$

$= \dfrac{-10b-9(a-2b)}{15}$

$= \dfrac{-10b-9a+18b}{15}$

$= \dfrac{-9a-10b+18b}{15}$

$= \dfrac{-9a+8b}{15}$

(f) $\dfrac{2(x+3)}{5} - \dfrac{1}{2} + \dfrac{3x-4}{4} = \dfrac{8(x+3)}{20} - \dfrac{10}{20} + \dfrac{5(3x-4)}{20}$

$= \dfrac{8(x+3)-10+5(3x-4)}{20}$

$= \dfrac{8x+24-10+15x-20}{20}$

$= \dfrac{8x+15x+24-10-20}{20}$

$= \dfrac{23x-6}{20}$

(g) $\dfrac{a+1}{2} - \dfrac{a+3}{3} - \dfrac{5a-2}{4}$

$= \dfrac{6(a+1)}{12} - \dfrac{4(a+3)}{12} - \dfrac{3(5a-2)}{12}$

$= \dfrac{6(a+1)-4(a+3)-3(5a-2)}{12}$

$= \dfrac{6a+6-4a-12-15a+6}{12}$

$= \dfrac{6a-4a-15a+6-12+6}{12}$

$= -\dfrac{13}{12}a$

(h) $\dfrac{x+1}{2} + \dfrac{x+3}{3} - \dfrac{5x-1}{6} = \dfrac{3(x+1)}{6} + \dfrac{2(x+3)}{6} - \dfrac{5x-1}{6}$

$= \dfrac{3(x+1)+2(x+3)-(5x-1)}{6}$

$= \dfrac{3x+3+2x+6-5x+1}{6}$

$= \dfrac{3x+2x-5x+3+6+1}{6}$

$= \dfrac{10}{6}$

$= \dfrac{5}{3}$

$= 1\dfrac{2}{3}$

(i) $\dfrac{2(a-b)}{7} - \dfrac{2a+3b}{14} + \dfrac{a+b}{2}$

$= \dfrac{4(a-b)}{14} - \dfrac{2a+3b}{14} + \dfrac{7(a+b)}{14}$

$= \dfrac{4(a-b)-(2a+3b)+7(a+b)}{14}$

$= \dfrac{4a-4b-2a-3b+7a+7b}{14}$

$= \dfrac{4a-2a+7a-4b-3b+7b}{14}$

$= \dfrac{9}{14}a$

(j) $\dfrac{x+3}{3} + \dfrac{5(3x+4)}{6} + 1 = \dfrac{2(x+3)}{6} + \dfrac{5(3x+4)}{6} + \dfrac{6}{6}$

$= \dfrac{2(x+3)+5(3x+4)+6}{6}$

$= \dfrac{2x+6+15x+20+6}{6}$

$= \dfrac{2x+15x+6+20+6}{6}$

$= \dfrac{17x+32}{6}$

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7. **(a)** $-39b^2 - 13ab = -13b(3b + a)$

(b) $5x + 10x(b + c) = 5x[1 + 2(b + c)]$
$= 5x(1 + 2b + 2c)$

(c) $3xy - 6x(y - z) = 3x[y - 2(y - z)]$
$= 3x(y - 2y + 2z)$
$= 3x(-y + 2z)$

(d) $2x(7 + y) - 14x(y + 2) = 2x[7 + y - 7(y + 2)]$
$= 2x(7 + y - 7y - 14)$
$= 2x(y - 7y + 7 - 14)$
$= 2x(-6y - 7)$

(e) $-3a(2 + b) + 18a(b - 1) = 3a[-(2 + b) + 6(b - 1)]$
$= 3a(-2 - b + 6b - 6)$
$= 3a(-b + 6b - 2 - 6)$
$= 3a(5b - 8)$

(f) $-4y(x - 2) - 12y(3 - x) = 4y[-(x - 2) - 3(3 - x)]$
$= 4y(-x + 2 - 9 + 3x)$
$= 4y(-x + 3x + 2 - 9)$
$= 4y(2x - 7)$

8. **(a)** $\dfrac{5(p-q)}{2} - \dfrac{2q-p}{14} - \dfrac{2(p+q)}{7}$

$= \dfrac{35(p-q)}{14} - \dfrac{2q-p}{14} - \dfrac{4(p+q)}{14}$

$= \dfrac{35(p-q) - (2q-p) - 4(p+q)}{14}$

$= \dfrac{35p - 35q - 2q + p - 4p - 4q}{14}$

$= \dfrac{35p + p - 4p - 35q - 2q - 4q}{14}$

$= \dfrac{32p - 41q}{14}$

(b) $-\dfrac{2a+b}{3} - \left[\dfrac{3(a-3b)}{2} - \dfrac{4(a+2b)}{5}\right]$

$= -\dfrac{2a+b}{3} - \dfrac{3(a-3b)}{2} + \dfrac{4(a+2b)}{5}$

$= -\dfrac{10(2a+b)}{30} - \dfrac{45(a-3b)}{30} + \dfrac{24(a+2b)}{30}$

$= \dfrac{-10(2a+b) - 45(a-3b) + 24(a+2b)}{30}$

$= \dfrac{-20a - 10b - 45a + 135b + 24a + 48b}{30}$

$= \dfrac{-20a - 45a + 24a - 10b + 135b + 48b}{30}$

$= \dfrac{-41a + 173b}{30}$

(c) $\dfrac{3(f-h)}{4} - \dfrac{7(h+k)}{6} + \dfrac{5(k-f)}{2}$

$= \dfrac{9(f-h)}{12} - \dfrac{14(h+k)}{12} + \dfrac{30(k-f)}{12}$

$= \dfrac{9(f-h) - 14(h+k) + 30(k-f)}{12}$

$= \dfrac{9f - 9h - 14h - 14k + 30k - 30f}{12}$

$= \dfrac{9f - 30f - 9h - 14h - 14k + 30k}{12}$

$= \dfrac{-21f - 23h + 16k}{12}$

(d) $4 - \dfrac{x-y}{3} - \dfrac{3(y+4z)}{4} + \dfrac{5(x+3z)}{8}$

$= \dfrac{96}{24} - \dfrac{8(x-y)}{24} - \dfrac{18(y+z)}{24} + \dfrac{15(x+3z)}{24}$

$= \dfrac{96 - 8(x-y) - 18(y+z) + 15(x+3z)}{24}$

$= \dfrac{96 - 8x + 8y - 18y - 18z + 15x + 45z}{24}$

$= \dfrac{96 - 8x + 15x + 8y - 18y - 18z + 45z}{24}$

$= \dfrac{96 + 7x - 10y + 27z}{24}$

Review Exercise 4

1. **(a)** $4a + 5b = 4(-2) + 5(7)$
$= -8 + 35$
$= 27$

(b) $2a^2 = 2(-2)^2$
$= 8$

(c) $(2a)^2 = [2(-2)]^2$
$= (-4)^2$
$= 16$

(d) $a(b - a) = (-2)[7 - (-2)]$
$= (-2)(7 + 2)$
$= (-2)(9)$
$= -18$

(e) $b - a^2 = 7 - (-2)^2$
$= 7 - 4$
$= 3$

(f) $(b - a)^2 = [7 - (-2)]^2$
$= (7 + 2)^2$
$= 9^2$
$= 81$

2. $\dfrac{3x - 5y^2 - 2xyz}{\dfrac{x}{y} - \dfrac{y^2}{z}} = \dfrac{3(3) - 5(-4)^2 - 2(3)(-4)(2)}{\dfrac{3}{-4} - \dfrac{(-4)^2}{2}}$

$= \dfrac{9 - 80 + 48}{-\dfrac{3}{4} - \dfrac{16}{2}}$

$= \dfrac{-23}{-\dfrac{3}{4} - \dfrac{32}{4}}$

$= \dfrac{-23}{-\dfrac{35}{4}}$

$= 2\dfrac{22}{35}$

3. **(a)** $3ab - 5xy + 4ab + 2yx = 3ab + 4ab - 5xy + 2yx$
$= 7ab - 3xy$

(b) $4(3p - 5q) + 6(2q - 5p) = 12p - 20q + 12q - 30p$
$= 12p - 30p - 20q + 12q$
$= -18p - 8q$

(c) $2a + 3[a - (b - a)] + 7(2b - a) = 2a + 3(a - b + a) + 7(2b - a)$
$= 2a + 3(a + a - b) + 7(2b - a)$
$= 2a + 3(2a - b) + 7(2b - a)$
$= 2a + 6a - 3b + 14b - 7a$
$= 2a + 6a - 7a - 3b + 14b$
$= a + 11b$

(d) $-2[3x - (4x - 5y) - 2(3x - 4y)] = -2(3x - 4x + 5y - 6x + 8y)$
$= -2(3x - 4x - 6x + 5y + 8y)$
$= -2(-7x + 13y)$
$= 14x - 26y$

(e) $4\{h - 3[f - 6(f - h)]\} = 4[h - 3(f - 6f + 6h)]$
$= 4[h - 3(-5f + 6h)]$
$= 4(h + 15f - 18h)$
$= 4(15f + h - 18h)$
$= 4(15f - 17h)$
$= 60f - 68h$

(f) $5(x + 5y) - \{2x - [3x - 3(x - 2y) + y]\}$
$= 5(x + 5y) - [2x - (3x - 3x + 6y + y)]$
$= 5(x + 5y) - (2x - 7y)$
$= 5x + 25y - 2x + 7y$
$= 5x - 2x + 25y + 7y$
$= 3x + 32y$

4. (a) $\frac{2x}{3} + \frac{5-x}{4} = \frac{8x}{12} + \frac{3(5-x)}{12}$
$= \frac{8x + 3(5-x)}{12}$
$= \frac{8x + 15 - 3x}{12}$
$= \frac{8x - 3x + 15}{12}$
$= \frac{5x + 15}{12}$

(b) $\frac{x-y}{8} - \frac{3x-2y}{12} = \frac{3(x-y)}{24} - \frac{2(3x-2y)}{24}$
$= \frac{3(x-y) - 2(3x-2y)}{24}$
$= \frac{3x - 3y - 6x + 4y}{24}$
$= \frac{3x - 6x - 3y + 4y}{24}$
$= \frac{-3x + y}{24}$

(c) $\frac{4(2a-b)}{3} - \frac{2(3a+b)}{5} = \frac{20(2a-b)}{15} - \frac{6(3a+b)}{15}$
$= \frac{20(2a-b) - 6(3a+b)}{15}$
$= \frac{40a - 20b - 18a - 6b}{15}$
$= \frac{40a - 18a - 20b - 6b}{15}$
$= \frac{22a - 26b}{15}$

(d) $\frac{h+f}{3} - \frac{f+k}{2} + \frac{4h-k}{5}$
$= \frac{10(h+f)}{30} - \frac{15(f+k)}{30} + \frac{6(4h-k)}{30}$
$= \frac{10(h+f) - 15(f+k) + 6(4h-k)}{30}$
$= \frac{10h + 10f - 15f - 15k + 24h - 6k}{30}$
$= \frac{10f - 15f + 10h + 24h - 15k - 6k}{30}$
$= \frac{-5f + 34h - 21k}{30}$

(e) $3q - \frac{4p-3q}{5} - \frac{q-4p}{6}$
$= \frac{90q}{30} - \frac{6(4p-3q)}{30} - \frac{5(q-4p)}{30}$
$= \frac{90q - 6(4p-3q) - 5(q-4p)}{30}$
$= \frac{90q - 24p + 18q - 5q + 20p}{30}$
$= \frac{-24p + 20p + 90q + 18q - 5q}{30}$
$= \frac{-4p + 103q}{30}$

(f) $\frac{4(x-5)}{7} - \left[\frac{5(x-y)}{6} + \frac{7x-y}{21}\right]$
$= \frac{4(x-5)}{7} - \frac{5(x-y)}{6} - \frac{7x-y}{21}$
$= \frac{24(x-5)}{42} - \frac{35(x-y)}{42} - \frac{2(7x-y)}{42}$
$= \frac{24(x-5) - 35(x-y) - 2(7x-y)}{42}$
$= \frac{24x - 120 - 35x + 35y - 14x + 2y}{42}$
$= \frac{24x - 35x - 14x + 35y + 2y - 120}{42}$
$= \frac{-25x + 37y - 120}{42}$

5. (a) $21pq + 14q - 28qr = 7q(3p + 2 - 4r)$

(b) $4x - 8(y - 2z) = 4[x - 2(y - 2z)]$
$= 4(x - 2y + 4z)$

6. (a) Total value of 5-cent coins = $5x$ cents

(b) Total value of 10-cent coins = $(3x \times 10)$ cents
$= 30x$ cents

(c) Number of 10-cent coins = $\frac{3}{7}x$

Total value of coins = $\left(5x + \frac{3}{7}x \times 10\right)$ cents
$= \left(5x + \frac{30}{7}x\right)$ cents
$= \left(\frac{35}{7}x + \frac{30}{7}x\right)$ cents
$= \frac{65}{7}x$ cents

7. Distance Farhan can cycle in 1 minute $= \dfrac{x}{3 \times 60}$

$= \dfrac{x}{180}$ km

Distance Farhan can cycle in y minutes $= \dfrac{xy}{180}$ km

8. **(a)** Required difference $= 3y \times 60 - 25y$
$= 180y - 25y$
$= 155y$ seconds

(b) Required sum $= 50(3z - 2) \times 60 + 4(z + 1) \times 3600$
$= 3000(3z - 2) + 14\,400(z + 1)$
$= 9000z - 6000 + 14\,400z + 14\,400$
$= 9000z + 14\,400z - 6000 + 14\,400$
$= (23\,400z + 8400)$ seconds

9. **(i)** Total amount Shirley earned $= \$[(25 - 5) \times x + 5 \times 1.5x]$
$= \$(20 \times x + 5 \times 1.5x)$
$= \$(20x + 7.5x)$
$= \$27.5x$

Total amount Kate earned $= \$[(18 - 4) \times y + 4 \times 1.5y]$
$= \$(14 \times y + 4 \times 1.5y)$
$= \$(14y + 6y)$
$= \$20y$

Total amount they earned $= \$(27.5x + 20y)$

(ii) Amount Kate was paid per hour $= \$5.50 + \$0.50$
$= \$6$

Total amount they earned $= \$[27.5(5.5) + 20(6)]$
$= \$(151.25 + 120)$
$= \$271.25$

10. **(i)** Total score obtained by Michael in the first two papers
$= p - 3q + 13 + 3p + 5q - 4$
$= p + 3p - 3q + 5q + 13 - 4$
$= (4p + 2q + 9)$ marks

(ii) Score obtained by Michael in the third paper
$= 10p + 5q - (4p + 2q + 9)$
$= 10p + 5q - 4p - 2q - 9$
$= 10p - 4p + 5q - 2q - 9$
$= (6p + 3q - 9)$ marks

(iii) $6p + 3q - 9 = 3(2p + q - 3)$

Challenge Yourself

1. Let the number of heads up in the pile of 5 be x.
Then the number of tails up in the pile of 5 is $5 - x$,
the number of heads up in the pile of 7 is $5 - x$.
After the teacher flips over all the coins in the pile of 5,
the number of heads up in that pile is $5 - x$.
Hence, both piles now have the same number of heads up. (shown)

2. The only possible set of values is $\{x = 2, y = 3, z = 6\}$.
Proofs

If $x = 2$ and $y \geqslant 4$, then $z \geqslant 5$ and $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} < 1$.

If $x \geqslant 3$, then $y, z > 3$ and $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} < 1$.

3. Let the two numbers be $\overline{xy}$ and $\overline{xz}$, where $y + z = 10$.
$\overline{xy} = 10x + y$
$\overline{xz} = 10x + z$
$\therefore\ \overline{xy} \times \overline{xz} = (10x + y)(10x + z)$
$= 10x(10x + z) + y(10x + z)$
$= 100x^2 + 10xz + 10xy + yz$
$= 100x^2 + 10x(y + z) + yz$
$= 100x^2 + 10x(10) + yz$
$= 100x^2 + 100x + yz$
$= 100x(x + 1) + yz$

Revision Exercise A1

1. **(a)** $42 = 2 \times 3 \times 7$

$66 = 2 \times 3 \times 11$

$78 = 2 \times 3 \times 13$

HCF of 42, 66 and 78 $= 2 \times 3$

$= 6$

(b) $7 = 7$

$13 = 13$

$14 = 2 \times 7$

LCM of 7, 13 and 14 $= 2 \times 7 \times 13$

$= 182$

2. **(i)** Greatest whole number which is a factor of both 405 and 1960

= HCF of 405 and 1960

= 5

(ii) Smallest whole number that is divisible by both 405 and 1960

= LCM of 405 and 1960

$= 2^3 \times 3^4 \times 5 \times 7^2$

= 158 760

3. **(i)** $105 = 3 \times 5 \times 7$

$126 = 2 \times 3^2 \times 7$

HCF of 105 and 126 $= 3 \times 7$

$= 21$

Greatest number of students that the refreshment can cater to

= 21

(ii) Number of bags of crisps each student will receive $= 105 \div 21$

$= 5$

(iii) Number of packets of fruit juice each student will receive

$= 126 \div 21$

$= 6$

4. **(i)** Pairs of cards that have a sum of 4 = {–2, 6}, {–1, 5}, {1, 3}

(ii) Pairs of cards that have a product of 2 = {–2, –1}, {1, 2}

(iii) Groups of three cards that have a sum of 10

= {–1, 5, 6}, {1, 3, 6}, {1, 4, 5}, {2, 3, 5}

5. **(a)** $101 \times \sqrt{80.7} \approx 100 \times \sqrt{81}$

$= 100 \times 9$

$= 900$

(b) $\sqrt[3]{26} \times 502 \div 49 \approx \sqrt[3]{27} \times 500 \div 50$

$= 3 \times 500 \div 50$

$= 1500 \div 50$

$= 30$

(c) $\sqrt{65} \times \sqrt[3]{63} \div 17 \approx \sqrt{64} \times \sqrt[3]{64} \div 16$

$= 8 \times 4 \div 16$

$= 32 \div 16$

$= 2$

6. Average speed $= \dfrac{628}{6.8 \times 60}$

$\approx \dfrac{630}{7 \times 60}$ m/s

7. **(a)** $\dfrac{a^2bd}{3ac - d} = \dfrac{1^2(2)(-3)}{3(1)(0) - (-3)}$

$= \dfrac{-6}{0 + 3}$

$= \dfrac{-6}{3}$

$= -2$

(b) $\dfrac{bc + d^2}{a + b} = \dfrac{2(0) + (-3)^2}{1 + 2}$

$= \dfrac{0 + 9}{3}$

$= \dfrac{9}{3}$

$= 3$

(c) $a^2 + b^2 - c^2 + d^2 = 1^2 + 2^2 - 0^2 + (-3)^2$

$= 1 + 4 - 0 + 9$

$= 14$

(d) $-a^3 - b^3 + c^3 - d^3 = -1^3 - 2^3 + 0^3 - (-3)^3$

$= -1 - 8 + 0 - (-27)$

$= -9 + 0 - (-27)$

$= -9 - (-27)$

$= -9 + 27$

$= 18$

8. Cost of a pear $= (a + b)$ cents

Total cost $= \$\left[\dfrac{10a}{100} + \dfrac{12(a + b)}{100}\right]$

$= \$\left[\dfrac{10a + 12(a + b)}{100}\right]$

$= \$\left(\dfrac{10a + 12a + 12b}{100}\right)$

$= \$\left(\dfrac{22a + 12b}{100}\right)$

$= \$\left[\dfrac{2(11a + 6b)}{100}\right]$

$= \$\left(\dfrac{11a + 6b}{50}\right)$

Revision Exercise A2

1. **(a)** $54 = 2 \times 3^3$

$126 = 2 \times 3^2 \times 7$

$342 = 2 \times 3^2 \times 19$

HCF of 54, 126 and 342 $= 2 \times 3^2$

$= 18$

(b) $16 = 2^4$

$28 = 2^2 \times 7$

$44 = 2^2 \times 11$

$68 = 2^2 \times 17$

LCM of 16, 18, 44 and 68 $= 2^4 \times 7 \times 11 \times 17$

$= 20\,944$

2. **(a)** $9216 = 2^{10} \times 3^2$

$\therefore -\sqrt{9216} = -\sqrt{2^{10} \times 3^2}$

$= -(2^5 \times 3)$

$= -96$

(b) $8000 = 2^6 \times 5^3$

$\therefore \sqrt[3]{8000} = \sqrt[3]{2^6 \times 5^3}$

$= 2^2 \times 5$

$= 20$

3. $1764 = 2^2 \times 3^2 \times 7^2$

$36 = 2^2 \times 3^2$

$8820 = 2^2 \times 3^2 \times 5 \times 7^2$

$\therefore$ Value of $p = 2^2 \times 3^2 \times 5$

$= 180$

4. **(i)** Temperature of town at 6 p.m. $= -6\ °\text{C} + 8°\ \text{C} - 4\ °\text{C}$

$= -2\ °\text{C}$

(ii) Overall increase $= -2\ °\text{C} - (-6\ °\text{C})$

$= -2\ °\text{C} + 6\ °\text{C}$

$= 4\ °\text{C}$

5. **(a)** $\frac{2}{3} - \left(-3\frac{1}{20}\right) + \left(-\frac{4}{5}\right) = \frac{2}{3} + 3\frac{1}{20} - \frac{4}{5}$

$= \frac{2}{3} + \frac{61}{20} - \frac{4}{5}$

$= \frac{40}{60} + \frac{183}{60} - \frac{48}{60}$

$= \frac{40 + 183 - 48}{60}$

$= \frac{175}{60}$

$= \frac{35}{12}$

$= 2\frac{11}{12}$

(b) **(i)** $[-4.749 - 6.558 \times (-2.094)^3] \div \sqrt[3]{-1.999}$

$= -44.030$ (to 3 d.p.)

(ii) $\left\{\left(\frac{1}{3}\right)^2 - \sqrt[3]{\frac{8}{33}} \times \left[-\sqrt{\frac{5}{6}} - (-0.375)^3\right]\right\} \times [-\pi \div (-6.5)]$

$= 0.313$ (to 3 d.p.)

6. Number of buttons in Box B after Kate transfers 15 buttons from Box A to Box B

$= 35 + 15$

$= 50$

Number of buttons in Box A after Kate transfers 15 buttons from Box A to Box B

$= 50 \div \frac{5}{7}$

$= {}^{10}\cancel{50} \times \frac{7}{\cancel{5}_1}$

$= 70$

Initial number of buttons in Box $A = 70 + 15$

$= 85$

7. **(i)** Area of carpet $= 4\frac{1}{10} \times 2\frac{9}{10}$

$\approx (4 \times 3)\ \text{m}^2$

(ii) Cost of carpet $\approx 4 \times 3 \times \$89.75$

$\approx \$(4 \times 3 \times 90)$

8. Required answer $= -8x + 9 + 15 - 4x - (-7x + 4 + 5x + 7)$

$= -8x + 9 + 15 - 4x - (-7x + 5x + 4 + 7)$

$= -8x + 9 + 15 - 4x - (-2x + 11)$

$= -8x + 9 + 15 - 4x + 2x - 11$

$= -8x - 4x + 2x + 9 + 15 - 11$

$= -10x + 13$

Chapter 5 Linear Equations and Simple Inequalities

TEACHING NOTES

Suggested Approach

Since many Secondary 1 students are still in the concrete operational stage (according to Piaget), teaching students how to solve linear equations in one variable with the use of algebra discs on a balance can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use this approach in examinations, and partly because they cannot use this approach to solve linear equations which consist of algebraic terms that have large or fractional coefficients (see Section 5.1). After students learn how to solve linear equations, they will learn how to evaluate an unknown in a formula and formulate linear equations to solve problems in real-world contexts. Since the concept of inequality is harder than that of equation, students only learn how to solve inequalities towards the end of this chapter.

Section 5.1: Linear Equations

Students have learnt how to complete mathematical sentences such as $7 + \square = 13$ in primary school. Teachers can introduce equations by telling students that when we replace $\square$ with x, we have $7 + x = 13$, which is an equation. Teachers should illustrate the meaning of 'solving an equation' using appropriate examples. Students should know the difference between linear expressions and linear equations.

Teachers can use the 'Balance Method' to show how to solve linear equations which do not involve any brackets before illustrating how to solve those which involve brackets. As this approach cannot be used to solve linear equations which consist of algebraic terms that have large or fractional coefficients, so there is a need to help students consolidate what they have learnt in Worked Examples 1, 2 and 3. The thinking time on page 115 of the textbook reinforces students' understanding of the concept of equation. For example, since $x + 3 = 6$, $2x + 3 = 9$ and $10x - 4 = 5x + 11$ are equivalent equations that can be obtained from $x = 3$, then the value of x in each of the equations is 3.

Section 5.2: Formulae

Teachers can use simple formulae such as $A = lb$, where A, l and b are the area, the length and the breadth of the rectangle respectively, to let students understand that a formula makes use of variables to write instructions for performing a calculation. Teachers may get students to provide examples of formulae which they have encountered in mathematics and the sciences.

Section 5.3: Applications of Linear Equations in Real-World Contexts

Teachers should illustrate how a word problem is solved using the model method before showing how the same problem can be solved using the algebraic method. Students should observe how the algebraic method is linked to the model method. Also, students should be aware why they need to learn the algebraic method. In this section, students are given ample opportunities to formulate linear equations to solve problems in real-world contexts.

Section 5.4: Simple Inequalities

In the investigation on page 126 of the textbook, students are required to work with numerical examples before generalising the conclusions for some properties of inequalities. In Secondary 1, students only need to know how to solve linear inequalities of the form $ax \leqslant b$, $ax \geqslant b$, $ax < b$ and $ax > b$, where a and b are integers and $a > 0$. Teachers should get students to formulate inequalities based on real-world contexts (see the journal writing on page 128 of the textbook).

WORKED SOLUTIONS

Journal Writing (Page 113)

1. To solve an equation in x means to find the value of x so that the values on both sides of the equation are equal, i.e. x satisfies the equation.

2. The operations should be applied to both sides of the equation such that the equation is simplified to the form $ax = b$, where a and b are constants. Thus $x = \frac{b}{a}$.

Teachers may wish to point out common mistakes that students may make in solving a linear equation in order to extract their understanding of the process.

Thinking Time (Page 115)

Some equivalent equations that have the solution $y = -1$:

- $y = -1$
- $y + 1 = 0$
- $y - 1 = -2$
- $3y + 8 = 5$
- $2y - 1 = -3$
- $2y - 1 = -3$
- $10y + 2 = 13y + 5$
- $2(2y - 3) = 5(y - 1)$

Investigation (Properties of Inequalities)

1.

Cases	Working	Inequality	Is the inequality sign reversed?	Conclusion
Multiplication by a *positive* number on both sides of the inequality $10 > 6$	LHS $= 10 \times 5$ $= 50$ RHS $= 6 \times 5$ $= 30$	$50 > 30$	No	If $x > y$ and $c > 0$, then $cx > cy$.
Division by a *positive* number on both sides of the inequality $10 > 6$	LHS $= 10 \div 5$ $= 2$ RHS $= 6 \div 5$ $= 1.2$	$2 > 1.2$	No	If $x > y$ and $c > 0$, then $\frac{x}{c} > \frac{y}{c}$.

Table 5.3

2. Yes, the conclusions drawn from Table 5.3 apply to $10 \geqslant 6$.

The following conclusions hold for $x \geqslant y$:

- If $x \geqslant y$ and $c > 0$, then $cx \geqslant cy$ and $\frac{x}{c} \geqslant \frac{y}{c}$.

The following conclusions hold for $x < y$:

- If $x < y$ and $c > 0$, then $cx < cy$ and $\frac{x}{c} < \frac{y}{c}$.

The following conclusions hold for $x \leqslant y$:

- If $x \leqslant y$ and $c > 0$, then $cx \leqslant cy$ and $\frac{x}{c} \leqslant \frac{y}{c}$.

Journal Writing (Page 128)

- A bowl of rice contains 5 g of protein. A teenager needs a minimum of 49 g of protein each day. It is given that he only eats rice on a particular day. The inequality which we need to set up to find the least number of bowls of rice he needs to eat in order to meet his minimum protein requirement that day is:
$$5x \geqslant 49,$$
where x represents the number of bowls of rice he needs to eat that day.
- The flag-down fare of a taxi is \$5. The taxi charges \$0.30 for each 385 m it travels. A person has not more than \$50 to spend on his taxi ride. The inequality which we need to set up to find the maximum distance that he can travel on the taxi is:
$$30x \leqslant 4500,$$
where x is the number of blocks of 385 m.

Teachers may wish to note that the list is not exhaustive.

Practise Now (Page 110)

(a)
$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 \\ \therefore x &= 4 \end{aligned}$$

(b)
$$\begin{aligned} x - 7 &= 6 \\ x - 7 + 7 &= 6 + 7 \\ \therefore x &= 13 \end{aligned}$$

(c)
$$\begin{aligned} x + 3 &= -7 \\ x + 3 - 3 &= -7 - 3 \\ \therefore x &= -10 \end{aligned}$$

(d)
$$\begin{aligned} x - 2 &= -3 \\ x - 2 + 2 &= -3 + 2 \\ \therefore x &= -1 \end{aligned}$$

Practise Now (Page 111)

(a)
$$\begin{aligned} 2x - 5 &= 5 \\ 2x - 5 + 5 &= 5 + 5 \\ 2x &= 10 \\ \therefore x &= 5 \end{aligned}$$

(b)
$$\begin{aligned} 3x + 4 &= 7 \\ 3x + 4 - 4 &= 7 - 4 \\ 3x &= 3 \\ \therefore x &= 1 \end{aligned}$$

(c)
$$\begin{aligned} -3x + 3 &= 9 \\ -3x + 3 - 3 &= 9 - 3 \\ -3x &= 6 \\ 3x &= -6 \\ \therefore x &= -2 \end{aligned}$$

(d)
$$\begin{aligned} -5x - 2 &= 13 \\ -5x - 2 + 2 &= 13 + 2 \\ -5x &= 15 \\ 5x &= -15 \\ \therefore x &= -3 \end{aligned}$$

Practise Now (Page 112)

(a)
$$
\begin{aligned}
3x + 4 &= x - 10 \\
3x - x + 4 &= x - x - 10 \\
2x + 4 &= -10 \\
2x + 4 - 4 &= -10 - 4 \\
2x &= -14 \\
\therefore x &= -7
\end{aligned}
$$

(b)
$$
\begin{aligned}
4x - 2 &= x + 7 \\
4x - x - 2 &= x - x + 7 \\
3x - 2 &= 7 \\
3x - 2 + 2 &= 7 + 2 \\
3x &= 9 \\
\therefore x &= 3
\end{aligned}
$$

(c)
$$
\begin{aligned}
3x - 2 &= -x + 14 \\
3x + x - 2 &= -x + x + 14 \\
4x - 2 &= 14 \\
4x - 2 + 2 &= 14 + 2 \\
4x &= 16 \\
\therefore x &= 4
\end{aligned}
$$

(d)
$$
\begin{aligned}
-2x - 5 &= 5x - 12 \\
-2x - 5x - 5 &= 5x - 5x - 12 \\
-7x - 5 &= -12 \\
-7x - 5 + 5 &= -12 + 5 \\
-7x &= -7 \\
7x &= 7 \\
x &= 1
\end{aligned}
$$

Practise Now (Page 113)

(a)
$$
\begin{aligned}
2(x - 3) &= -3x + 4 \\
2x - 6 &= -3x + 4 \\
2x + 3x - 6 &= -3x + 3x + 4 \\
5x - 6 &= 4 \\
5x - 6 + 6 &= 4 + 6 \\
5x &= 10 \\
\therefore x &= 2
\end{aligned}
$$

(b)
$$
\begin{aligned}
2(x + 3) &= 5x - 9 \\
2x + 6 &= 5x - 9 \\
2x - 5x + 6 &= 5x - 5x - 9 \\
-3x + 6 &= -9 \\
-3x + 6 - 6 &= -9 - 6 \\
-3x &= -15 \\
3x &= 15 \\
\therefore x &= 5
\end{aligned}
$$

(c)
$$
\begin{aligned}
-2(x + 2) &= 3x - 9 \\
-2x - 4 &= 3x - 9 \\
-2x - 3x - 4 &= 3x - 3x - 9 \\
-5x - 4 &= -9 \\
-5x - 4 + 4 &= -9 + 4 \\
-5x &= -5 \\
5x &= 5 \\
\therefore x &= 1
\end{aligned}
$$

(d)
$$
\begin{aligned}
-2(3x - 4) &= 4(2x + 5) \\
-6x + 8 &= 8x + 20 \\
-6x - 8x + 8 &= 8x - 8x + 20 \\
-14x + 8 &= 20 \\
-14x + 8 - 8 &= 20 - 8 \\
-14x &= 12 \\
14x &= -12 \\
\therefore x &= -\frac{6}{7}
\end{aligned}
$$

Practise Now 1

1. (a)
$$
\begin{aligned}
x + 9 &= 4 \\
x + 9 - 9 &= 4 - 9 \\
\therefore x &= -5
\end{aligned}
$$

(b)
$$
\begin{aligned}
3x - 2 &= 4 \\
3x - 2 + 2 &= 4 + 2 \\
3x &= 6 \\
\frac{3x}{3} &= \frac{6}{3} \\
\therefore x &= 2
\end{aligned}
$$

(c)
$$
\begin{aligned}
7x + 2 &= 2x - 13 \\
7x - 2x + 2 &= 2x - 2x - 13 \\
5x + 2 &= -13 \\
5x + 2 - 2 &= -13 - 2 \\
5x &= -15 \\
\frac{5x}{5} &= \frac{-15}{5} \\
\therefore x &= -3
\end{aligned}
$$

(d)
$$
\begin{aligned}
3(3y + 4) &= 2(2y + 1) \\
9y + 12 &= 4y + 2 \\
9y - 4y + 12 &= 4y - 4y + 2 \\
5y + 12 &= 2 \\
5y + 12 - 12 &= 2 - 12 \\
5y &= -10 \\
\frac{5y}{5} &= \frac{-10}{5} \\
\therefore y &= -2
\end{aligned}
$$

(e)
$$
\begin{aligned}
2(y - 1) + 3(y - 1) &= 4 - 2y \\
2y - 2 + 3y - 3 &= 4 - 2y \\
2y + 3y - 2 - 3 &= 4 - 2y \\
5y - 5 &= 4 - 2y \\
5y + 2y - 5 &= 4 - 2y + 2y \\
7y - 5 &= 4 \\
7y - 5 + 5 &= 4 + 5 \\
7y &= 9 \\
\frac{7y}{7} &= \frac{9}{7} \\
\therefore y &= 1\frac{2}{7}
\end{aligned}
$$

2. **(a)**
$$x + 0.7 = 2.7$$
$$x + 0.7 - 0.7 = 2.7 - 0.7$$
$$\therefore x = 2$$

(b)
$$2y - 1.3 = 2.8$$
$$2y - 1.3 + 1.3 = 2.8 + 1.3$$
$$2y = 4.1$$
$$\frac{2y}{2} = \frac{4.1}{2}$$
$$\therefore y = 2.05$$

Practise Now 2

(a)
$$\frac{x}{2} + 9 = 5$$
$$\frac{x}{2} + 9 - 9 = 5 - 9$$
$$\frac{x}{2} = -4$$
$$2 \times \frac{x}{2} = 2 \times (-4)$$
$$\therefore x = -8$$

(b)
$$\frac{5}{7}y + 2 = \frac{1}{2}y + 3\frac{1}{4}$$
$$\frac{5}{7}y - \frac{1}{2}y + 2 = \frac{1}{2}y - \frac{1}{2}y + 3\frac{1}{4}$$
$$\frac{3}{14}y + 2 = 3\frac{1}{4}$$
$$\frac{3}{14}y + 2 - 2 = 3\frac{1}{4} - 2$$
$$\frac{3}{14}y = 1\frac{1}{4}$$
$$\frac{14}{3} \times \frac{3}{14}y = \frac{14}{3} \times 1\frac{1}{4}$$
$$\therefore y = 5\frac{5}{6}$$

(c)
$$\frac{3z - 1}{2} = \frac{z - 4}{3}$$
$$6 \times \frac{3z - 1}{2} = 6 \times \frac{z - 4}{3}$$
$$3(3z - 1) = 2(z - 4)$$
$$9z - 3 = 2z - 8$$
$$9z - 2z - 3 = 2z - 2z - 8$$
$$7z - 3 = -8$$
$$7z - 3 + 3 = -8 + 3$$
$$7z = -5$$
$$\frac{7z}{7} = -\frac{5}{7}$$
$$\therefore z = -\frac{5}{7}$$

Practise Now 3

(a)
$$\frac{8}{2x - 3} = 4$$
$$(2x - 3) \times \frac{8}{2x - 3} = (2x - 3) \times 4$$
$$8 = 4(2x - 3)$$
$$8 = 8x - 12$$
$$8x - 12 = 8$$
$$8x - 12 + 12 = 8 + 12$$
$$8x = 20$$
$$\frac{8x}{8} = \frac{20}{8}$$
$$\therefore x = 2\frac{1}{2}$$

(b)
$$\frac{y - 3}{y + 4} = \frac{3}{2}$$
$$2(y + 4) \times \frac{y - 3}{y + 4} = 2(y + 4) \times \frac{3}{2}$$
$$2(y - 3) = 3(y + 4)$$
$$2y - 6 = 3y + 12$$
$$2y - 3y - 6 = 3y - 3y + 12$$
$$-y - 6 = 12$$
$$-y - 6 + 6 = 12 + 6$$
$$-y = 18$$
$$\therefore y = -18$$

Practise Now 4

$F = ma$

(a) When $m = 1000$, $a = 0.05$,
$F = 1000(0.05)$
$= 50$ N
Net force acting on body = 50 N

(b) When $F = 100$, $a = 0.1$,
$100 = m(0.1)$
$$\therefore m = \frac{100}{0.1}$$
$= 1000$ kg
Mass of body = 1000 kg

Practise Now 5

1. $\dfrac{2x+y-3z}{y+3x} = \dfrac{x}{2y}$

When $x = 1$, $y = 4$,

$$\frac{2(1)+4-3z}{4+3(1)} = \frac{1}{2(4)}$$
$$\frac{2+4-3z}{4+3} = \frac{1}{8}$$
$$\frac{6-3z}{7} = \frac{1}{8}$$
$$8(6-3z) = 7$$
$$48-24z = 7$$
$$-24z = 7-48$$
$$-24z = -41$$
$$\therefore z = \frac{-41}{-24}$$
$$= 1\frac{17}{24}$$

2. $t = \dfrac{v-u}{a}$

When $t = 3$, $v = 2\frac{1}{2}$, $u = 1\frac{1}{3}$,

$$3 = \frac{2\frac{1}{2}-1\frac{1}{3}}{a}$$
$$3 = \frac{1\frac{1}{6}}{a}$$
$$3a = 1\frac{1}{6}$$
$$\therefore a = 1\frac{1}{6} \div 3$$
$$= \frac{7}{18}$$

Practise Now 6

(i) $A = \dfrac{1}{2}\pi r^2$

(ii) When $r = 5$,

$$A = \frac{1}{2}(3.142)(5)^2$$
$$= 39.275 \text{ cm}^2$$

Area of semicircle = 39.275 cm^2

Practise Now 7

1. Let the smaller number be x.
Then the larger number is $5x$.

$$x + 5x = 24$$
$$6x = 24$$
$$x = \frac{24}{6}$$
$$= 4$$

$\therefore$ The two numbers are 4 and 20.

2. Let the number of marks Lixin obtains be x.
Then the number of marks Devi obtains is $x + 15$.

$$x + 15 = 2x$$
$$x - 2x = -15$$
$$-x = -15$$
$$\therefore x = 15$$

Lixin obtains 15 marks.

Practise Now 8

Let the number be x.

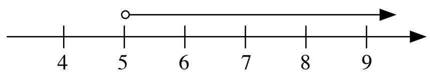
$$\frac{1}{5}x + 3\frac{7}{10} = 7$$
$$\frac{1}{5}x = 7 - 3\frac{7}{10}$$
$$\frac{1}{5}x = 3\frac{3}{10}$$
$$\therefore x = 5 \times 3\frac{3}{10}$$
$$= 16\frac{1}{2}$$

The number is $16\frac{1}{2}$.

Practise Now 9

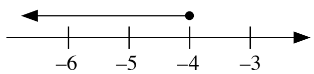
1. (a) $15x > 75$

$$x > \frac{75}{15}$$
$$\therefore x > 5$$



(b) $4x \leqslant -16$

$$x \leqslant \frac{-16}{4}$$
$$x \leqslant -4$$



2. $6x > 7$

$$x > \frac{7}{6}$$
$$x > 1\frac{1}{6}$$

$\therefore$ The smallest integer value of x is 2.

Practise Now 10

Let the number of buses that are needed to ferry 520 people be x.
Then $45x \geqslant 520$

$$x \geqslant \frac{520}{45}$$
$$x \geqslant 11\frac{5}{9}$$

$\therefore$ The minimum number of buses that are needed to ferry 520 students is 12.

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Exercise 5A

1. **(a)** $x + 8 = 15$

$x + 8 - 8 = 15 - 8$

$\therefore x = 7$

(b) $x + 9 = -5$

$x + 9 - 9 = -5 - 9$

$\therefore x = -14$

(c) $x - 5 = 17$

$x - 5 + 5 = 17 + 5$

$\therefore x = 22$

(d) $y - 7 = -3$

$y - 7 + 7 = -3 + 7$

$\therefore y = 4$

(e) $y + 0.4 = 1.6$

$y + 0.4 - 0.4 = 1.6 - 0.4$

$\therefore y = 1.2$

(f) $y - 2.4 = 3.6$

$y - 2.4 + 2.4 = 3.6 + 2.4$

$y = 6$

(g) $-2.7 + a = -6.4$

$-2.7 + 2.7 + a = -6.4 + 2.7$

$\therefore a = -3.7$

2. **(a)** $4x = -28$

$$\frac{4x}{4} = \frac{-28}{4}$$

$\therefore x = -7$

(b) $-24x = -144$

$24x = 144$

$$\frac{24x}{24} = \frac{144}{24}$$

$\therefore x = 6$

(c) $3x - 4 = 11$

$3x - 4 + 4 = 11 + 4$

$3x = 15$

$$\frac{3x}{3} = \frac{15}{3}$$

$\therefore x = 5$

(d) $9x + 4 = 31$

$9x + 4 - 4 = 31 - 4$

$9x = 27$

$$\frac{9x}{9} = \frac{27}{9}$$

$\therefore x = 3$

(e) $12 - 7x = 5$

$12 - 12 - 7x = 5 - 12$

$-7x = -7$

$7x = 7$

$$\frac{7x}{7} = \frac{7}{7}$$

$\therefore x = 1$

(f) $3 - 7y = -18$

$3 - 3 - 7y = -18 - 3$

$-7y = -21$

$7y = 21$

$$\frac{7y}{7} = \frac{21}{7}$$

$\therefore y = 3$

(g) $4y - 1.9 = 6.3$

$4y - 1.9 + 1.9 = 6.3 + 1.9$

$4y = 8.2$

$$\frac{4y}{4} = \frac{8.2}{4}$$

$\therefore y = 2.05$

(h) $-3y - 7.8 = -9.6$

$-3y - 7.8 + 7.8 = -9.6 + 7.8$

$-3y = -1.8$

$3y = 1.8$

$$\frac{3y}{3} = \frac{1.8}{3}$$

$\therefore y = 0.6$

(i) $7y - 2\frac{3}{4} = \frac{1}{2}$

$$7y - 2\frac{3}{4} + 2\frac{3}{4} = \frac{1}{2} + 2\frac{3}{4}$$

$$7y = 3\frac{1}{4}$$

$$\frac{7y}{7} = 3\frac{1}{4} \div 7$$

$$\therefore y = \frac{13}{28}$$

(j) $1\frac{1}{2} - 2y = \frac{1}{4}$

$$1\frac{1}{2} - 1\frac{1}{2} - 2y = \frac{1}{4} - 1\frac{1}{2}$$

$$-2y = -1\frac{1}{4}$$

$$2y = 1\frac{1}{4}$$

$$\frac{2y}{2} = 1\frac{1}{4} \div 2$$

$$\therefore y = \frac{5}{8}$$

3. **(a)** $3x - 7 = 4 - 8x$

$3x + 8x - 7 = 4 - 8x + 8x$

$11x - 7 = 4$

$11x - 7 + 7 = 4 + 7$

$11x = 11$

$$\frac{11x}{11} = \frac{11}{11}$$

$\therefore x = 1$

(b) $4x - 10 = 5x + 7$

$4x - 5x - 10 = 5x - 5x + 7$

$-x - 10 = 7$

$-x - 10 + 10 = 7 + 10$

$-x = 17$

$$\therefore x = -17$$

(c) $30 + 7y = -2y - 6$

$$30 + 7y + 2y = -2y + 2y - 6$$
$$30 + 9y = -6$$
$$30 - 30 + 9y = -6 - 30$$
$$9y = -36$$
$$\frac{9y}{9} = \frac{-36}{9}$$
$$\therefore y = -4$$

(d) $2y - 7 = 7y - 27$

$$2y - 7y - 7 = 7y - 7y - 27$$
$$-5y - 7 = -27$$
$$-5y - 7 + 7 = -27 + 7$$
$$-5y = -20$$
$$5y = 20$$
$$\frac{5y}{5} = \frac{20}{5}$$
$$\therefore y = 4$$

4. **(a)** $2(x + 3) = 8$

$$2x + 6 = 8$$
$$2x + 6 - 6 = 8 - 6$$
$$2x = 2$$
$$\frac{2x}{2} = \frac{2}{2}$$
$$\therefore x = 1$$

(b) $5(x - 7) = -15$

$$5x - 35 = -15$$
$$5x - 35 + 35 = -15 + 35$$
$$5x = 20$$
$$\frac{5x}{5} = \frac{20}{5}$$
$$\therefore x = 4$$

(c) $7(-2x + 4) = -4x$

$$-14x + 28 = -4x$$
$$-14x + 4x + 28 = -4x + 4x$$
$$-10x + 28 = 0$$
$$-10x + 28 - 28 = 0 - 28$$
$$-10x = -28$$
$$10x = 28$$
$$\frac{10x}{10} = \frac{28}{10}$$
$$\therefore x = 2\frac{4}{5}$$

(d) $3(2 - 0.4x) = 18$

$$6 - 1.2x = 18$$
$$6 - 6 - 1.2x = 18 - 6$$
$$-1.2x = 12$$
$$1.2x = -12$$
$$\frac{1.2x}{1.2} = \frac{-12}{1.2}$$

$$\therefore x = -10$$

(e) $2(2x - 2.2) = 4.6$

$$4x - 4.4 = 4.6$$
$$4x - 4.4 + 4.4 = 4.6 + 4.4$$
$$4x = 9$$
$$\frac{4x}{4} = \frac{9}{4}$$
$$\therefore x = 2.25$$

(f) $4(3y + 4.1) = 7.6$

$$12y + 16.4 = 7.6$$
$$12y + 16.4 - 16.4 = 7.6 - 16.4$$
$$12y = -8.8$$
$$\frac{12y}{12} = \frac{-8.8}{12}$$
$$\therefore y = -\frac{11}{15}$$

(g) $3(2y + 3) = 4y + 3$

$$6y + 9 = 4y + 3$$
$$6y - 4y + 9 = 4y - 4y + 3$$
$$2y + 9 = 3$$
$$2y + 9 - 9 = 3 - 9$$
$$2y = -6$$
$$\frac{2y}{2} = \frac{-6}{2}$$
$$\therefore y = -3$$

(h) $3(y + 1) = 4y - 21$

$$3y + 3 = 4y - 21$$
$$3y - 4y + 3 = 4y - 4y - 21$$
$$-y + 3 = -21$$
$$-y + 3 - 3 = -21 - 3$$
$$-y = -24$$
$$\therefore y = 24$$

(i) $3(y + 2) = 2(y + 4)$

$$3y + 6 = 2y + 8$$
$$3y - 2y + 6 = 2y - 2y + 8$$
$$y + 6 = 8$$
$$y + 6 - 6 = 8 - 6$$
$$\therefore y = 2$$

(j) $5(5y - 6) = 4(y - 7)$

$$25y - 30 = 4y - 28$$
$$25y - 4y - 30 = 4y - 4y - 28$$
$$21y - 30 = -28$$
$$21y - 30 + 30 = -28 + 30$$
$$21y = 2$$
$$\frac{21y}{21} = \frac{2}{21}$$
$$\therefore y = \frac{2}{21}$$

(k) $2(3b - 4) = 5(b + 6)$

$$6b - 8 = 5b + 30$$
$$6b - 5b - 8 = 5b - 5b + 30$$
$$b - 8 = 30$$
$$b - 8 + 8 = 30 + 8$$

$$\therefore b = 38$$

(l) $3(2c + 5) = 4(c - 3)$

$$6c + 15 = 4c - 12$$
$$6c - 4c + 15 = 4c - 4c - 12$$
$$2c + 15 = -12$$
$$2c + 15 - 15 = -12 - 15$$
$$2c = -27$$
$$\frac{2c}{2} = \frac{-27}{2}$$
$$\therefore c = -13\frac{1}{2}$$

(m) $9(2d + 7) = 11(d + 14)$

$$18d + 63 = 11d + 154$$
$$18d - 11d + 63 = 11d - 11d + 154$$
$$7d + 63 = 154$$
$$7d + 63 - 63 = 154 - 63$$
$$7d = 91$$
$$\frac{7d}{7} = \frac{91}{7}$$
$$\therefore d = 13$$

(n) $5(7f - 3) = 28(f - 1)$

$$35f - 15 = 28f - 28$$
$$35f - 28f - 15 = 28f - 28f - 28$$
$$7f - 15 = -28$$
$$7f - 15 + 15 = -28 + 15$$
$$7f = -13$$
$$\frac{7f}{7} = \frac{-13}{7}$$
$$\therefore f = -1\frac{6}{7}$$

5. (a) $\frac{1}{3}x = 7$

$$3 \times \frac{1}{3}x = 3 \times 7$$
$$\therefore x = 21$$

(b) $\frac{3}{4}x = -6$

$$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times (-6)$$
$$\therefore x = -8$$

(c) $\frac{1}{3}x + 3 = 4$

$$\frac{1}{3}x + 3 - 3 = 4 - 3$$
$$\frac{1}{3}x = 1$$
$$3 \times \frac{1}{3}x = 3 \times 1$$
$$\therefore x = 3$$

(d) $\frac{y}{4} - 8 = -2$

$$\frac{y}{4} - 8 + 8 = -2 + 8$$
$$\frac{y}{4} = 6$$
$$4 \times \frac{y}{4} = 4 \times 6$$
$$\therefore y = 24$$

(e) $3 - \frac{1}{4}y = 2$

$$3 - 3 - \frac{1}{4}y = 2 - 3$$
$$-\frac{1}{4}y = -1$$
$$\frac{1}{4}y = 1$$
$$4 \times \frac{1}{4}y = 4 \times 1$$
$$\therefore y = 4$$

(f) $15 - \frac{2}{5}y = 11$

$$15 - 15 - \frac{2}{5}y = 11 - 15$$
$$-\frac{2}{5}y = -4$$
$$\frac{2}{5}y = 4$$
$$\frac{5}{2} x \frac{2}{5}y = \frac{5}{2} \times 4$$
$$\therefore y = 10$$

6. (a) $x = 12 - \frac{1}{3}x$

$$x + \frac{1}{3}x = 12 - \frac{1}{3}x + \frac{1}{3}x$$
$$\frac{4}{3}x = 12$$
$$\frac{3}{4} \times \frac{4}{3}x = \frac{3}{4} \times 12$$
$$\therefore x = 9$$

(b) $\frac{3}{5}x = \frac{1}{2}x + \frac{1}{2}$

$$\frac{3}{5}x - \frac{1}{2}x = \frac{1}{2}x - \frac{1}{2}x + \frac{1}{2}$$
$$\frac{1}{10}x = \frac{1}{2}$$
$$10 \times \frac{1}{10}x = 10 \times \frac{1}{2}$$
$$\therefore x = 5$$

(c) $$\frac{y}{2} - \frac{1}{5} = 2 - \frac{y}{3}$$
$$\frac{y}{2} + \frac{y}{3} - \frac{1}{5} = 2 - \frac{y}{3} + \frac{y}{3}$$
$$\frac{5y}{6} - \frac{1}{5} = 2$$
$$\frac{5y}{6} - \frac{1}{5} + \frac{1}{5} = 2 + \frac{1}{5}$$
$$\frac{5y}{6} = 2\frac{1}{5}$$
$$\frac{6}{5} \times \frac{5y}{6} = \frac{6}{5} \times 2\frac{1}{5}$$
$$\therefore y = 2\frac{16}{25}$$

(d) $$\frac{2}{3}y - \frac{3}{4} = 2y + \frac{5}{8}$$
$$\frac{2}{3}y - 2y - \frac{3}{4} = 2y - 2y + \frac{5}{8}$$
$$-\frac{4}{3}y - \frac{3}{4} = \frac{5}{8}$$
$$-\frac{4}{3}y - \frac{3}{4} + \frac{3}{4} = \frac{5}{8} + \frac{3}{4}$$
$$-\frac{4}{3}y = 1\frac{3}{8}$$
$$\frac{4}{3}y = -1\frac{3}{8}$$
$$\frac{3}{4} \times \frac{4}{3}y = \frac{3}{4} \times \left(-1\frac{3}{8}\right)$$
$$\therefore y = -1\frac{1}{32}$$

7. **(a)** $$\frac{2}{x} = \frac{4}{5}$$
$$5x \times \frac{2}{x} = 5x \times \frac{4}{5}$$
$$10 = 4x$$
$$4x = 10$$
$$\frac{4x}{4} = \frac{10}{4}$$
$$\therefore x = 2\frac{1}{2}$$

(b) $$\frac{12}{y-1} = \frac{2}{3}$$
$$3(y-1) \times \frac{12}{y-1} = 3(y-1) \times \frac{2}{3}$$
$$36 = 2(y-1)$$
$$36 = 2y - 2$$
$$2y - 2 = 36$$
$$2y - 2 + 2 = 36 + 2$$
$$2y = 38$$
$$\frac{2y}{2} = \frac{38}{2}$$
$$\therefore y = 19$$

8. **(a)** $$-3(2 - x) = 6x$$
$$-6 + 3x = 6x$$
$$-6 + 3x - 6x = 6x - 6x$$
$$-6 - 3x = 0$$
$$-6 + 6 - 3x = 0 + 6$$
$$-3x = 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$\therefore x = -2$$

(b) $$5 - 3x = -6(x + 2)$$
$$5 - 3x = -6x - 12$$
$$5 - 3x + 6x = -6x + 6x - 12$$
$$5 + 3x = -12$$
$$5 - 5 + 3x = -12 - 5$$
$$3x = -17$$
$$\frac{3x}{3} = \frac{-17}{3}$$
$$\therefore x = -5\frac{2}{3}$$

(c) $$-3(9y + 2) = 2(-4y - 7)$$
$$-27y - 6 = -8y - 14$$
$$-27y + 8y - 6 = -8y + 8y - 14$$
$$-19y - 6 = -14$$
$$-19y - 6 + 6 = -14 + 6$$
$$-19y = -8$$
$$19y = 8$$
$$\frac{19y}{19} = \frac{8}{19}$$
$$\therefore y = \frac{8}{19}$$

(d) $$-3(4y - 5) = -7(-5 - 2y)$$
$$-12y + 15 = 35 + 14y$$
$$-12y - 14y + 15 = 35 + 14y - 14y$$
$$-26y + 15 = 35$$
$$-26y + 15 - 15 = 35 - 15$$
$$-26y = 20$$
$$26y = -20$$
$$\frac{26y}{26} = \frac{-20}{26}$$
$$\therefore y = -\frac{10}{13}$$

(e) $$3(5 - h) - 2(h - 2) = -1$$
$$15 - 3h - 2h + 4 = -1$$
$$15 + 4 - 3h - 2h = -1$$
$$19 - 5h = -1$$
$$19 - 19 - 5h = -1 - 19$$
$$-5h = -20$$
$$5h = 20$$
$$\frac{5h}{5} = \frac{20}{5}$$
$$\therefore h = 4$$

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9. (a)

$$\frac{5x+1}{3} = 7$$
$$3 \times \frac{5x+1}{3} = 3 \times 7$$
$$5x + 1 = 21$$
$$5x + 1 - 1 = 21 - 1$$
$$5x = 20$$
$$\frac{5x}{5} = \frac{20}{5}$$
$$\therefore x = 4$$

(b)

$$\frac{2x-3}{4} = \frac{x-3}{3}$$
$$12 \times \frac{2x-3}{4} = 12 \times \frac{x-3}{3}$$
$$3(2x-3) = 4(x-3)$$
$$6x - 9 = 4x - 12$$
$$6x - 4x - 9 = 4x - 4x - 12$$
$$2x - 9 = -12$$
$$2x - 9 + 9 = -12 + 9$$
$$2x = -3$$
$$\frac{2x}{2} = \frac{-3}{2}$$
$$\therefore x = -1\frac{1}{2}$$

(c)

$$\frac{3x-1}{5} = \frac{x-1}{3}$$
$$15 \times \frac{3x-1}{5} = 15 \times \frac{x-1}{3}$$
$$3(3x-1) = 5(x-1)$$
$$9x - 3 = 5x - 5$$
$$9x - 5x - 3 = 5x - 5x - 5$$
$$4x - 3 = -5$$
$$4x - 3 + 3 = -5 + 3$$
$$4x = -2$$
$$\frac{4x}{4} = \frac{-2}{4}$$
$$\therefore x = -\frac{1}{2}$$

(d)

$$\frac{1}{4}(5y+4) = \frac{1}{3}(2y-1)$$
$$12 \times \frac{1}{4}(5y+4) = 12 \times \frac{1}{3}(2y-1)$$
$$3(5y+4) = 4(2y-1)$$
$$15y + 12 = 8y - 4$$
$$15y - 8y + 12 = 8y - 8y - 4$$
$$7y + 12 = -4$$
$$7y + 12 - 12 = -4 - 12$$
$$7y = -16$$
$$\frac{7y}{7} = \frac{-16}{7}$$
$$\therefore y = -2\frac{2}{7}$$

(e)

$$\frac{2y-1}{5} - \frac{y+3}{7} = 0$$
$$\frac{2y-1}{5} - \frac{y+3}{7} + \frac{y+3}{7} = 0 + \frac{y+3}{7}$$
$$\frac{2y-1}{5} = \frac{y+3}{7}$$
$$35 \times \frac{2y-1}{5} = 35 \times \frac{y+3}{7}$$
$$7(2y-1) = 5(y+3)$$
$$14y - 7 = 5y + 15$$
$$14y - 5y - 7 = 5y - 5y + 15$$
$$9y - 7 = 15$$
$$9y - 7 + 7 = 15 + 7$$
$$9y = 22$$
$$\frac{9y}{9} = \frac{22}{9}$$
$$\therefore y = 2\frac{4}{9}$$

(f)

$$\frac{2y+3}{4} + \frac{y-5}{6} = 0$$
$$\frac{2y+3}{4} + \frac{y-5}{6} - \frac{y-5}{6} = 0 - \frac{5-y}{6}$$
$$\frac{2y+3}{4} = -\frac{5-y}{6}$$
$$\frac{2y+3}{4} = \frac{y-5}{6}$$
$$12 \times \frac{2y+3}{4} = 12 \times \frac{y-5}{6}$$
$$3(2y+3) = 2(5-y)$$
$$6y + 9 = 10 - 2y$$
$$6y + 2y + 9 = 10 - 2y + 2y$$
$$8y + 9 = 10$$
$$8y + 9 - 9 = 10 - 9$$
$$8y = 1$$
$$\frac{8y}{8} = \frac{1}{8}$$
$$\therefore y = \frac{1}{8}$$

10. (a)

$$\frac{12}{x+3} = 2$$
$$(x+3) \times \frac{12}{x+3} = (x+3) \times 2$$
$$12 = 2(x+3)$$
$$12 = 2x + 6$$
$$2x + 6 = 12$$
$$2x + 6 - 6 = 12 - 6$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$\therefore x = 3$$

(b)
$$\frac{11}{2x-1} = 4$$
$$(2x-1)\times\frac{11}{2x-1} = (2x-1)\times 4$$
$$11 = 4(2x-1)$$
$$11 = 8x-4$$
$$8x-4 = 11$$
$$8x-4+4 = 11+4$$
$$8x = 15$$
$$\frac{8x}{8} = \frac{15}{8}$$
$$\therefore\ x = 1\frac{7}{8}$$

(c)
$$\frac{32}{2x-5} - 3 = \frac{1}{4}$$
$$\frac{32}{2x-5} - 3 + 3 = \frac{1}{4} + 3$$
$$\frac{32}{2x-5} = \frac{13}{4}$$
$$4(2x-5)\times\frac{32}{2x-5} = 4(2x-5)\times\frac{13}{4}$$
$$128 = 13(2x-5)$$
$$128 = 26x-65$$
$$26x-65 = 128$$
$$26x-65+65 = 128+65$$
$$26x = 193$$
$$\frac{26x}{26} = \frac{193}{26}$$
$$\therefore\ x = 7\frac{11}{26}$$

(d)
$$\frac{1}{2} = \frac{1}{x+2} - 1$$
$$\frac{1}{x+2} - 1 = \frac{1}{2}$$
$$\frac{1}{x+2} - 1 + 1 = \frac{1}{2} + 1$$
$$\frac{1}{x+2} = \frac{3}{2}$$
$$2(x+2)\times\frac{1}{x+2} = 2(x+2)\times\frac{3}{2}$$
$$2 = 3(x+2)$$
$$2 = 3x+6$$
$$3x+6 = 2$$
$$3x+6-6 = 2-6$$
$$3x = -4$$
$$\frac{3x}{3} = \frac{-4}{3}$$
$$\therefore\ x = -1\frac{1}{3}$$

(e)
$$\frac{y+5}{y-6} = \frac{5}{4}$$
$$4(y-6)\times\frac{y+5}{y-6} = 4(y-6)\times\frac{5}{4}$$
$$4(y+5) = 5(y-6)$$
$$4y+20 = 5y-30$$
$$4y-5y+20 = 5y-5y-30$$
$$-y+20 = -30$$
$$-y+20-20 = -30-20$$
$$-y = -50$$
$$\therefore\ y = 50$$

(f)
$$\frac{2y+1}{3y-5} = \frac{4}{7}$$
$$7(3y-5)\times\frac{2y+1}{3y-5} = 7(3y-5)\times\frac{4}{7}$$
$$7(2y+1) = 4(3y-5)$$
$$14y+7 = 12y-20$$
$$14y-12y+7 = 12y-12y-20$$
$$2y+7 = -20$$
$$2y+7-7 = -20-7$$
$$2y = -27$$
$$\frac{2y}{2} = \frac{-27}{2}$$
$$\therefore\ y = -13\frac{1}{2}$$

(g)
$$\frac{2}{y-2} = \frac{3}{y+6}$$
$$(y-2)(y+6)\times\frac{2}{y-2} = (y-2)(y+6)\times\frac{3}{y+6}$$
$$2(y+6) = 3(y-2)$$
$$2y+12 = 3y-6$$
$$2y-3y+12 = 3y-3y-6$$
$$-y+12 = -6$$
$$-y+12-12 = -6-12$$
$$-y = -18$$
$$\therefore\ y = 18$$

(h)
$$\frac{2}{7y-3} = \frac{3}{9y-5}$$
$$(7y-3)(9y-5)\times\frac{2}{7y-3} = (7y-3)(9y-5)\times\frac{3}{9y-5}$$
$$2(9y-5) = 3(7y-3)$$
$$18y-10 = 21y-9$$
$$18y-21y-10 = 21y-21y-9$$
$$-3y-10 = -9$$
$$-3y-10+10 = -9+10$$
$$-3y = 1$$
$$3y = 1$$
$$\frac{3y}{3} = \frac{-1}{3}$$
$$\therefore\ y = -\frac{1}{3}$$

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11. (a)

$$10x - \frac{5x+4}{3} = 7$$
$$\frac{3(10x)-(5x+4)}{3} = 7$$
$$\frac{30x-5x-4}{3} = 7$$
$$\frac{25x-4}{3} = 7$$
$$3 \times \frac{25x-4}{3} = 3 \times 7$$
$$25x - 4 = 21$$
$$25x - 4 + 4 = 21 + 4$$
$$25x = 25$$
$$\frac{25x}{25} = \frac{25}{25}$$
$$\therefore\ x = 1$$

(b)

$$\frac{4x}{3} - \frac{x-1}{2} = 1\frac{1}{4}$$
$$\frac{2(4x)-3(x-1)}{6} = \frac{5}{4}$$
$$\frac{8x-3x+3}{6} = \frac{5}{4}$$
$$\frac{5x+3}{6} = \frac{5}{4}$$
$$12 \times \frac{5x+3}{6} = 12 \times \frac{5}{4}$$
$$2(5x+3) = 15$$
$$10x + 6 = 15$$
$$10x + 6 - 6 = 15 - 6$$
$$10x = 9$$
$$\frac{10x}{10} = \frac{9}{10}$$
$$\therefore\ x = \frac{9}{10}$$

(c)

$$\frac{x-1}{3} - \frac{x+3}{4} = -1$$
$$\frac{4(x-1)-3(x+3)}{12} = -1$$
$$\frac{4x-4-3x-9}{12} = -1$$
$$\frac{4x-3x-4-9}{12} = -1$$
$$\frac{x-13}{12} = -1$$
$$12 \times \frac{x-13}{12} = 12 \times (-1)$$
$$x - 13 = -12$$
$$x - 13 + 13 = -12 + 13$$
$$\therefore\ x = 1$$

(d)

$$1 - \frac{y+5}{3} = \frac{3(y-1)}{4}$$
$$\frac{3-(y+5)}{3} = \frac{3(y-1)}{4}$$
$$\frac{3-y-5}{3} = \frac{3(y-1)}{4}$$
$$\frac{-y+3-5}{3} = \frac{3(y-1)}{4}$$
$$\frac{-y-2}{3} = \frac{3(y-1)}{4}$$
$$12 \times \frac{-y-2}{3} = 12 \times \frac{3(y-1)}{4}$$
$$4(-y-2) = 9(y-1)$$
$$-4y - 8 = 9y - 9$$
$$-4y - 9y - 8 = 9y - 9y - 9$$
$$-13y - 8 = -9$$
$$-13y - 8 + 8 = -9 + 8$$
$$-13y = -1$$
$$13y = 1$$
$$\frac{13y}{13} = \frac{1}{13}$$
$$\therefore\ y = \frac{1}{13}$$

(e)

$$\frac{6(y-2)}{7} - 12 = \frac{2(y-7)}{3}$$
$$\frac{6(y-2)-84}{7} = \frac{2(y-7)}{3}$$
$$\frac{6y-12-84}{7} = \frac{2(y-7)}{3}$$
$$\frac{6y-96}{7} = \frac{2(y-7)}{3}$$
$$21 \times \frac{6y-96}{7} = 21 \times \frac{2(y-7)}{3}$$
$$3(6y-96) = 14(y-7)$$
$$18y - 288 = 14y - 98$$
$$18y - 14y - 288 = 14y - 14y - 98$$
$$4y - 288 = -98$$
$$4y - 288 + 288 = -98 + 288$$
$$4y = 190$$
$$\frac{4y}{4} = \frac{190}{4}$$
$$\therefore\ y = 47\frac{1}{2}$$

(f) $$\frac{7-2y}{2} - \frac{2}{5}(2-y) = 1\frac{1}{4}$$
$$\frac{5(7-2y)-4(2-y)}{10} = \frac{5}{4}$$
$$\frac{35-10y-8+4y}{10} = \frac{5}{4}$$
$$\frac{-10y+4y+35-8}{10} = \frac{5}{4}$$
$$\frac{-6y+27}{10} = \frac{5}{4}$$
$$20 \times \frac{-6y+27}{10} = 20 \times \frac{5}{4}$$
$$2(-6y+27) = 25$$
$$-12y+54 = 25$$
$$-12y+54-54 = 25-54$$
$$-12y = -29$$
$$12y = 29$$
$$\frac{12y}{12} = \frac{29}{12}$$
$$\therefore y = 2\frac{5}{12}$$

12. When $x = \frac{19}{20}$,

$$\text{LHS} = 2\left(\frac{19}{20}\right) - \frac{3}{4}$$
$$= 1\frac{9}{10} - \frac{3}{4}$$
$$= 1\frac{3}{20}$$
$$\text{RHS} = \frac{1}{3}\left(\frac{19}{20}\right) + \frac{5}{6}$$
$$= \frac{19}{60} + \frac{5}{6}$$
$$= 1\frac{3}{20} = \text{LHS}$$

$\therefore x = \frac{19}{20}$ is the solution of the equation

$2x - \frac{3}{4} = \frac{1}{3}x + \frac{5}{6}$.

13. $$4x + y = 3x + 5y$$
$$4x - 3x + y = 3x - 3x + 5y$$
$$x + y = 5y$$
$$x + y - y = 5y - y$$
$$x = 4y$$
$$\frac{3}{16y} \times x = \frac{3}{16y} \times 4y$$
$$\therefore \frac{3x}{16y} = \frac{3}{4}$$

14. $$\frac{3x-5y}{7x-4y} = \frac{3}{4}$$
$$4(7x-4y) \times \frac{3x-5y}{7x-4y} = 4(7x-4y) \times \frac{3}{4}$$
$$4(3x-5y) = 3(7x-4y)$$
$$12x - 20y = 21x - 12y$$
$$12x - 21x - 20y = 21x - 21x - 12y$$
$$-9x - 20y = -12y$$
$$-9x - 20y + 20y = -12y + 20y$$
$$-9x = 8y$$
$$9x = -8y$$
$$\frac{9x}{9} = \frac{-8y}{9}$$
$$x = -\frac{8y}{9}$$
$$\frac{1}{y} \times x = \frac{1}{y} \times \left(-\frac{8y}{9}\right)$$
$$\therefore \frac{x}{y} = -\frac{8}{9}$$

Exercise 5B

1. $y = \frac{3}{5}x + 26$

When $x = 12$,

$$y = \frac{3}{5}(12) + 26$$
$$= 33\frac{1}{5}$$

2. $a = \frac{y^2 - xz}{5}$

When $x = 2$, $y = -1$, $z = -3$,

$$a = \frac{(-1)^2 - 2(-3)}{5}$$
$$= \frac{1+6}{5}$$
$$= \frac{7}{5}$$
$$= 1\frac{2}{5}$$

3. $S = 4\pi r^2$

(i) When $r = 10\frac{1}{2}$,

$$S = 4\left(\frac{22}{7}\right)\left(10\frac{1}{2}\right)^2$$
$$= 1386$$

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(ii) When $S = 616$,

$$616 = 4\left(\frac{22}{7}\right)r^2$$

$$616 = \frac{88}{7}r^2$$

$$\frac{88}{7}r^2 = 616$$

$$r^2 = \frac{7}{88} \times 616$$

$$r^2 = 49$$

$$\therefore r = \pm\sqrt{49}$$

$$= \pm 7$$

$$= 7 \text{ or } -7 \text{ (N.A. since } r > 0)$$

4. $A = \frac{1}{2}bh$

(i) When $b = 20$, $h = 45$,

$$A = \frac{1}{2}(20)(45)$$

$$= 450 \text{ cm}^2$$

Area of triangle = 450 cm^2

(ii) When $A = 30$, $b = 10$,

$$30 = \frac{1}{2}(10)h$$

$$30 = 5h$$

$$5h = 30$$

$$\therefore h = 6$$

Height of triangle = 6 cm

5. **(a)** $P = xyz$

(b) $S = p^2 + q^3$

(c) $A = \dfrac{m + n + p + q}{4}$

(d) $T = 60a + b$

6. $k = \dfrac{p + 2q}{3}$

When $k = 7$, $q = 9$,

$$7 = \frac{p + 2(9)}{3}$$

$$7 = \frac{p + 18}{3}$$

$$3 \times 7 = p + 18$$

$$21 = p + 18$$

$$p + 18 = 21$$

$$\therefore p = 21 - 18$$

$$= 3$$

7. $U = \pi(r + h)$

When $U = 16\frac{1}{2}$, $h = 2$,

$$16\frac{1}{2} = \frac{22}{7}(r + 2)$$

$$\frac{22}{7}(r + 2) = 16\frac{1}{2}$$

$$r + 2 = \frac{7}{22} \times 16\frac{1}{2}$$

$$r + 2 = 5\frac{1}{4}$$

$$\therefore r = 5\frac{1}{4} - 2$$

$$= 3\frac{1}{4}$$

8. $v^2 = u^2 + 2gs$

When $v = 25$, $u = 12$, $g = 10$,

$$25^2 = 12^2 + 2(10)s$$

$$625 = 144 + 20s$$

$$144 + 20s = 625$$

$$20s = 625 - 144$$

$$20s = 481$$

$$\therefore s = \frac{481}{20}$$

$$= 24\frac{1}{20}$$

9. $\dfrac{a}{b} - d = \dfrac{2c}{b}$

When $a = 3$, $b = 4$, $d = -5$,

$$\frac{3}{4} - (-5) = \frac{2c}{4}$$

$$5\frac{3}{4} = \frac{c}{2}$$

$$\frac{c}{2} = 5\frac{3}{4}$$

$$\therefore c = 2 \times 5\frac{3}{4}$$

$$= 11\frac{1}{2}$$

10. $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c} + \dfrac{1}{d}$

When $a = \frac{1}{2}$, $b = \frac{1}{4}$, $d = -\frac{1}{5}$,

$$\frac{1}{\frac{1}{2}} - \frac{1}{\frac{1}{4}} = \frac{1}{c} + \frac{1}{\left(\frac{-1}{5}\right)}$$

$$2 - 4 = \frac{1}{c} - 5$$

$$-2 = \frac{1}{c} - 5$$

$$-2 + 5 = \frac{1}{c}$$

$$3 = \frac{1}{c}$$

$$3c = 1$$

$$\therefore c = \frac{1}{3}$$

11. $N = \dfrac{m}{x + q}$

When $N = 1\frac{4}{5}$, $m = 9$, $x = 2$,

$$1\frac{4}{5} = \frac{9}{2 + q}$$
$$\frac{9}{5} = \frac{9}{2 + q}$$
$$\frac{1}{5} = \frac{1}{2 + q}$$
$$2 + q = 5$$
$$\therefore q = 5 - 2$$
$$= 3$$

12. $c = \dfrac{a}{b} - \dfrac{d - e}{f - d}$

When $a = 3$, $b = 4$, $c = –6$, $d = –5$ and $e = 2$,

$$-6 = \frac{3}{4} - \frac{-5 - 2}{f - (-5)}$$
$$-6 = \frac{3}{4} - \frac{-7}{f + 5}$$
$$-6 = \frac{3}{4} + \frac{7}{f + 5}$$
$$-6 - \frac{3}{4} = \frac{7}{f + 5}$$
$$-6\frac{3}{4} = \frac{7}{f + 5}$$
$$-\frac{27}{4} = \frac{7}{f + 5}$$
$$-27(f + 5) = 4 \times 7$$
$$-27(f + 5) = 28$$
$$-27f - 135 = 28$$
$$-27f = 28 + 135$$
$$-27f = 163$$
$$\therefore f = \frac{163}{-27}$$
$$= -6\frac{1}{27}$$

13. $a = \dfrac{b}{c - b}$

When $a = 3$, $c = 10$,

$$3 = \frac{b}{10 - b}$$
$$3(10 - b) = b$$
$$30 - 3b = b$$
$$-3b - b = -30$$
$$-4b = -30$$
$$\therefore b = \frac{-30}{-4}$$
$$= 7\frac{1}{2}$$

14. $\dfrac{m(nx^2 - y)}{z} = 5n$

When $m = 6$, $x = –2$, $y = –3$, $z = –5$,

$$\frac{6[n(-2)^2 - (-3)]}{-5} = 5n$$
$$\frac{6(4n + 3)}{-5} = 5n$$
$$6(4n + 3) = -25n$$
$$24n + 18 = -25n$$
$$24n + 25n = -18$$
$$49n = -18$$
$$\therefore n = -\frac{18}{49}$$

15. **(i)** Let the smallest odd number be n.

The next odd number will be $n + 2$.

The greatest odd number will be $(n + 2) + 2 = n + 4$.

$$\therefore S = n + (n + 2) + (n + 4)$$
$$= n + n + n + 2 + 4$$
$$= 3n + 6$$

(ii) When the greatest odd number is –101,

$$n + 4 = -101$$
$$\therefore n = -101 - 4$$
$$= -105$$
$$\therefore S = 3(-105) + 6$$
$$= -309$$

16. **(i)** $T = c \times d + e \times \dfrac{f}{100}$

$$= cd + \frac{ef}{100}$$

(ii) $e = \dfrac{-145c}{4 - c}$

When $e = 150$,

$$150 = \frac{-145c}{4 - c}$$
$$150(4 - c) = -145c$$
$$600 - 150c = -145c$$
$$-150c + 145c = -600$$
$$-5c = -600$$
$$\therefore c = \frac{-600}{-5}$$
$$= 120$$

$d = \dfrac{f + 5}{50}$

When $d = 3$,

$$3 = \frac{f + 5}{50}$$
$$50 \times 3 = f + 5$$
$$150 = f + 5$$
$$f + 5 = 150$$
$$\therefore f = 150 - 5$$
$$= 145$$
$$\therefore T = 120(3) + \frac{150(145)}{100}$$
$$= 577.50$$

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17. $y = (x - 32) \times \frac{5}{9}$

(i) When $x = 134$,

$y = (134 - 32) \times \frac{5}{9}$

$= 56.7$ (to 3 s.f.)

Required temperature = 56.7 °C

(ii) When $x = 0$,

$y = (0 - 32) \times \frac{5}{9}$

$= -17.8$ (to 3 s.f.)

Since 0 °F = –17.8 °C, it is less common for the temperature to fall below 0 °F because 0 °F is much lower than 0 °C.

(iii) When $y = -62.1$,

$$-62.1 = (x - 32) \times \frac{5}{9}$$

$$(x - 32) \times \frac{5}{9} = -62.1$$

$$x - 32 = -62.1 \times \frac{9}{5}$$

$$x - 32 = -111.78$$

$$\therefore x = -111.78 + 32$$

$$= -79.78$$

$$= -79.8 \text{ (to 3 s.f.)}$$

Required temperature = –79.8 °F

Exercise 5C

1. Let the mass of the empty lorry be x kg.

Then the mass of the bricks is $3x$ kg.

$$x + 3x = 11\,600$$
$$4x = 11\,600$$
$$x = \frac{11\,600}{4}$$
$$= 2900$$

$\therefore$ The mass of the bricks is 3(2900) = 8700 kg.

2. Let the smallest odd number be n.

The next odd number will be $n + 2$.

Then the next odd number will be $(n + 2) + 2 = n + 4$.

The greatest odd number will be $(n + 4) + 2 = n + 6$.

$$n + (n + 2) + (n + 4) + (n + 6) = 56$$
$$n + n + n + n + 2 + 4 + 6 = 56$$
$$4n + 12 = 56$$
$$4n = 56 - 12$$
$$4n = 44$$
$$n = \frac{44}{4}$$
$$= 11$$

$\therefore$ The greatest of the 4 numbers is 11 + 6 = 17.

3. Let Priya's age be x years old.

Then Amirah's age is $(x + 4)$ years,

Shirley's age is $(x - 2)$ years.

$$x + (x + 4) + (x - 2) = 47$$
$$x + x + x + 4 - 2 = 47$$
$$3x + 2 = 47$$
$$3x = 47 - 2$$
$$3x = 45$$
$$x = \frac{45}{3}$$
$$= 15$$

$\therefore$ Priya is 15 years old, Amirah is 15 + 4 = 19 years old and Shirley is 15 – 2 = 13 years old.

4. Let the greater number be x.

Then the smaller number is $\frac{2}{3}x$.

$$x + \frac{2}{3}x = 45$$
$$\frac{5}{3}x = 45$$
$$x = \frac{3}{5} \times 45$$
$$= 27$$

$\therefore$ The smaller number is $\frac{2}{3}(27) = 18$.

5. Let the number be x.

$$3x = x + 28$$
$$3x - x = 28$$
$$2x = 28$$
$$\therefore x = \frac{28}{2}$$
$$= 14$$

The number is 14.

6. Let the number of people going on the holiday be x.

$$15x = 84 + 12x$$
$$15x - 12x = 84$$
$$3x = 84$$
$$\therefore x = \frac{84}{3}$$
$$= 28$$

There are 28 people going on the holiday.

7. Let the number of boys who play badminton be x.

Then the number of boys who play soccer is $3x$.

$$3x - 12 = x + 12$$
$$3x - x = 12 + 12$$
$$2x = 24$$
$$\therefore x = \frac{24}{2}$$
$$= 12$$

There are 12 boys who play badminton.

8. Let the number be x.

$$\frac{1}{2}x + 49 = \frac{9}{4}x$$
$$\frac{1}{2}x - \frac{9}{4}x = -49$$
$$-\frac{7}{4}x = -49$$
$$\therefore x = -\frac{4}{7} \times (-49)$$
$$= 28$$

The number is 28.

9. Let the number be x.

$$68 - 4x = 3(x + 4)$$
$$68 - 4x = 3x + 12$$
$$-4x - 3x = 12 - 68$$
$$-7x = -56$$
$$\therefore x = \frac{-56}{-7}$$
$$= 8$$

The number is 8.

10. Let the son's age be x years.

Then the man's age is $6x$ years.

$$6x + 20 = 2(x + 20)$$
$$6x + 20 = 2x + 40$$
$$6x - 2x = 40 - 20$$
$$4x = 20$$
$$x = \frac{20}{4}$$
$$= 5$$

$\therefore$ The man was $6(5) - 5 = 25$ years old when his son was born.

11. Let the cost of a mooncake with one egg yolk be $\$x$.

Then the cost of a mooncake with two egg yolks is $\$(x + 2)$.

$$6(x + 2) + 5x = 130.8$$
$$6x + 12 + 5x = 130.8$$
$$6x + 5x = 130.8 - 12$$
$$11x = 118.8$$
$$x = \frac{118.8}{11}$$
$$= 10.8$$

$\therefore$ The cost of a mooncake with two egg yolks is $\$(10.8 + 2) = \$12.80$.

12. Let the number of 20-cent coins Jun Wei has be x.

Then the number of 10-cent coins he has is $x + 12$.

$$10(x + 12) + 20x = 540$$
$$10x + 120 + 20x = 540$$
$$10x + 20x = 540 - 120$$
$$30x = 420$$
$$x = \frac{420}{30}$$
$$= 14$$

$\therefore$ Jun Wei has $14 + (14 + 12) = 40$ coins.

13. Let Kate's average speed for the first part of her journey be x km/h. Then her average speed for the second part of her journey is $(x - 15)$ km/h.

Time taken for first part of journey $= \frac{350}{x}$ hours.

Time taken for second part of journey $= \frac{470 - 350}{x - 15}$

$= \frac{120}{x - 15}$ hours.

$$\frac{350}{x} = \frac{120}{x - 15}$$
$$350(x - 15) = 120x$$
$$350x - 5250 = 120x$$
$$350x - 120x = 5250$$
$$230x = 5250$$
$$x = \frac{5250}{230}$$
$$= 22\frac{19}{23}$$

$\therefore$ Kate's average speed for the second part of her journey is

$22\frac{19}{23} - 15 = 7\frac{19}{23}$ km/h.

14. Let the denominator of the fraction be x.

Then the numerator of the fraction is $x - 5$.

$\therefore$ The fraction is $\frac{x - 5}{x}$.

$$\frac{x - 5 + 1}{x + 1} = \frac{2}{3}$$
$$\frac{x - 4}{x + 1} = \frac{2}{3}$$
$$3(x - 4) = 2(x + 1)$$
$$3x - 12 = 2x + 2$$
$$3x - 2x = 2 + 12$$
$$x = 14$$

$\therefore$ The fraction is $\frac{14 - 5}{14} = \frac{9}{14}$.

15. Let the number in the tens place be x.

Then the number in the ones place is $2.5x$.

$\therefore$ The number is $10x + 2.5x = 12.5x$.

$\therefore$ The number obtained when the digits are reversed is $10(2.5x) + x = 25x + x = 26x$.

$$26x - 12.5x = 27$$
$$13.5x = 27$$
$$x = \frac{27}{13.5}$$
$$= 2$$

The number is $12.5(2) = 25$.

Exercise 5D

1. **(a)** If $x > y$, then $5x > 5y$.

(b) If $x < y$, then $\frac{x}{20} < \frac{y}{20}$.

(c) If $x \geqslant y$, then $3x \geqslant 3y$.

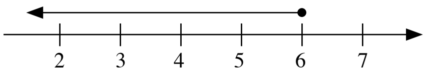
(d) If $x \leqslant y$, then $\frac{x}{10} \leqslant \frac{y}{10}$.

(e) If $15 > 5$ and $5 > x$, then $15 > x$.

(f) If $x < 50$ and $50 < y$, then $x < y$.

2. **(a)** $3x \leqslant 18$

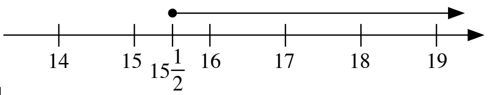
$x \leqslant \frac{18}{3}$



$\therefore x \leqslant 6$

(b) $4x \geqslant 62$

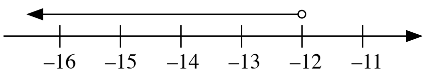
$x \geqslant \frac{62}{4}$



$\therefore x \geqslant 15\frac{1}{2}$

(c) $3y < -36$

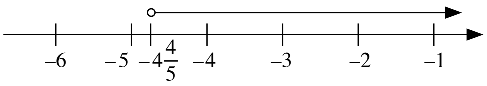
$\therefore y < \frac{-36}{3}$



$y < -12$

(d) $5y > -24$

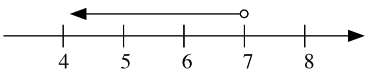
$y > \frac{-24}{5}$



$\therefore y > -4\frac{4}{5}$

(e) $4x < 28$

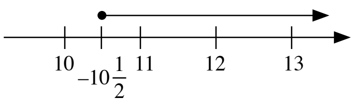
$x < \frac{28}{4}$



$\therefore x < 7$

(f) $12x \geqslant 126$

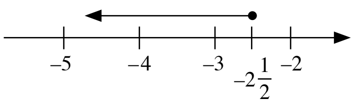
$x \geqslant \frac{126}{12}$



$\therefore x \geqslant 10\frac{1}{2}$

(g) $2y \leqslant -5$

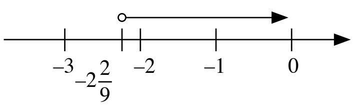
$y \leqslant -\frac{5}{2}$



$y \leqslant -2\frac{1}{2}$

(h) $9y > -20$

$y > -\frac{20}{9}$



$\therefore y > -2\frac{2}{9}$

3. Let the number of vans that are needed to ferry 80 people be x.
Then $12x \geqslant 80$

$x \geqslant \frac{80}{12}$

$x \geqslant 6\frac{2}{3}$

$\therefore$ The minimum number of vans that are needed to ferry 80 people is 7.

4. $8 \leqslant 7y$

$7y \geqslant 8$

$y \geqslant \frac{8}{7}$

$y \geqslant 1\frac{1}{7}$

$\therefore$ The smallest rational value of y is $1\frac{1}{7}$.

5. $20x > 33$

$x > \frac{33}{20}$

$x > 1\frac{13}{20}$

$\therefore$ The smallest value of x if x is a prime number is 2.

6. $3x < -105$

$x < \frac{-105}{3}$

$x < -35$

$\therefore$ The greatest odd integer value of x is –37.

7. $5y < 20$ and $2y \geqslant -6$

$y < \frac{20}{5}$ $\qquad y \geqslant -\frac{6}{2}$

$y < 4$ $\qquad y \geqslant -3$

$\therefore$ The possible values are –3, –2, –1, 0, 1, 2 and 3.

Review Exercise 5

1. **(a)**

$$x - 1 = \frac{1}{2}x$$
$$x - \frac{1}{2}x = 1$$
$$\frac{1}{2}x = 1$$
$$\therefore x = 2$$

(b)

$$\begin{aligned} 2(x-1) + 3(x+1) &= 4(x+4) \\ 2x - 2 + 3x + 3 &= 4x + 16 \\ 5x + 1 &= 4x + 16 \\ 5x - 4x &= 16 - 1 \\ \therefore x &= 15 \end{aligned}$$

(c)

$$\begin{aligned} 2y - [7 - (5y - 4)] &= 6 \\ 2y - (7 - 5y + 4) &= 6 \\ 2y - (11 - 5y) &= 6 \\ 2y - 11 + 5y &= 6 \\ 7y - 11 &= 6 \\ 7y &= 6 + 11 \\ 7y &= 17 \\ \therefore y &= \frac{17}{7} \\ &= 2\frac{3}{7} \end{aligned}$$

(d) $\frac{3}{4}x - 5 = 0.5x$

$$\frac{3}{4}x - 0.5x = 5$$
$$\frac{1}{4}x = 5$$
$$\therefore x = 4 \times 5$$
$$= 20$$

(e) $\frac{2y + 7}{4} = 12$

$$2y + 7 = 4 \times 12$$
$$2y + 7 = 48$$
$$2y = 48 - 7$$
$$2y = 41$$
$$\therefore y = \frac{41}{2}$$
$$= 20\frac{1}{2}$$

(f) $\frac{4y - 1}{5y + 1} = \frac{5}{7}$

$$7(4y - 1) = 5(5y + 1)$$
$$28y - 7 = 25y + 5$$
$$28y - 25y = 5 + 7$$
$$3y = 12$$
$$\therefore y = \frac{12}{3}$$
$$= 4$$

(g) $\frac{a + 1}{4} + \frac{a - 1}{3} = 4$

$$\frac{3(a + 1) + 4(a - 1)}{12} = 4$$
$$\frac{3a + 3 + 4a - 4}{12} = 4$$
$$\frac{7a - 1}{12} = 4$$
$$7a - 1 = 12 \times 4$$
$$7a - 1 = 48$$
$$7a = 48 + 1$$
$$7a = 49$$
$$\therefore a = \frac{49}{7}$$
$$= 7$$

(h) $\frac{b - 4}{3} - \frac{2b + 1}{6} = \frac{5b - 1}{2}$

$$\frac{2(b - 4) - (2b + 1)}{6} = \frac{5b - 1}{2}$$
$$\frac{2b - 8 - 2b - 1}{6} = \frac{5b - 1}{2}$$
$$\frac{-9}{6} = \frac{5b - 1}{2}$$
$$\frac{-3}{2} = \frac{5b - 1}{2}$$
$$-3 = 5b - 1$$
$$5b - 1 = -3$$
$$5b = -2$$
$$\therefore b = -\frac{2}{5}$$

(i) $\frac{2c}{9} - \frac{c - 1}{6} = \frac{c + 3}{12}$

$$\frac{2(2c) - 3(c - 1)}{18} = \frac{c + 3}{12}$$
$$\frac{4c - 3c + 3}{18} = \frac{c + 3}{12}$$
$$\frac{c + 3}{18} = \frac{c + 3}{12}$$
$$12(c + 3) = 18(c + 3)$$
$$2(c + 3) = 3(c + 3)$$
$$2c + 6 = 3c + 9$$
$$2c - 3c = 9 - 6$$
$$-c = 3$$
$$\therefore c = -3$$

(j) $\frac{2(3 - 4d)}{3} - \frac{3(d + 7)}{2} = 5d + \frac{1}{6}$

$$\frac{4(3 - 4d) - 9(d + 7)}{6} = \frac{6(5d) + 1}{6}$$
$$\frac{12 - 16d - 9d - 63}{6} = \frac{30d + 1}{6}$$
$$\frac{-25d - 51}{6} = \frac{30d + 1}{6}$$
$$-25d - 51 = 30d + 1$$
$$-25d - 30d = 1 + 51$$
$$-55d = 52$$
$$\therefore d = -\frac{52}{55}$$

2. (a) $18x < -25$

$$x < \frac{-25}{18}$$
$$\therefore x < -1\frac{7}{18}$$

(b) $10y \geqslant -24$

$$y \geqslant -\frac{24}{10}$$
$$\therefore y \geqslant -2\frac{2}{5}$$

3. $3(x - 1) - 5(x - 4) = 8$

$$3x - 3 - 5x + 20 = 8$$
$$-2x + 17 = 8$$
$$-2x = 8 - 17$$
$$-2x = -9$$
$$x = \frac{-9}{-2}$$
$$= 4\frac{1}{2}$$
$$\therefore x - 5\frac{1}{2} = 4\frac{1}{2} - 5\frac{1}{2}$$
$$= -1$$

4. $4x \geqslant 11$

$$x \geqslant \frac{11}{4}$$
$$x \geqslant 2\frac{3}{4}$$

$\therefore$ The smallest integer value of x is 3.

5. $3y < -24$

$y < \frac{-24}{3}$

$y < -8$

$\therefore$ The greatest integer value of y is -9.

6. $5x < 125$

$x < \frac{125}{5}$

$x < 25$

$\therefore$ The greatest value of x if x is divisible by 12 is 24.

7. $5y \geqslant 84$

$y \geqslant \frac{84}{5}$

$y \geqslant 16\frac{4}{5}$

$\therefore$ The smallest value of y if y is a prime number is 17.

8. $V = \frac{4}{3}\pi r^3$

(i) When $r = 7$,

$V = \frac{4}{3}\left(\frac{22}{7}\right)(7)^3$

$= 1437\frac{1}{3}$

(ii) When $V = 113\frac{1}{7}$,

$113\frac{1}{7} = \frac{4}{3}\left(\frac{22}{7}\right)r^3$

$113\frac{1}{7} = \frac{88}{21}r^3$

$\frac{88}{21}r^3 = 113\frac{1}{7}$

$r^3 = \frac{21}{88} \times 113\frac{1}{7}$

$r^3 = 27$

$\therefore r = \sqrt[3]{27}$

$= 3$

9. $n - 2y = \frac{3y - n}{m}$

When $y = 5$, $m = -3$,

$n - 2(5) = \frac{3(5) - n}{-3}$

$n - 10 = \frac{15 - n}{-3}$

$-3(n - 10) = 15 - n$

$-3n + 30 = 15 - n$

$-3n + n = 15 - 30$

$-2n = -15$

$\therefore n = \frac{-15}{-2}$

$= 7\frac{1}{2}$

10. Let the smaller odd number be n.

Then the greater number is $n + 2$.

$n + 2 + 5n = 92$

$6n + 2 = 92$

$6n = 92 - 2$

$6n = 90$

$n = \frac{90}{6}$

$= 15$

$\therefore$ The two consecutive odd numbers are 15 and 17.

11. Let the mass of Object B be x kg.

Then the mass of Object A is $(x + 5)$ kg,

the mass of Object C is $2(x + 5)$ kg.

$(x + 5) + x + 2(x + 5) = 255$

$x + 5 + x + 2x + 10 = 255$

$4x + 15 = 255$

$4x = 255 - 15$

$4x = 240$

$x = \frac{240}{4}$

$= 60$

$\therefore$ The mass of Object C is $2(60 + 5) = 130$ kg.

12. Let Farhan's present age be x years.

Then Farhan's cousin's present age is $(38 - x)$ years.

$x - 7 = 3(38 - x - 7)$

$x - 7 = 3(31 - x)$

$x - 7 = 93 - 3x$

$x + 3x = 93 + 7$

$4x = 100$

$\therefore x = \frac{100}{4}$

$= 25$

Farhan is 25 years old now.

13. Let Raj's present age be x years.

Then Nora's present age is $2x$ years,

Ethan's present age be is $2(2x)$ years.

$2(2x) + 22 = 2(x + 22)$

$4x + 22 = 2x + 44$

$4x - 2x = 44 - 22$

$2x = 22$

$x = \frac{22}{2}$

$= 11$

$\therefore$ Nora is $2(11) = 22$ years old now.

14. Let the number of sweets that the man has to give to his son be x.

$55 + x = 4(25 - x)$

$55 + x = 100 - 4x$

$x + 4x = 100 - 55$

$5x = 45$

$\therefore x = \frac{45}{5}$

$= 9$

The man has to give 9 sweets to his son.

15. Let the original price of each apple be x cents.

$$24x = (24 + 6)(x - 5)$$
$$24x = 30(x - 5)$$
$$24x = 30x - 150$$
$$24x - 30x = -150$$
$$-6x = -150$$
$$\therefore x = \frac{-150}{-6}$$
$$= 25$$

The original price of each apple is 25 cents.

16. Let the distance between Town A and Town B be x km.

45 minutes = $\frac{45}{60}$ hour = $\frac{3}{4}$ hour

$$\frac{x}{4} + \frac{x}{6} = \frac{3}{4}$$
$$\frac{6x + 4x}{24} = \frac{3}{4}$$
$$\frac{10x}{24} = \frac{3}{4}$$
$$\frac{5x}{12} = \frac{3}{4}$$
$$5x = 12 \times \frac{3}{4}$$
$$5x = 9$$
$$x = \frac{9}{5}$$
$$= 1\frac{4}{5}$$

$\therefore$ The main travels a total distance of $2 \times 1\frac{4}{5} = 3\frac{3}{5}$ km.

17. Let the denominator of the fraction be x.

Then the numerator of the fraction is $x - 2$.

$\therefore$ The fraction is $\frac{x-2}{x}$.

$$\frac{x - 2 - 3}{x - 3} = \frac{3}{4}$$
$$\frac{x - 5}{x - 3} = \frac{3}{4}$$
$$4(x - 5) = 3(x - 3)$$
$$4x - 20 = 3x - 9$$
$$4x - 3x = -9 + 20$$
$$x = 11$$

$\therefore$ The fraction is $\frac{11-2}{11} = \frac{9}{11}$.

18. Let the number of sets of multimedia equipment that can be bought with \$35 000 be x.

Then $1900x \leqslant 35\,000$

$$x \leqslant \frac{35\,000}{1900}$$
$$x \leqslant 18\frac{8}{19}$$

$\therefore$ The maximum number of sets of multimedia equipment that can be bought with \$35 000 is 18.

19. Let the number of tickets Jun Wei can buy be x.

Then $12.50x \leqslant 250$

$$x \leqslant \frac{250}{12.5}$$
$$x \leqslant 20$$

$\therefore$ The maximum number of tickets Jun Wei can buy with \$250 is 20.

20. Let the first integer be x.

Then the second integer will be $(x + 1)$.

$$x + (x + 1) < 42$$
$$2x < 41$$
$$x < \frac{41}{2}$$
$$x < 20.5$$

$\therefore$ The largest possible integer x can be is 20.

$20 + 1 = 21$

$21^2 = 441$

$\therefore$ The square of largest possible integer is 441.

21. Let Nora's age be x years.

Then Kate's age is $(x - 4)$ years.

$$x + (x - 4) \leqslant 45$$
$$2x \leqslant 45 + 4$$
$$x \leqslant \frac{49}{2}$$
$$x \leqslant 24.5$$

$\therefore$ Maximum possible age of Nora is 24 years.

$24 - 4 = 20$

$\therefore$ The maximum possible age of Kate is 20 years.

22. Let the number of ships needed to carry 400 passengers be .

$$60x \geqslant 400$$
$$x \geqslant \frac{400}{60}$$
$$x \geqslant 6\frac{2}{3}$$

$\therefore$ The minimum number of ships needed to carry 400 passengers is 7.

23. Let the number of pencils that can be bought with \$27 be x.

$$2.50x \leqslant 27$$
$$x \leqslant \frac{27}{2.5}$$
$$x \leqslant 10\frac{4}{5}$$

$\therefore$ The maximum number of pencils that can be bought with \$27 is 10.

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Challenge Yourself

1. $\sqrt{x} + 2 = 0$

$\sqrt{x} = -2$

There is no solution since $\sqrt{x}$ cannot be a negative number.

2. Since $(x + 2)^2$ and $(y - 3)^2$ cannot be negative.

$(x + 2)^2 = 0$ and $(y - 3)^2 = 0$

$x + 2 = 0$ and $y - 3 = 0$

$x = -2$ and $y = 3$

$\therefore x + y = -2 + 3$

$= 1$

3. $A + B = 8$ — (1)

$B + C = 11$ — (2)

$B + D = 13$ — (3)

$C + D = 14$ — (4)

(2) – (3): $B + C - B - D = 11 - 13$

$C - D = -2$ — (5)

(4) + (5): $C + D + C - D = 14 + (-2)$

$2C = 12$

$\therefore C = \frac{12}{2}$

$= 6$

Substitute $C = 6$ into (4): $6 + D = 14$

$\therefore D = 14 - 6$

$= 8$

Substitute $C = 6$ into (2): $B + 6 = 11$

$\therefore B = 11 - 6$

$= 5$

Substitute $B = 5$ into (1): $A + 5 = 8$

$\therefore A = 8 - 5$

$= 3$

4. $A \times B = 8$ — (1)

$B \times C = 28$ — (2)

$C \times D = 63$ — (3)

$B \times D = 36$ — (4)

(2) ÷ (3): $\frac{B \times C}{C \times D} = \frac{28}{63}$

Since C cannot be equal to 0, then $\frac{B}{D} = \frac{4}{9}$, — (5)

(4) × (5): $B \times D \times \frac{B}{D} = 36 \times \frac{4}{9}$

Since D cannot be equal to 0, then $B^2 = 16$.

$\therefore B = \pm\sqrt{16}$

$= 4$ or -4 (N.A. since $B > 0$)

Substitute $B = 4$ into (1): $A \times 4 = 8$

$\therefore A = \frac{8}{4}$

$= 2$

Substitute $B = 4$ into (2): $4 \times C = 28$

$\therefore C = \frac{28}{4}$

$= 7$

Substitute $B = 4$ into (4): $4 \times D = 36$

$\therefore D = \frac{36}{4}$

$= 9$

Chapter 6 Functions and Linear Graphs

TEACHING NOTES

Suggested Approach

Although the topic on functions and linear graphs is new to most students, they do encounter examples of their applications in their daily lives, e.g. maps show the usage of Cartesian coordinates; escalators and moving walkways illustrate the concept of steepness. Teachers can get students to discuss about in detail these real-life examples. When students are able to appreciate their uses, teachers can proceed to introduce the concept of functions and linear graphs.

Section 6.1: Cartesian Coordinates

Teachers can build upon prerequisites, namely number lines to introduce the horizontal axis (x-axis) and the vertical axis (y-axis). Teachers can introduce this concept by playing a game (see Class Discussion: Battleship Game (Two Players)) to arouse students' interest.

Teachers should teach students not only on how to draw horizontal and vertical axes and plot the given points, but also to determine the position of points. Teachers can impress upon students that the first number in each ordered pair is with reference to the horizontal scale while the second number is with reference to the vertical scale. As such, students need to take note that the point (3, 4) has a different position compared to the point (4, 3).

Section 6.2: Functions

Teachers can use the Function Machine (see Investigation: Function Machine) to explore the concept of a function with the students and show that when a function is applied to any input x, it will produce exactly one output y. Once the students have understood the relationship between the input x and the output y, they are then able to represent the function using an equation, a table and a graph.

Section 6.3: Graphs of Linear Functions

Teachers should illustrate how a graph of a linear function is drawn on a sheet of graph paper. Teachers can impress upon students that when they draw a graph, the graph has to follow the scale stated for both the x-axis and y-axis and the graph is only drawn for the values of x stated in the range.

Section 6.4: Applications of Linear Graphs in Real-World Contexts

Teachers can give examples of linear graphs used in many daily situations and explain what each of the graphs is used for. Through Worked Example 2, students will learn how functions and linear graphs are applied in real-world contexts and solve similar problems

The thinking time on page 151 of the textbook requires students to think further and consider if a negative value is possible or logical in the real world. Teachers should get the students to apply the answer of it to the other problems in real-world contexts.

Challenge Yourself

To further guide pupils to better understand the concept, teachers may modify the question to giving the x-coordinate of C.

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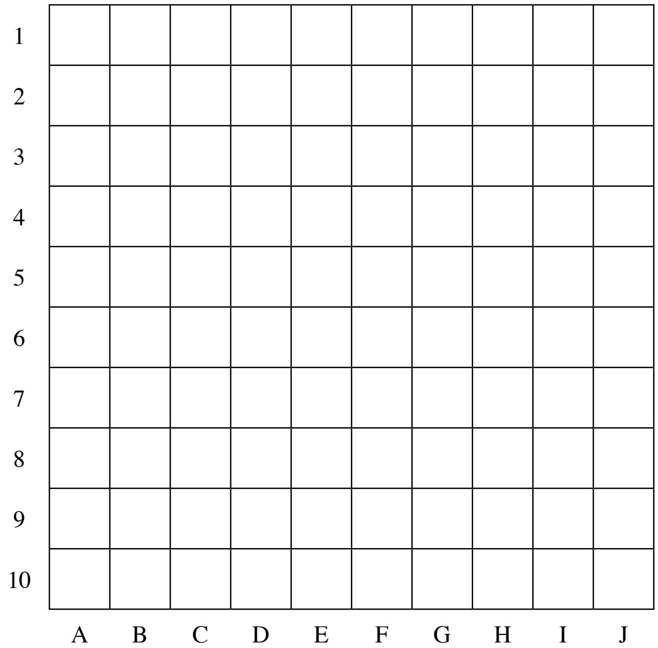
WORKED SOLUTIONS

Class Discussion (Battleship Game (Two Players))

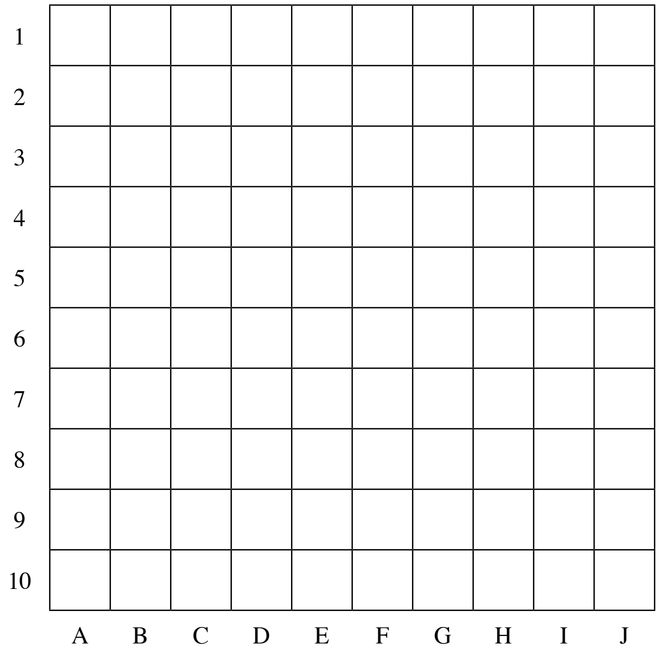
The purpose of this Battleship Game is to introduce students to the use of 2D Cartesian coordinates to specify points through an interesting and engaging activity.

Teachers may wish to emphasise to students that they should call out a location on the grid by calling the letter before calling the number, e.g. D7 instead of 7D.

Teachers may wish to use the grids provided (similar to that in Fig. 6.1) to conduct this activity.



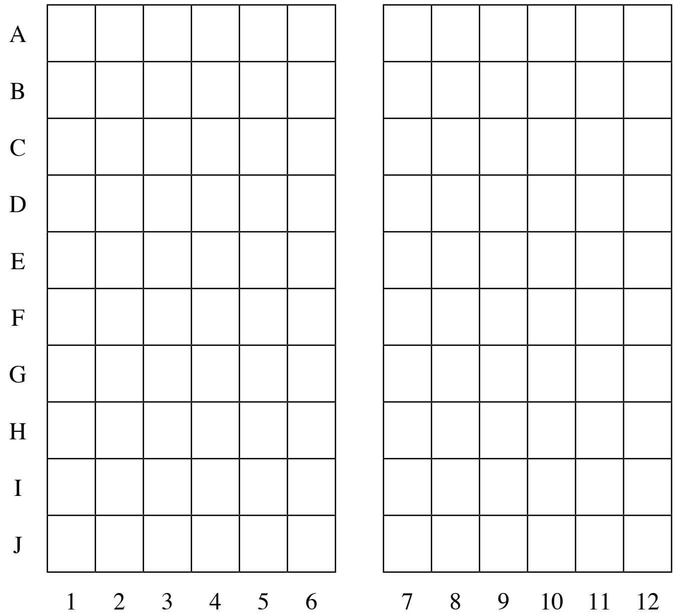
Self



Opponent

Class Discussion (Ordered Pairs)

1. A single number is not sufficient to describe the exact position of a student in the classroom seating plan. For example, when the number 1 is used to indicate the position of a student in the classroom, the student could be either in row 1 or column 1. From Fig. 6.2, we can see that there are 11 possible positions of the student.
   Similarly, the location of a seat in a cinema cannot be represented by a single number. An example of a seating plan of a cinema is as shown:



   From the seating plan shown, both the number and the letter are required to represent the location of a seat in the cinema.
2. The order in which two numbers are written are important, i.e. (5, 3) and (3, 5) do not indicate the same position.

Journal Writing (Page 137)

1. **Guiding Questions:**
   - How do you determine the locations of your house, a bus stop and a shopping mall in your neighbourhood on the map?
   - How can you obtain the approximate distances between your house, a bus stop and a shopping mall in your neighbourhood?
2. **Guiding Questions:**
   - What types of shops can normally be found on the ground floor of a shopping mall?
   - What is the size of each shop? How many spaces on the map should each shop occupy?
   - Are there any other considerations, e.g. walkways and washrooms, when designing the map?
3. **Guiding Questions:**
   - What types of horizontal and vertical scales are commonly used for the seating plan of a cinema in Singapore?
   - What are the different types of seats, e.g. wheelchair berths and couple seats, which can be found in a cinema?

Investigation (Function Machine)

1. $y = x + 3$

2. **(a)** Input $x = 4 \rightarrow$ Output $y = 4 + 3 = 7$

(b) Input $x = -7 \rightarrow$ Output $y = -7 + 3 = -4$

3. **(a)** Input $x = 9 - 3 = 6 \rightarrow$ Output $y = 9$

(b) Input $x = 0 - 3 = -3 \rightarrow$ Output $y = 0$

4.

x	–7	–3	2	4	6
y	–4	0	5	7	9

Table 6.1

5.

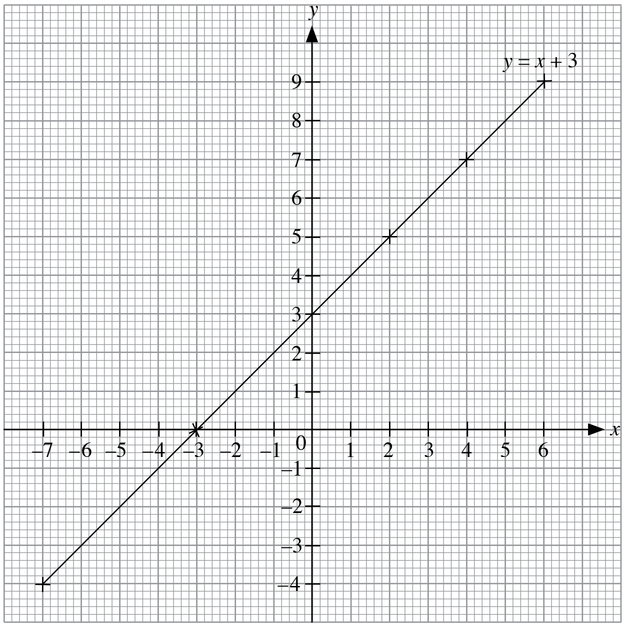


Fig. 6.6

The coordinates of every point on the straight line in Fig. 6.6 satisfy the equation of the function $y = x + 3$.

6. Every input x produces exactly one output y.

7. $y = -2x - 1$

8.

x	–1	–0.5	0	2	3
y	1	0	–1	–5	–7

Table 6.2

9.

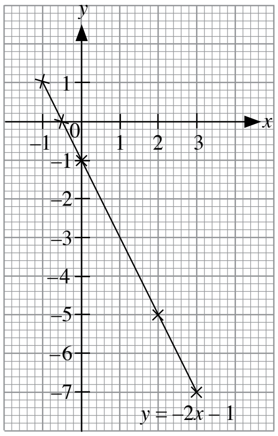


Fig. 6.8

10. Every input x produces exactly one output y.

Thinking Time (Page 143)

1. $y^2 = x$ is *not* the equation of a function because

- there are two values of y for every positive value of x, e.g. if the input $x = 9$, then the output $y = \pm 3$,
- there is no value for the output y if the input x is negative.

2. It is possible for a function to have two input values x with the same output value y. Consider the equation of the function $y = x^2$. If the input $x = -3$ or 3, then the output $y = 9$.

Class Discussion (Equation of a Function)

1. Since the point A lies on the graph of the function $y = 2x$, its coordinates satisfy the equation of the function $y = 2x$.
Since the point B do not lie on the graph of the function $y = 2x$, its coordinates do not satisfy the equation of the function $y = 2x$.

2. Examples of coordinates of points that satisfy the equation of the function $y = 2x$ include (2, 4), (3, 6), (4, 8), (0.5, 1) and (1.25, 2.5).

3. Amirah is correct to say that 'the coordinates of every point on the line satisfy the equation of the function $y = 2x$'. Since a graph is a way to display a function, the coordinates of every point on the graph satisfy the equation of the function.

Thinking Time (Page 147)

(i) The y-coordinate of each point that lies on the line $y = 2x$ is twice its x-coordinate, i.e. the coordinates are given by $(x, 2x)$

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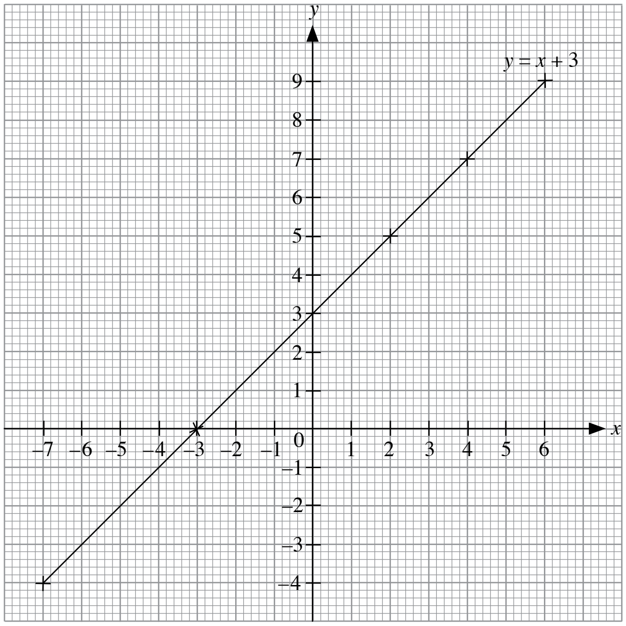
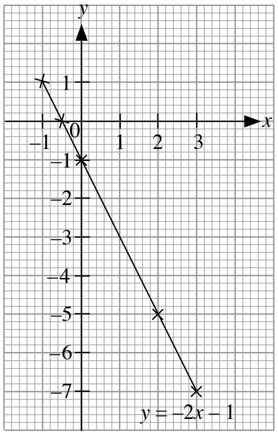
(ii)





Yes, I agree with Lixin. As shown above, the graphs of the functions $y = x + 3$ and $y = -2x - 1$ are straight lines. Hence, they are linear.

Thinking Time (Page 151)

(i) When $x = -2$,

$y = 100 - (-2) \times 3$
$= 100 + 6$
$= 106$

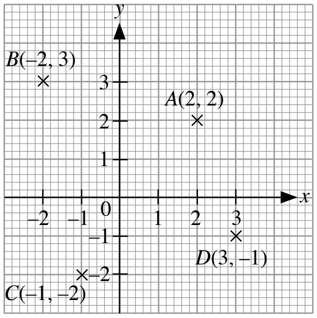
This means that 2 days before Nora receives her monthly allowance, she has \$106. However, in the real world, it is not possible for her to have more money before she receives her monthly allowance than when she receives her monthly allowance.

(ii) When $x = 35$,

$y = 100 - 35 \times 3$
$= 100 - 105$
$= -5$

This means that 35 days after Nora receives her monthly allowance, she has –\$5. Logically speaking, she should not have a negative amount of money. However, in the real world, it is possible for her to have –\$5 as she may have borrowed \$5 from her friends.

Practise Now (Page 138)



Practise Now (Page 143)

1. **(i)** When $x = 4$,

$y = 2(4) - 3$
$= 8 - 3$
$= 5$

(ii) When $y = -5$,

$-5 = 2x - 3$
$-5 + 3 = 2x$
$-2 = 2x$
$\therefore x = -1$

2. **(i)** When $x = 0$,

$$y = -\frac{1}{3}(0) - \frac{2}{5}$$
$$= 0 - \frac{2}{5}$$
$$= -\frac{2}{5}$$

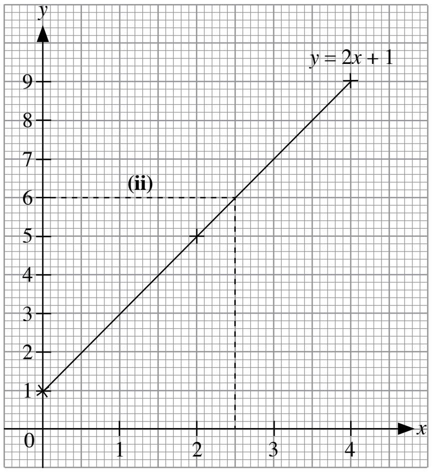
(ii) When $y = -\frac{2}{3}$,

$$-\frac{2}{3} = -\frac{1}{3}x - \frac{2}{5}$$
$$-\frac{2}{3} + \frac{2}{5} = -\frac{1}{3}x$$
$$-\frac{4}{15} = -\frac{1}{3}x$$
$$\therefore x = \frac{4}{5}$$

Practise Now 1

1. **(i)**

x	0	2	4
$y = 2x + 1$	1	5	9

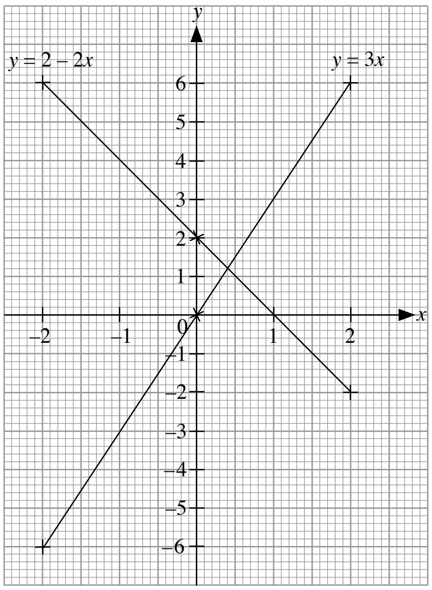


(ii) From the graph in **(i)**,
when $y = 6$,
$q = x = 2.5$

2.

x	–2	0	2
$y = 3x$	–6	0	6

x	–2	0	2
$y = 2 - 2x$	6	2	–2



Practise Now 2

(a) **(i)** Amount of money the passenger has to pay if taxi travels 3 km
= \$3 + 3 × \$0.50
= \$3 + \$1.50
= \$4.50

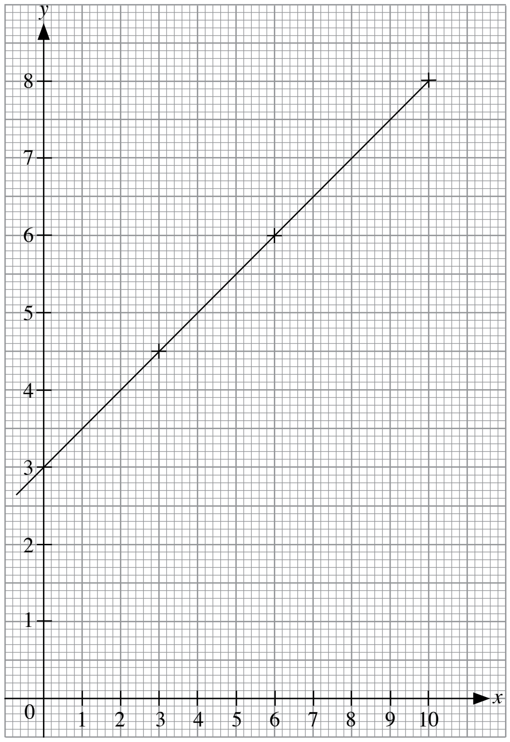
(ii) Amount of money the passenger has to pay if taxi travels 6 km
= \$3 + 6 × \$0.50
= \$3 + \$3
= \$6

(iii) Amount of money the passenger has to pay if taxi travels 10 km
= \$3 + 10 × \$0.50
= \$3 + \$5
= \$8

(b)

x	3	6	10
y	4.50	6	8

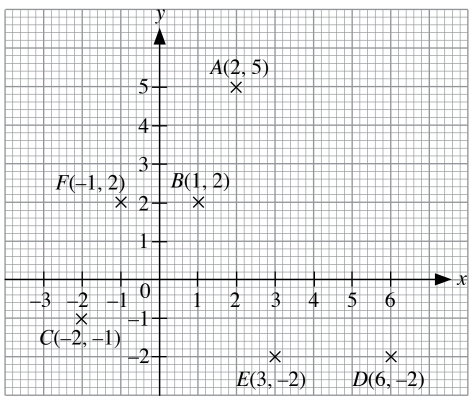
(c)



Exercise 6A

1. $A(-4, -3)$, $B(-2, 4)$, $C(3, 4)$, $D(4, 2)$, $E(1, 1)$, $F(3, -3)$

2.



3. **(i)** When $x = 3$,
$y = 4(3) + 5$
$= 12 + 5$
$= 17$

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(ii) When $x = -2$,

$y = 4(-2) + 5$

$= -8 + 5$

$= -3$

4. **(i)** When $y = 34$,

$34 = 25 - 3x$

$3x = 25 - 34$

$3x = -9$

$\therefore x = -3$

(ii) When $y = -5$,

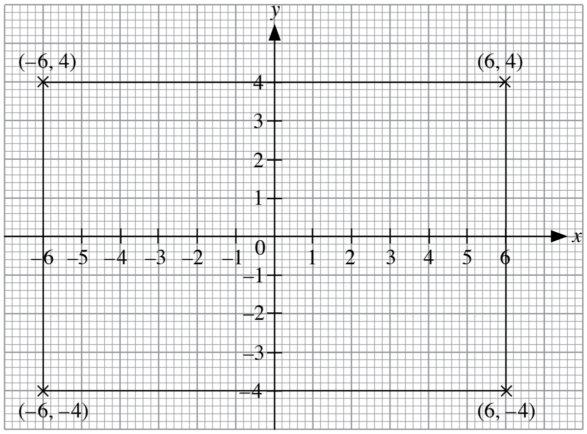
$-5 = 25 - 3x$

$3x = 25 + 5$

$3x = 30$

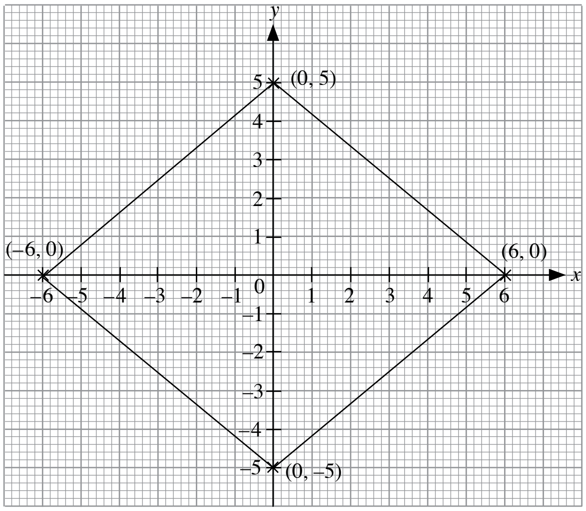
$\therefore x = 10$

5. **(a)**



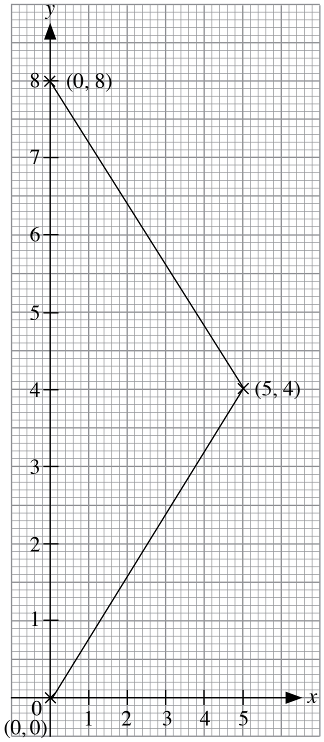
The figure shows a rectangle.

(b)



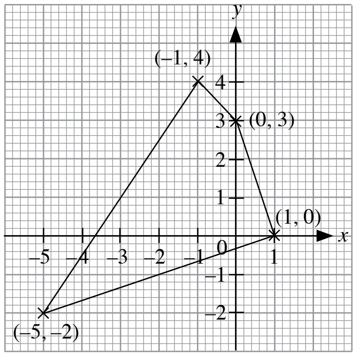
The figure shows a rhombus.

(c)



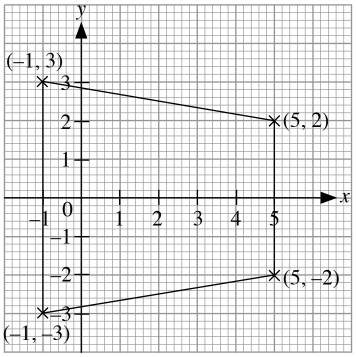
The figure shows an isosceles triangle.

(d)

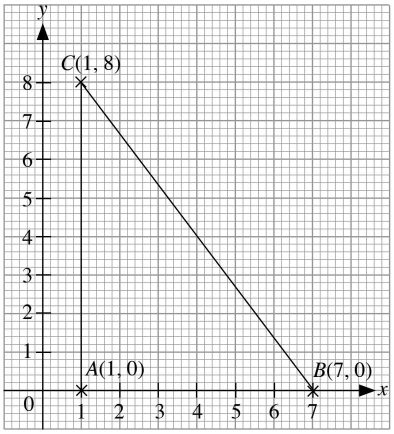


The figure shows a quadrilateral.

(e)

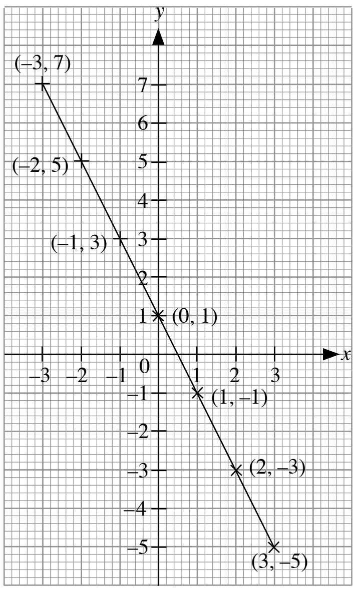


The figure shows a trapezium.

6. 

Area of $\Delta ABC = \frac{1}{2} \times 6 \times 8$

$= 24 \text{ units}^2$

7. 

The points lie on a straight line.

8. **(a)** **(i)** When $x = -3$,

$$y = \frac{2}{3}(-3) + \frac{1}{3}$$
$$= -2 + \frac{1}{3}$$
$$= -1\frac{2}{3}$$

(ii) When $x = 1\frac{1}{2}$,

$$y = \frac{2}{3}\left(1\frac{1}{2}\right) + \frac{1}{3}$$
$$= 1 + \frac{1}{3}$$
$$= 1\frac{1}{3}$$

(b) **(i)** When $y = 1$,

$$1 = \frac{2}{3}x + \frac{1}{3}$$
$$1 - \frac{1}{3} = \frac{2}{3}x$$
$$\frac{2}{3} = \frac{2}{3}x$$
$$\therefore x = 1$$

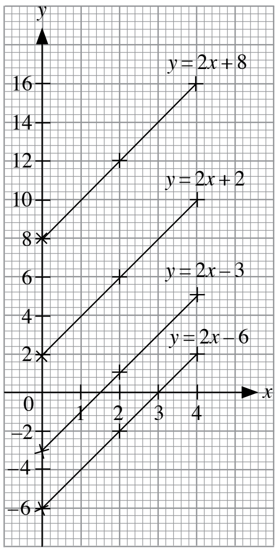
(ii) When $y = -\frac{1}{6}$,

$$-\frac{1}{6} = \frac{2}{3}x + \frac{1}{3}$$
$$-\frac{1}{6} - \frac{1}{3} = \frac{2}{3}x$$
$$-\frac{1}{2} = \frac{2}{3}x$$
$$\therefore x = -\frac{3}{4}$$

Exercise 6B

1. **(a)**

x	0	2	4
$y = 2x + 8$	8	12	16
$y = 2x + 2$	2	6	10
$y = 2x - 3$	–3	1	5
$y = 2x - 6$	–6	–2	2



(b) They are parallel lines.

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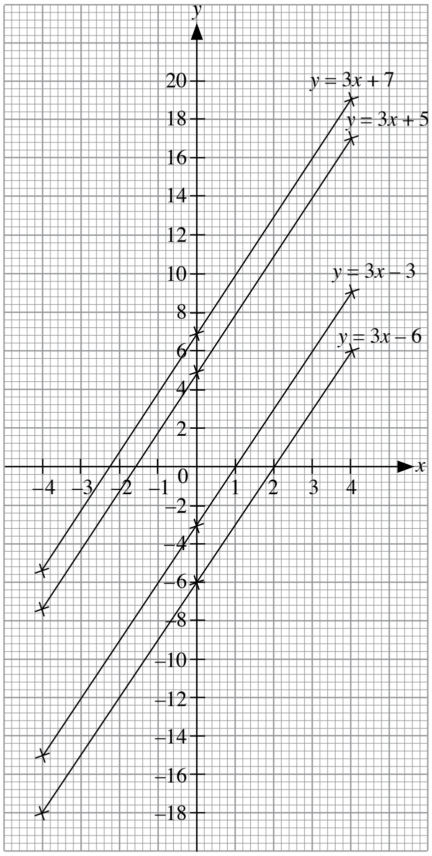
2. **(a)**

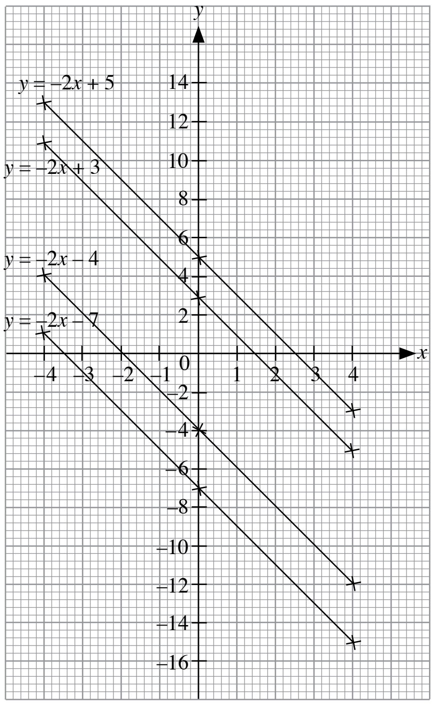
x	–4	0	4
$y = 3x + 7$	–5	7	19
$y = 3x + 5$	–7	5	17
$y = 3x - 3$	–15	–3	9
$y = 3x - 6$	–18	–6	6



(b) They are parallel lines.

3. **(a)**

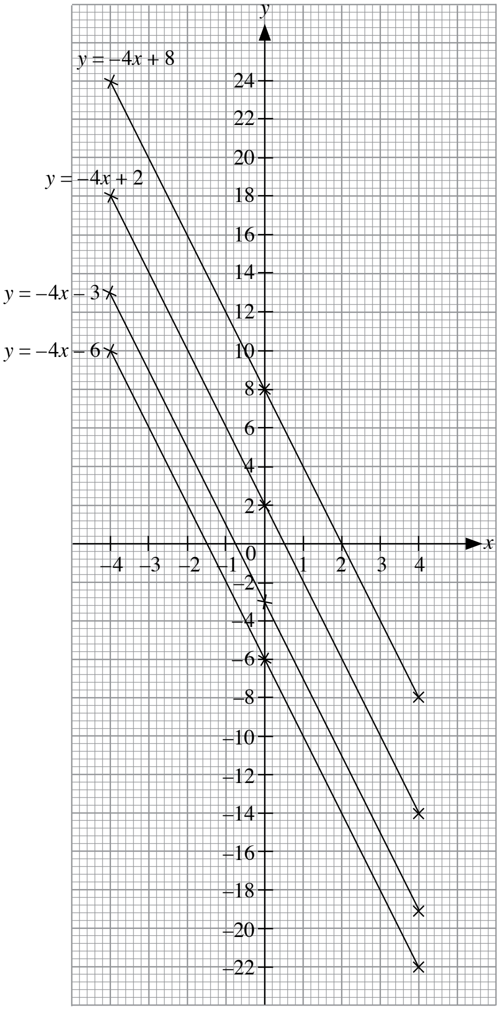
x	–4	0	4
$y = -2x + 5$	13	5	–3
$y = -2x + 3$	11	3	–5
$y = -2x - 4$	4	–4	–12
$y = -2x - 7$	1	–7	–15



(b) They are parallel lines.

4. (a)

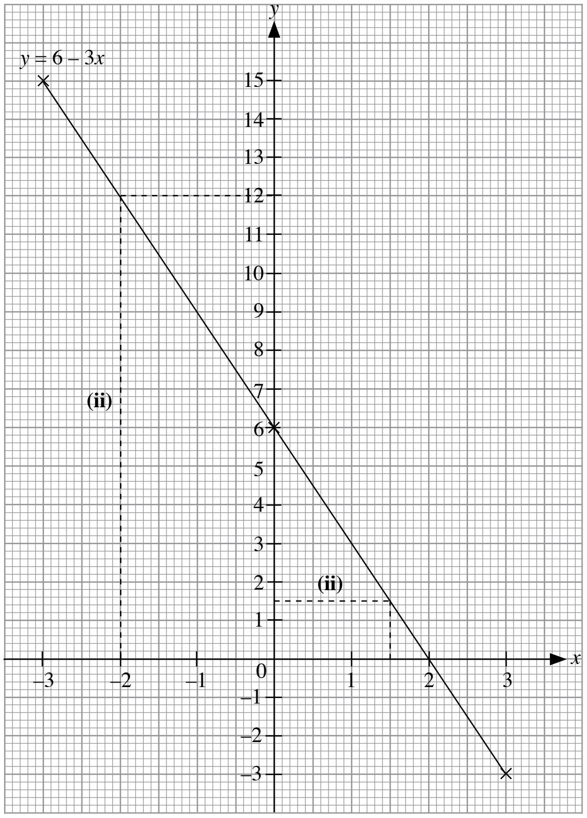
x	–4	0	4
$y = -4x + 8$	24	8	–8
$y = -4x + 2$	18	2	–14
$y = -4x - 3$	13	–3	–19
$y = -4x - 6$	10	–6	–22



(b) $y = mx + c_1$, $y = mx + c_2$, $y = mx + c_3$, $y = mx + c_4$,
where m, c_1, c_2, c_3 and c_4 are constants

5. (i)

x	–3	0	3
$y = 6 - 3x$	15	6	–3



(ii) From the graph in **(i)**,
when $y = 0$,
$a = x = 2$
From the graph in **(i)**,
when $x = -2$,
$b = y = 12$
From the graph in **(i)**,
when $y = 1.5$,
$c = x = 1.5$

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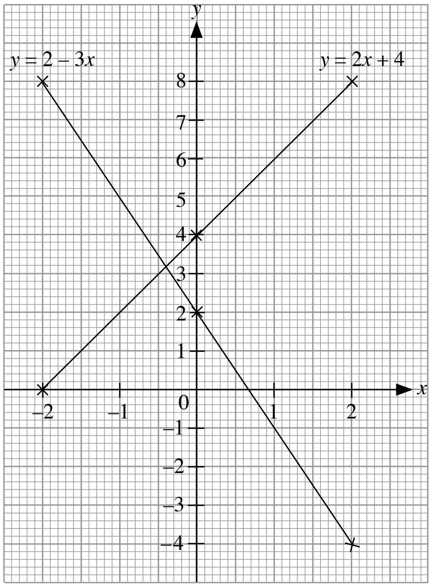
6.

x	-2	0	2
$y = 2x + 4$	0	4	8
$y = 2 - 3x$	8	2	-4



Exercise 6C

1. **(a)** **(i)** Amount of money left after 3 days $= \$120 - 3 \times \$5$
$= \$120 - \$15$
$= \$105$

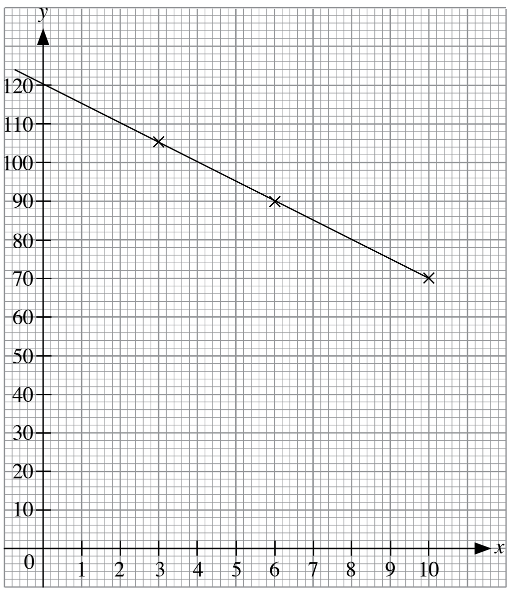
(ii) Amount of money left after 6 days $= \$120 - 6 \times \$5$
$= \$120 - \$30$
$= \$90$

(iii) Amount of money left after 10 days $= \$120 - 10 \times \$5$
$= \$120 - \$50$
$= \$70$

(b)

x	3	6	10
y	105	90	70

(c)



2. **(a)** **(i)** Distance car can travel if it has 3 l of petrol = 27 km

(ii) Distance car can travel if it has 5.2 l of petrol = 47 km

(b) Amount of petrol required to travel 36 km = 4 l

$\therefore$ Cost of petrol required to travel 36 km $= 4 \times \$1.40$
$= \$5.60$

3. **(i)**

N	10	30	50	70
C	100	200	300	400

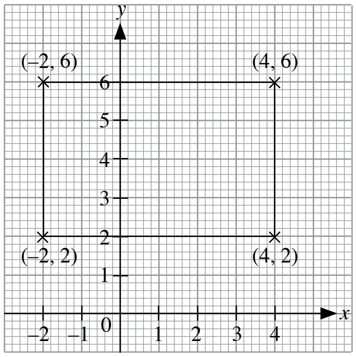
(ii) There is a fixed overhead of \$50.

(iii) Amount of money Devi has to pay for 68 T-shirts = \$390

(iv) Number of T-shirts Devi can order with \$410 = 72

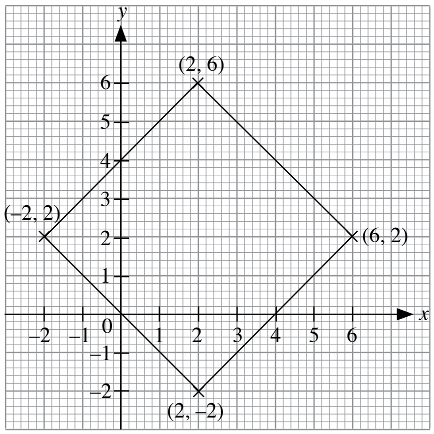
Review Exercise 6

1. **(a)**



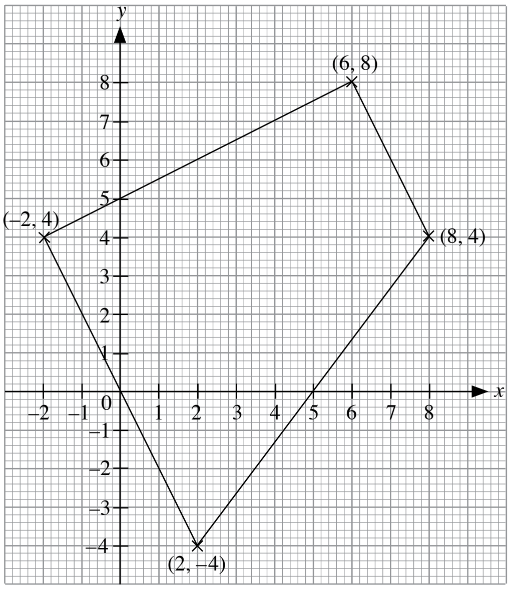
The figure shows a rectangle.

(b)



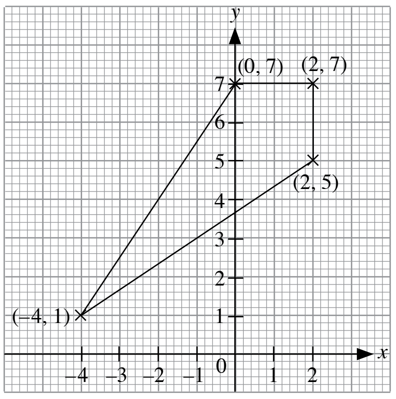
The figure shows a square.

(c)



The figure shows a trapezium.

(d)



The figure shows a kite.

2. **(a)** $A(-5, 0)$, $B(-4, 3)$, $C(-3, 4)$, $D(0, 5)$, $E(3, 4)$, $F(4, 3)$, $G(4, -3)$, $H(3, -4)$, $I(-3, -4)$, $J(-4, -3)$

(b) **(i)** H

(ii) G

3. **(i)** When $x = 12$,

$$y = 4(12) - 1\frac{1}{2}$$
$$= 48 - 1\frac{1}{2}$$
$$= 46\frac{1}{2}$$

(ii) When $x = 2\frac{1}{2}$,

$$y = 4\left(2\frac{1}{2}\right) - 1\frac{1}{2}$$
$$= 10 - 1\frac{1}{2}$$
$$= 8\frac{1}{2}$$

(iii) When $x = -\frac{1}{2}$,

$$y = 4\left(-\frac{1}{2}\right) - 1\frac{1}{2}$$
$$= -2 - 1\frac{1}{2}$$
$$= -3\frac{1}{2}$$

4. **(i)** When $y = 150$,

$$150 = 250 - 20x$$
$$150 - 250 = -20x$$
$$-100 = -20x$$
$$\therefore x = 5$$

(ii) When $y = 450$,

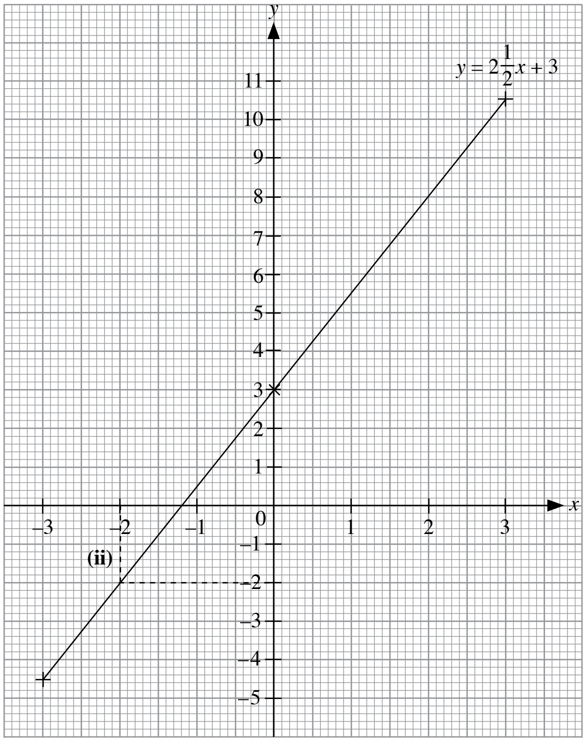
$$450 = 250 - 20x$$
$$450 - 250 = -20x$$
$$200 = -20x$$
$$\therefore x = -10$$

(iii) When $y = -1150$,

$$-1150 = 250 - 20x$$
$$-1150 - 250 = -20x$$
$$-1400 = -20x$$
$$\therefore x = 70$$

5. **(i)**

x	–3	0	3
$y = 2\frac{1}{2}x + 3$	–4.5	3	10.5



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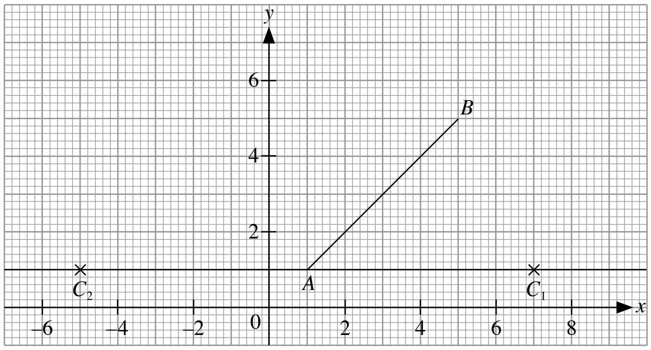
(ii) From the graph in **(i)**,
when $x = -2$,
$a = y = -2$
From the graph in **(i)**,
when $y = 3$,
$b = x = 0$

Challenge Yourself



Teachers can guide students to first draw the line $y = 1$ on a piece of graph paper.

The base of the triangle will be AC and its height will be the perpendicular height from B to AC i.e 4 units. Hence, the base has to be 6 units. Counting 6 units to the right of A will give the point $C_1(7, 1)$, and counting 6 units to the left of A will give the point $C_2(-5, 1)$.

Chapter 7 Number Patterns

TEACHING NOTES

Suggested Approach

Students have done word problems involving number sequences and patterns in primary school. These word problems required the students to recognise simple patterns from various number sequences and determine either the next few terms or a specific term. However, they were not taught to use algebra to solve problems involving number patterns. Teachers can arouse students' interest in this topic by bringing in real-life applications (see chapter opener on page 157 and Investigation: Fibonacci Sequence).

Section 7.1: Number Sequences

In primary school, students were only asked how to find the next few terms and a specific term of number sequences but they have not been taught how to state the rule. Teachers can build upon this by getting students to work in pairs to state the rules of number sequences and then write down the next few terms (see Class Discussion: Number Sequences). Students should learn that they can add, subtract, multiply or divide or use a combination of arithmetic operations to get the next term of a number sequence.

Section 7.2: General Term of a Number Sequence

Teachers can build upon what students have learnt in Chapter 4 (Basic Algebra and Manipulation) and teach students how to observe a number sequence and look for a pattern so that they can use algebra and find a formula for the general term, $T_n = n^{\text{th}}$ term.

Teachers can get students to work in pairs to find a formula for the general term and hence find a specific term for different number sequences (see Class Discussion: Generalising Simple Sequences). After the students have learnt how to generalise simple sequences, they should know that the aim is not to simply solve the problem but to represent it so that it becomes a general expression which can be used to find specific terms.

Section 7.3: Number Patterns

In primary school, students have attempted questions involving number patterns. In this section, teachers can ask the students to apply what they have learnt for number sequences on number patterns.

Teachers can get students to work in pairs to find a formula for the general term and hence find a specific term for different number patterns (see Class Discussion: The Triangular Number Sequence). Through this class discussion, students should learn that they need not use a large number of coins to find the total number of coins needed to form a triangle with a base that has 100 coins. They need only to find the formula for the general term and they are able to find the total number of coins by substituting $n = 100$ into the formula. They should also learn that with the formula, they can find T_n easily for any n.

Section 7.4: Number Patterns in Real-World Contexts

Teachers may get students to discover number patterns in real-world contexts (e.g. shells, pine cones, rocks, wallpaper, floor tiles) and ask them to represent that number pattern into a general expression.

Through Worked Example 5, students will learn that in the real world, which in this case in Chemistry, the general term of a number sequence is important and advantageous in finding specific terms. In this worked example, finding the general term of the number of hydrogen atoms allowed one to find the member number, number of carbon atom(s) and number of hydrogen atoms easily without going through tedious workings, especially if the value of the specific term is large. For other figures, students should consider drawing the next figure in the sequence so as to identify the pattern.

Challenge Yourself

Some of the questions (e.g. Questions 3, 4 and 5) are not easy for average students while others (Questions 1 and 2) should be manageable if teachers guide them as follows:

Questions 1 and 2: Teachers can get the students to draw a table and write down the first 6 terms. The students have to observe carefully how each term in the sequence can be obtained and find a formula for the general term to get the final answer to the question.

Questions 3, 4 and 5: Teachers have to get the students to think beyond just the four operations. The students have to consider more ways and observe carefully how each term in the sequence is obtained. Once they have figured this out, they are able to search on the Internet to find out the names for the sequences.

WORKED SOLUTIONS

Class Discussion (Number Sequences)

1.

	Sequence	Rule
Positive even numbers	2, 4, 6, 8, 10, 12, 14, … +2 +2 +2 +2 +2 +2	Start with 2, then add 2 to each term to get the next term.
Positive odd numbers	1, 3, 5, 7, 9, 11, 13, … +2 +2 +2 +2 +2 +2	Start with 1, then add 2 to each term to get the next term.
Multiples of 3	3, 6, 9, 12, 15, 18, 21, … +3 +3 +3 +3 +3 +3	Start with 3, then add 3 to each term to get the next term.
Powers of 2	1, 2, 4, 8, 16, 32, 64, … ×2 ×2 ×2 ×2 ×2 ×2	Start with 1, then multiply each term by 2 to get the next term.
Powers of 3	1, 3, 9, 27, 81, 243, 729, … ×3 ×3 ×3 ×3 ×3 ×3	Start with 1, then multiply each term by 3 to get the next term.

Table 7.1

2. The sequence of positive odd numbers can be obtained by subtracting 1 from each term of the sequence 2, 4, 6, 8, 10, ….

Teachers may wish to note that there are other possible answers to this question.

3. **(a)** Rule: Find the square of the position of each term. The next two terms are 36 and 49.

(b) Rule: Find the cube of the position of each term. The next two terms are 216 and 343.

Class Discussion (Generalising Simple Sequences)

(a) Hence, $T_n = 3n$.

100^{th} term, $T_{100} = 3 \times 100$
$= 300$

(b) Hence, $T_n = n^2$.

100^{th} term, $T_{100} = 100^2$
$= 10\ 000$

(c) Hence, $T_n = n^3$.

100^{th} term, $T_{100} = 100^3$
$= 1\ 000\ 000$

Class Discussion (The Triangular Number Sequence)

1.

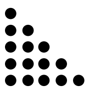


Figure 5

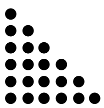


Figure 6

2.

Figure Number, n	Number of Coins at the Base of the Triangle, n	Total Number of Coins, T_n
1	1	$1 = 1 = \frac{1 \times 2}{2}$
2	2	$1 + 2 = 3 = \frac{2 \times 3}{2}$
3	3	$1 + 2 + 3 = 6 = \frac{3 \times 4}{2}$
4	4	$1 + 2 + 3 + 4 = 10 = \frac{4 \times 5}{2}$
5	5	$1 + 2 + 3 + 4 + 5 = 15 = \frac{5 \times 6}{2}$
6	6	$1 + 2 + 3 + 4 + 5 + 6 = 21 = \frac{6 \times 7}{2}$
⋮	⋮	⋮
n	n	$1 + 2 + 3 + 4 + \cdots + n = \frac{1}{2}n(n + 1)$

Table 7.6

3. When $n = 100$,

$$\frac{1}{2}n(n+1) = \frac{1}{2} \times 100 \times (100 + 1)$$
$$= \frac{1}{2} \times 100 \times 101$$
$$= 5050$$

Total number of coins needed to form a triangle with a base that has 100 coins = 5050

Investigation (Fibonacci Sequence)

1. 1; 5; 13; 21
2. 3, 5, 8, 13, 21, 34
3. Michaelmas Daisy has 55 petals.
4. 4, 6; 7, 10

Journal Writing (Page 169)

Pascal's Triangle was developed by the French Mathematician Blaise Pascal. It is formed by starting with the number 1. Each number in the subsequent rows is obtained by finding the sum of the number which is diagonally above it to the left and that which is diagonally above it to the right. 0 is used as a substitute in the absence of a number in either of the two positions.

1 1 2 3 5 8 13 21

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

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The Fibonacci sequence is a set of numbers that begins with 1 and 1, and each subsequent term is the sum of the previous two terms, i.e. 1, 1, 2, 3, 5, 8, 13, 21, … The sums of the numbers on the diagonals of Pascal's Triangle form the Fibonacci sequence, as illustrated.

Teachers may wish to get students to describe the symmetry in Pascal's Triangle and to identify other patterns that can be observed from the triangle.

Practise Now 1

1. **(a)** Rule: Add 5 to each term to get the next term. The next two terms are 28 and 33.

(b) Rule: Subtract 6 from each term to get the next term. The next two terms are –50 and –56.

(c) Rule: Multiply each term by 3 to get the next term. The next two terms are 1215 and 3645.

(d) Rule: Divide each term by –3 to get the next term. The next two terms are –18 and 6.

2. **(a)** 22, 29

(b) 15, 11

Practise Now 2

(i) $T_4 = 4(4) + 7$
$= 16 + 7$
$= 23$

(ii) $T_7 = 4(7) + 7$
$= 28 + 7$
$= 35$

Sum of 4th term and 7th term of sequence $= T_4 + T_7$
$= 23 + 35$
$= 58$

Practise Now 3

1. **(a)** Since the common difference is 4, $T_n = 4n + ?$.
The term before T_1 is $c = T_0$
$= 5 – 4$
$= 1.$
$\therefore$ General term of sequence, $T_n = 4n + 1$

(b) Since the common difference is 5, $T_n = 5n + ?$.
The term before T_1 is $c = T_0$
$= 7 – 5$
$= 2.$
$\therefore$ General term of sequence, $T_n = 5n + 2$

(c) Since the common difference is 6, $T_n = 6n + ?$.
The term before T_1 is $c = T_0$
$= 2 – 6$
$= –4.$
$\therefore$ General term of sequence, $T_n = 6n – 4$

(d) Since the common difference is 3, $T_n = 3n + ?$.
The term before T_1 is $c = T_0$
$= 1 – 3$
$= –2.$
$\therefore$ General term of sequence, $T_n = 3n – 2$

2. **(i)** 23, 27

(ii) Since the common difference is 4, $T_n = 4n + ?$.
The term before T_1 is $c = T_0$
$= 3 – 4$
$= –1.$
$\therefore$ General term of sequence, $T_n = 4n – 1$

(iii) $T_{50} = 4(50) – 1$
$= 200 – 1$
$= 199$

Practise Now 4

1. **(i)**

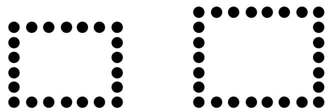

Figure 5 Figure 6

(ii)

Figure Number	Number of Dots
1	$2 + 1 \times 4 = 6$
2	$2 + 2 \times 4 = 10$
3	$2 + 3 \times 4 = 14$
4	$2 + 4 \times 4 = 18$
5	$2 + 5 \times 4 = 22$
6	$2 + 6 \times 4 = 26$
⋮	⋮
n	$2 + n \times 4 = 4n + 2$

(iii) When $n = 2013$,
$4n + 2 = 4(2013) + 2$
$= 8054$
Number of dots in 2013th figure = 8054

2. **(i)** 8th line: $72 = 8 \times 9$

(ii) Since $110 = 10 \times 11 = 10(10 + 1)$,
$k = 10$.

Practise Now 5

(i)

Member Number	Number of carbon atoms	Number of hydrogen atoms
1	2	4
2	3	6
3	4	8
4	5	10
5	6	12
6	7	14
⋮	⋮	⋮
n	$n + 1$	$2n + 2$

(ii) Let $h + 1 = 55$.

$h = 55 - 1$
$= 54$

When $n = h = 54$,

$2n + 2 = 2(54) + 2$
$= 110$

Number of hydrogen atoms the member has = 110

(iii) Let $2k + 2 = 120$.

$2k = 120 - 2$
$= 118$
$k = 59$

When $n = k = 59$,

$n + 1 = 59 + 1$
$= 60$

Number of carbon atoms the member has = 60

Exercise 7A

1. **(a)** Rule: Add 5 to each term to get the next term. The next two terms are 39 and 44.

(b) Rule: Subtract 8 from each term to get the next term. The next two terms are 40 and 32.

(c) Rule: Multiply each term by 2 to get the next term. The next two terms are 384 and 768.

(d) Rule: Divide each term by 2 to get the next term. The next two terms are 50 and 25.

(e) Rule: Divide each term by –4 to get the next term. The next two terms are 16 and –4.

(f) Rule: Multiply each term by –2 to get the next term. The next two terms are –288 and 576.

(g) Rule: Subtract 7 from each term to get the next term. The next two terms are –87 and –94.

(h) Rule: Add 10 to each term to get the next term. The next two terms are –50 and –40.

(i) Rule: Add 10 to each term to get the next term. The next two terms are 50 and 60.

(j) Rule: Add 7 to each term to get the next term. The next two terms are 80 and 87.

(k) Rule: Multiply each term by 3 to get the next term. The next two terms are 324 and 972.

2. **(a)** 9, 15

(b) 12, 8

(c) –33, –32

(d) 88, 85

(e) 21, 28

3. **(a)** –67, –131

(b) 8, 13

(c) 144, 196

(d) –216, 343

(e) 81, 243

Exercise 7B

1. **(a)** Since the common difference is 6, $T_n = 6n + ?$.

The term before T_1 is $c = T_0$
$= 7 - 6$
$= 1$.

$\therefore$ General term of sequence, $T_n = 6n + 1$

(b) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0$
$= -4 - 3$
$= -7$.

$\therefore$ General term of sequence, $T_n = 3n - 7$

(c) Since the common difference is 7, $T_n = 7n + ?$.

The term before T_1 is $c = T_0$
$= 60 - 7$
$= 53$.

$\therefore$ General term of sequence, $T_n = 7n + 53$

(d) Since the common difference is –3, $T_n = -3n + ?$.

The term before T_1 is $c = T_0$
$= 14 + 3$
$= 17$.

$\therefore$ General term of sequence, $T_n = -3n + 17$

2. **(i)** $T_5 = 2(5) + 5$
$= 10 + 5$
$= 15$

(ii) $T_8 = 2(8) + 5$
$= 16 + 5$
$= 21$

(iii) $15 = 3 \times 5$
$21 = 3 \times 7$

LCM of 5th term and 8th term of sequence $= 3 \times 5 \times 7$
$= 105$

3. **(i)** 18, 21

(ii) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0$
$= 3 - 3$
$= 0$.

$\therefore$ General term of sequence, $T_n = 3n$

(iii) $T_{105} = 3(105)$
$= 315$

4. **(i)** 30, 34

(ii) Since the common difference is 4, $T_n = 4n + ?$.

The term before T_1 is $c = T_0$
$= 10 - 4$
$= 6$.

$\therefore$ General term of sequence, $T_n = 4n + 6$

(iii) $T_{200} = 4(200) + 6$
$= 800 + 6$
$= 806$

5. **(i)**

Number of points	1	2	3	4	5	6
Number of segments	$1 + 1$ $= 2$	$2 + 1$ $= 3$	$3 + 1$ $= 4$	$4 + 1$ $= 5$	$5 + 1$ $= 6$	$6 + 1$ $= 7$

(ii) Let the number of points be n.

Number of segments $= n + 1$.

When $n = 49$, number of segments $= 49 + 1$

$= 50$

(iii) $101 = n + 1$

$\therefore n = 101 - 1$

$= 100$

6. **(i)**

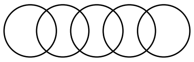

Figure 5

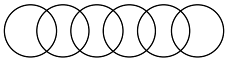

Figure 6

(ii)

Figure Number	**Number of Intersection(s) between the Circles**
1	0
2	1
3	2
4	3
5	4
6	5
⋮	⋮
n	$n - 1$

(iii) Let $n - 1 = 28$.

$n = 28 + 1$

$= 29$

7. **(a)** When $n = 1$,

$2n^2 + 1 = 2(1)^2 + 1$

$= 2 + 1$

$= 3$

When $n = 2$,

$2n^2 + 1 = 2(2)^2 + 1$

$= 8 + 1$

$= 9$

When $n = 3$,

$2n^2 + 1 = 2(3)^2 + 1$

$= 18 + 1$

$= 19$

When $n = 4$,

$2n^2 + 1 = 2(4)^2 + 1$

$= 32 + 1$

$= 33$

The first four terms of the sequence are 3, 9, 19 and 33.

(b) **(i)** General term of sequence, $T_n = 2n^2 + 1 - 2$

$= 2n^2 - 1$

(ii) $T_{388} = 2(388)^2 - 1$

$= 301\,088 - 1$

$= 301\,087$

8. **(i)**

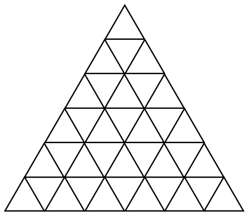

Figure 5

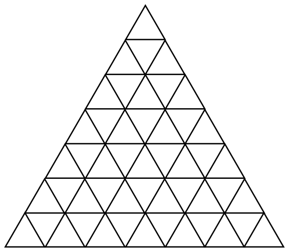

Figure 6

(ii)

Figure Number	**Number of Small Triangles**
1	4
2	9
3	16
4	25
5	36
6	49
⋮	⋮
n	$(n + 1)^2$

(iii) When $n = 20$,

$(n + 1)^2 = (20 + 1)^2$

$= 21^2$

$= 441$

Number of triangles in 20th figure = 441

(iv) Let $(n + 1)^2 = 121$.

$n + 1 = 11$ or $n + 1 = -11$

$n = 11 - 1$ or $n = -11 - 1$

$= 10$ or $= -12$ (N.A. since $n > 0$)

9. **(i)** 6th line: $54 = 6 \times 9$

(ii) Since $208 = 13 \times 16 = 13(13 + 3)$,

$k = 13$.

10. **(i)** 5th line: $1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2 = (5 + 1)^2$

(ii) $c = \sqrt{169}$

$= 13$

$d + 1 = 13$

$d = 13 - 1$

$= 12$

$a = 13 + 12$

$= 25$

11. (a) (i)

Number of people	4	6	8	10	12	14
Number of tables	$\frac{4-2}{2}=1$	$\frac{6-2}{2}=2$	$\frac{8-2}{2}=3$	$\frac{10-2}{2}=4$	$\frac{12-2}{2}=5$	$\frac{14-2}{2}=6$

(ii)

Number of tables	1	2	3	4	5	6
Number of people	$2(1)+2=4$	$2(2)+2=6$	$2(3)+2=8$	$2(4)+2=10$	$2(5)+2=12$	$2(6)+2=14$

(b) (i) From (a)(i): When $n = 20$,

$$\frac{n-2}{2} = \frac{20-2}{2}$$
$$= 9$$

$\therefore$ 9 tables will be needed to seat 20 people.

(ii) When $n = 30$,

$$\frac{n-2}{2} = \frac{30-2}{2}$$
$$= 14$$

$\therefore$ 14 tables will be needed to seat 30 people.

(c) (i) From (a)(ii): When $n = 22$,

$2(22) + 2 = 46$

$\therefore$ 46 people can be seated at 22 tables.

(ii) When n = 36,

$2(36) + 2 = 74$

$\therefore$ 74 people can be seated at 36 tables.

12. (i)

Number of points on the line segments *AB* (including the points *A* and *B*)	2	3	4	5	6	7
Number of possible line segments	$\frac{2\times(2-1)}{2}$ $= 1$	$\frac{3\times(3-1)}{2}$ $= 3$	$\frac{4\times(4-1)}{2}$ $= 6$	$\frac{5\times(5-1)}{2}$ $= 10$	$\frac{6\times(6-1)}{2}$ $= 15$	$\frac{7\times(7-1)}{2}$ $= 21$

(ii) Number of points including $AB = 18 + 2$
$= 20$

Number of possible line segments $= \frac{20\times(20-1)}{2}$
$= 190$

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13. (i) 1 5 10 10 5 1

(ii)

Row	Sum
1	$1 = 1 = 2^0$
2	$1 + 1 = 2 = 2^1$
3	$1 + 2 + 1 = 4 = 2^2$
4	$1 + 3 + 3 + 1 = 8 = 2^3$
5	$1 + 4 + 6 + 4 + 1 = 16 = 2^4$
6	$1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$
⋮	⋮
n	$1 + (n - 1) + \cdots + (n - 1) + 1 = 2^{n-1}$

14. (a)

Figure	1	2	3	4	5	6
Number of black squares (b)	1	2	3	4	5	6
Number of white squares (w)	$1 \times 2 + 1 = 3$	$2 \times 2 + 1 = 5$	$3 \times 2 + 1 = 7$	$4 \times 2 + 1 = 9$	$5 \times 2 + 1 = 11$	$6 \times 2 + 1 = 13$
Area of whole figure ($b + w$)	4	7	10	13	16	19
Perimeter of whole figure (cm)	$2(1 + 4) = 10$	$2(2 + 4) = 12$	$2(3 + 4) = 14$	$2(4 + 4) = 16$	$2(5 + 4) = 18$	$2(6 + 4) = 20$

(b) (i) Number of white squares in Figure 9 $= 9 \times 2 + 1$
$= 19$

(ii) Perimeter of Figure 9 $= 2(9 + 4)$
$= 26$ cm

(iii) Number of white squares in Figure $n = n(2 + 1)$
$= 2n + 1$

(iv) Perimeter of Figure $n = 2(n + 4)$
$= (2n + 8)$ cm

15. (i) 8th line: $\frac{2}{8 \times 9 \times 10} = \frac{1}{8} - \frac{2}{9} + \frac{1}{10}$

(ii) Based on the pattern, nth line:

$$\frac{2}{n(n+1)(n+2)} = \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}$$

$$\therefore \frac{1}{10} - \frac{2}{11} + \frac{1}{12} = \frac{2}{10 \times 11 \times 12}$$
$$= \frac{2}{1320}$$
$$= \frac{1}{660}$$

(iii)
$$\frac{2}{7980} = \frac{1}{p} - \frac{2}{p+1} + \frac{1}{p+2}$$
$$\frac{1}{p} - \frac{2}{p+1} + \frac{1}{p+2} = \frac{2}{p(p+1)(p+2)}$$
$$\therefore p(p + 1)(p + 2) = 7980$$
$$(p^2 + p)(p + 2) = 7980$$
$$p^3 + 2p^2 + p^2 + 2p - 7980 = 0$$
$$p^3 + 3p^2 + 2p - 7980 = 0$$
$\therefore p = 19$ or
$p = -11 + 17.292$ (5 s.f.) (reject, p is a whole number) or
$p = -11 - 17.292$ (5 s.f.) (reject, $p > 1$)

16. (a) (i) 11, 13
(ii) 24, 28
(iii) 84, 112
(iv) 85, 113

(b) 6th line: $13^2 + 84^2 = 85^2$
7th line: $15^2 + 112^2 = 113^2$

17. (i)

Member Number	Number of carbon atoms	Number of hydrogen atoms
1	3	4
2	4	6
3	5	8
4	6	10
5	7	12
6	8	14
⋮	⋮	⋮
n	$n + 2$	$2n + 2$

(ii) Let $h + 2 = 25$.
$h = 25 - 2$
$= 23$
When $n = h = 23$,
$2n + 2 = 2(23) + 2$
$= 48$
Number of hydrogen atoms the member has = 48

(iii) Let $2k + 2 = 64$.

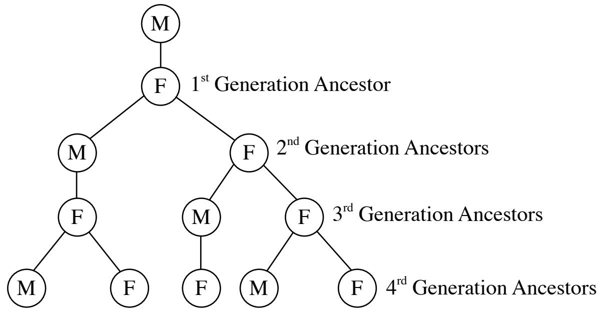
$$\begin{aligned} 2k &= 64 - 2 \\ &= 62 \\ k &= 31 \end{aligned}$$

When $n = k = 31$,

$$\begin{aligned} n + 2 &= 31 + 2 \\ &= 33 \end{aligned}$$

Number of carbon atoms the member has = 33

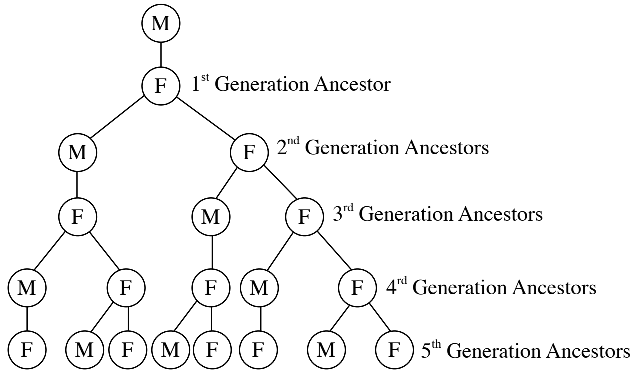
18. (i)



Number of 4th generation ancestors a male bee has = 5

(ii) The number of n^{th} generation ancestors forms a sequence: 1, 2, 3, 5, ... The first two numbers of the sequence are 1 and 2, and each subsequent term is the sum of the previous two terms.

(iii) Number of 5th generation ancestors a male bee has $= 3 + 5$
$= 8$



(iv) The sequence for the number of n^{th} generation ancestors is 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Number of 10th generation ancestors a male bee has $= 34 + 55$
$= 89$

Review Exercise 7

1. **(a)** 53, 44

(b) 28, 40

(c) $\frac{1}{27}$, $\frac{1}{81}$

(d) 121, 169

2. **(i)** 64, 81

(ii) General term of sequence, $T_n = (n + 2)^2$

(iii)
$$\begin{aligned} T_{25} &= (25 + 2)^2 \\ &= 27^2 \\ &= 729 \end{aligned}$$

3. (i)

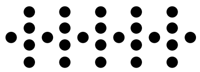


Figure 5

(ii)

Figure Number	Number of Buttons
1	$5 \times 1 + 1 = 6$
2	$5 \times 2 + 1 = 11$
3	$5 \times 3 + 1 = 16$
4	$5 \times 4 + 1 = 21$
5	$5 \times 5 + 1 = 26$
⋮	⋮
n	$5 \times n + 1 = 5n + 1$

(iii) When $n = 56$,

$$\begin{aligned} 5n + 1 &= 5(56) + 1 \\ &= 281 \end{aligned}$$

Number of buttons in 56th figure = 281

(iv) Let $5n + 1 = 583$.

$$\begin{aligned} 5n &= 583 - 1 \\ &= 582 \\ n &= 116\frac{2}{5} \end{aligned}$$

Since $n = 116\frac{2}{5} \notin \mathbb{Z}^+$, it is not possible for a figure in the sequence to be made up of 583 buttons.

4. (i)

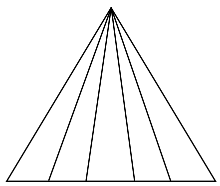


Figure 5

(ii)

Figure Number	Number of Triangles
1	$1 = \frac{1 \times 2}{2}$
2	$3 = \frac{2 \times 3}{2}$
3	$6 = \frac{3 \times 4}{2}$
4	$10 = \frac{4 \times 5}{2}$
5	$15 = \frac{5 \times 6}{2}$
⋮	⋮
n	$\frac{1}{2}n(n + 1)$

(iii) When $n = 77$,

$$\begin{aligned} \frac{1}{2}n(n + 1) &= \frac{1}{2} \times 77 \times (77 + 1) \\ &= \frac{1}{2} \times 77 \times 78 \\ &= 3003 \end{aligned}$$

Number of triangles in 77th figure = 3003

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(iv) Let $\frac{1}{2}n(n+1) = 66$.

$n(n+1) = 132$

Since $132 = 11 \times 12 = 11(11+1)$,

$n = 11$.

5. **(i)** 7^{th} line: $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$
$= 784 = (1 + 2 + 3 + 4 + 5 + 6 + 7)^2$

(ii) $1^3 + 2^3 + 3^3 + 4^3 + \cdots + 15^3 = (1 + 2 + 3 + 4 + \cdots + 15)^2$
$= 120^2$
$= 14\ 400$

(iii) Since $1^3 + 2^3 + 3^3 + 4^3 + \cdots + k^3 = 1296 = 36^2 = (1 + 2 + 3 + 4 + \cdots + 8)^2$,

$k = 8$.

6. **(i)** $a = \sqrt{12^2 + 35^2}$
$= \sqrt{1369}$
$= 37$

(ii) Since the common difference is 2, $T_{n_A} = 2n + ?$.

The term before T_{1_A} is $c = T_{0_A}$
$= 4 - 2$
$= 2$.

$\therefore$ General term of sequence A, $T_{n_A} = 2n + 2$

(iii) General term of sequence C, $T_{n_C} = \sqrt{T_{n_A}^2 + T_{n_B}^2}$
$= \sqrt{(2n+2)^2 + (n^2 + 2n)^2}$

$$T_{18_C} = \sqrt{\left[2(18)+2\right]^2 + \left[18^2 + 2(18)\right]^2}$$
$$= \sqrt{38^2 + 360^2}$$
$$= \sqrt{131\,044}$$
$$= 362$$

Challenge Yourself

1.

Value	3^1	3^2	3^3	3^4	3^5	3^6	$\cdots$
Last Digit	3	9	7	1	3	9	$\cdots$

$2015 \div 4 = 503$ R 3

$\therefore$ Last digit of $3^{2015} = 7$

2.

Number of People	**Number of Handshakes**
2	$1 = \frac{2 \times 1}{2}$
3	$3 = \frac{3 \times 2}{2}$
4	$6 = \frac{4 \times 3}{2}$
5	$10 = \frac{5 \times 4}{2}$
6	$15 = \frac{6 \times 5}{2}$
$\vdots$	$\vdots$
n	$\frac{1}{2}n(n-1)$

Number of handshakes that will take place = $\frac{1}{2}n(n-1)$

3. **(i)** 4, 9

(ii) The general term, T_n, of the sequence is obtained by continuously finding the sum of the digits of n^2 until a single-digit number is left, e.g. to obtain T_7,

$7^2 = 49 \rightarrow 4 + 9 = 13 \rightarrow 1 + 3 = 4$, $\therefore T_7 = 4$.

4. **(i)** 11, 18

(ii) For $n \geqslant 3$, $T_n = T_{n-1} + T_{n-2}$.

(iii) Lucas Numbers (which is different from Lucas Sequence)

5. **(i)** 10, 12

(ii) For $n \geqslant 4$, $T_n = T_{n-2} + T_{n-3}$.

(iii) Perrin Numbers (or Perrin Sequence)

Revision Exercise B1

1. **(a)** $0.15x + 2.35(x - 2) = 1.3$

$$\begin{aligned} 0.15x + 2.35x - 4.7 &= 1.3 \\ 2.5x - 4.7 &= 1.3 \\ 2.5x &= 1.3 + 4.7 \\ 2.5x &= 6 \\ \therefore x &= 2.4 \end{aligned}$$

(b) $\dfrac{5}{1-y} - \dfrac{7}{2-2y} = 4$

$$\begin{aligned} \frac{5}{1-y} - \frac{7}{2(1-y)} &= 4 \\ \frac{10-7}{2(1-y)} &= 4 \\ \frac{3}{2(1-y)} &= 4 \\ 3 &= 8(1-y) \\ 3 &= 8 - 8y \\ 3 - 8 &= -8y \\ -5 &= -8y \\ \therefore y &= \frac{5}{8} \end{aligned}$$

2. **(a)** $12x > 60$

$\therefore x > 5$

(b) $15y \leqslant -24$

$$\begin{aligned} y &\leqslant -\frac{24}{15} \\ \therefore y &\leqslant -1\frac{3}{5} \end{aligned}$$

3. $\dfrac{x-4y}{5x+y} = \dfrac{3}{5}$

$$\begin{aligned} 5(x-4y) &= 3(5x+y) \\ 5x - 20y &= 15x + 3y \\ 5x - 15x &= 3y + 20y \\ -10x &= 23y \\ \frac{x}{y} &= -\frac{23}{10} \\ \therefore \frac{x}{3y} &= -\frac{23}{30} \end{aligned}$$

4. Let the number of 20-cent coins in the box be x.

Then the number of 50-cent coins in the box is $54 - x$.

$$\begin{aligned} 0.2x + 0.5(54 - x) &= 20.7 \\ 0.2x + 27 - 0.5x &= 20.7 \\ -0.3x + 27 &= 20.7 \\ -0.3x &= 20.7 - 27 \\ -0.3x &= -6.3 \\ x &= 21 \end{aligned}$$

There are twenty-one 20-cent coins in the box.

5. Let the time the motorist spends on the expressway be x hours.

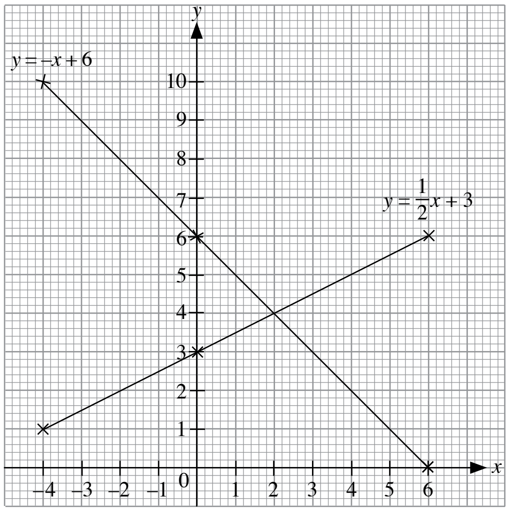
Then the time he spends on the stretch of road is $2x$ hours.

$$\begin{aligned} 95x + 65 \times 2x &= 375 \\ 95x + 130x &= 375 \\ 225x &= 375 \\ x &= 1\frac{2}{3} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total time taken} &= x + 2x \\ &= 3x \\ &= 3\left(1\frac{2}{3}\right) \\ &= 5 \text{ hours} \end{aligned}$$

6.

x	-4	0	6
$y = \frac{1}{2}x + 3$	1	3	6
$y = -x + 6$	10	6	0



7. **(i)** Since the common difference is 9, $T_n = 9n + ?$.

The term before T_1 is $c = T_0$

$$\begin{aligned} &= 6 - 9 \\ &= -3. \end{aligned}$$

$\therefore$ General term of sequence, $T_n = 9n - 3$

(ii) Let $9k - 3 = 159$.

$$\begin{aligned} 9k &= 159 + 3 \\ &= 162 \\ \therefore k &= 18 \end{aligned}$$

8. **(i)** 8^{th} line: $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2$

(ii) Since $144 = 12^2$,

$k = 12$.

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Revision Exercise B2

1. (a) $\frac{1}{3}(x-3) - x + 5 = 3(x-1)$

$\frac{1}{3}x - 1 - x + 5 = 3x - 3$

$-\frac{2}{3}x + 4 = 3x - 3$

$-\frac{2}{3}x - 3x = -3 - 4$

$-\frac{11}{3}x = -7$

$\therefore x = 1\frac{10}{11}$

(b) $\frac{2}{y} - \frac{3}{y} + 1 = 3$

$-\frac{1}{y} + 1 = 3$

$-\frac{1}{y} = 3 - 1$

$-\frac{1}{y} = 2$

$-y = \frac{1}{2}$

$\therefore y = -\frac{1}{2}$

2. (a) $14x \geqslant -110$

$\therefore x \geqslant -7\frac{6}{7}$

(b) $-18 < 3y$

$-\frac{18}{3} < y$

$\therefore y > -6$

3. Let the smallest even number be x.
Then the next 6 even numbers are $x + 2$, $x + 4$, $x + 6$, $x + 8$, $x + 10$ and $x + 12$.

$x + x + 2 + x + 4 + x + 6 + x + 8 + x + 10 + x + 12 = 336$

$7x + 42 = 336$

$7x = 336 - 42$

$7x = 294$

$\therefore x = 42$

The smallest of the 7 numbers is 42.

4. $8.5x + 3.6(2x + 5) = 206.4$

$8.5x + 7.2x + 18 = 206.4$

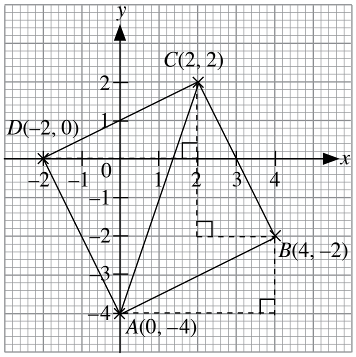
$15.7x + 18 = 206.4$

$15.7x = 206.4 - 18$

$15.7x = 188.4$

$\therefore x = 12$

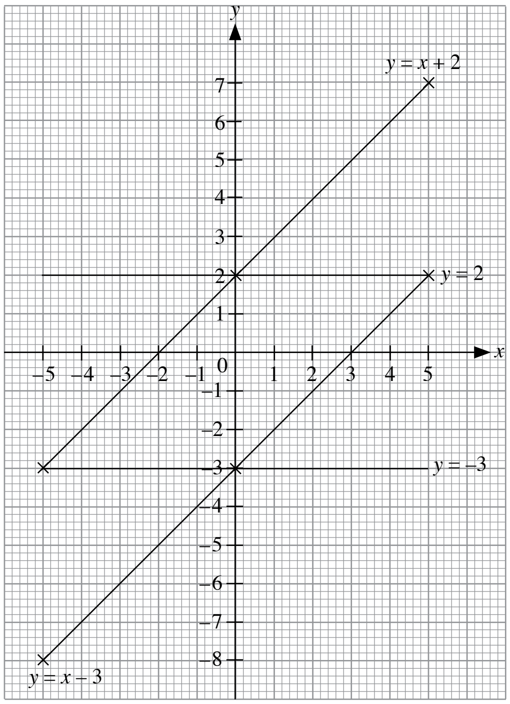
5. (i)



(ii) Coordinates of $D = (-2, 0)$

6. (a)

x	–5	0	5
$y = x + 2$	–3	2	7
$y = x - 3$	–8	–3	2



(b) Area enclosed by the four lines $= \frac{1}{2} \times 5 \times 5 + \frac{1}{2} \times 5 \times 5$

$= 12\frac{1}{2} + 12\frac{1}{2}$

$= 25 \text{ units}^2$

7. (i) Since the common difference is –3, $T_n = -3n + ?$.
The term before T_1 is $c = T_0$

$= 44 + 3$

$= 47.$

$\therefore$ General term of sequence, $T_n = -3n + 47$

(ii) Let $-3k + 47 = -13$.

$-3k = -13 - 47$

$= -60$

$\therefore k = 20$

8. (i) 6^{th} line: $6^2 - 2 \times 6 = 24$

(ii) Since $9^2 - 2 \times 9 = 63$,
$k = 9$.

Chapter 8 Percentage

TEACHING NOTES

Suggested Approach

Although students have learnt percentage in primary school (i.e. how to express a part of a whole as a percentage, write fractions and decimals as percentages, and vice versa, find a percentage part of a whole and solve up to 2-step word problems involving percentage), many may still struggle with percentage. Teachers can introduce percentage as fractions by going right back to the fundamentals. Teachers can give students practical applications of percentages and show the changes in fractions and proportions through the examples to give them a better understanding of the concept.

Section 8.1: Introduction to Percentage

Teachers can get students to work in pairs to find an advertisement/article in which percentage(s) can be found and discuss about it together (see Class Discussion: Percentages in Real Life). After the discussion, students should understand the meaning of percentage(s) better and interpret information more accurately. Students need to be able to comment critically on the usefulness of percentages before they can have a confident grasp of the topic.

Teachers can then build upon what students have learnt about percentage in primary school. Students may be able to learn how to accurately calculate a percentage but they might struggle to explain the meaning behind it. Teachers should emphasise on the basics of fractions and proportions before getting the students to calculate and interpret percentages.

In Worked Example 5, students should learn that it is easy to see that more people passed the entrance test in 2011 but it is not easy to see which year had a higher proportion of people passing the entrance test. Teachers can highlight to the students that two quantities can be easily compared using percentages because the proportions are converted to the same base i.e. 100.

Section 8.2: Percentage Change and Reverse Percentage

Teachers should guide students on how to use algebra in percentage change and reverse percentage. Students may draw models, wherever applicable, to help them understand the problem.

Through the worked examples in this section, students should be able to tackle percentage change and reverse percentage problems involving algebra. They should also learn how to identify whether the problem is a reverse percentage or a percentage change problem. Teachers can highlight to the students that percentage change is when they are given both the new value and the original value while a reverse percentage is when they need to find the original value given a quantity after a percentage increase or decrease.

WORKED SOLUTIONS

Class Discussion (Percentage in Real Life)

1. **Guiding Questions:**
   - What is the advertisement/article about?
   - Are the percentages found in the advertisement/article expressed using the percentage symbol or in words?
   - What do the percentages mean in the context of the advertisement/article?

   Teachers may use this to assess students' prior knowledge of percentage, e.g. whether students are able to relate percentages to fractions and to perform relevant calculations using the given percentages to illustrate the meaning of the percentages in the context of the advertisement/article. Teachers may also use this as a trigger to show students the need to learn percentage, and link back to the different scenarios in the advertisement/article.
   Alternatively, teachers may wish to use the article titled 'A smaller and cheaper iPad' (Today, 5 July 2012) and/or the apparel advertisement (Page 3, Today, 5 July 2012) for this question.

   The article 'A smaller and cheaper iPad?' is about Apple's apparent intention to launch an iPad which is smaller and less expensive. This is in order to counter its rivals' new products so as to maintain its stronghold in the tablet market. The percentage found in the article is 61 per cent, which is expressed in words and in the context of the article, it means that Apple has 61% of the tablet market share. A conclusion that can be drawn is that the iPad is the most popular tablet in the market as the total tablet market share is the base, i.e. 100%. The other brands have a total of 100% – 61% = 39% of the tablet market share.

   The advertisement shows that a particular brand of apparel is holding a storewide end-of-season sale. The percentages found in the advertisement are 70% and 10%, which are expressed using the percentage symbol. In the context of the advertisement, it means that shoppers can enjoy a discount of up to 70% on the items and those with UOB cards are entitled to an additional 10% off if they purchase a minimum of 3 items.

   Teachers may wish to ask students how the additional 10% off for UOB card members is calculated, i.e. whether a shopper with a UOB card gets to enjoy a maximum discount of 70% + 10% = 80% on selected items. Teachers may also wish to get students to use any amount,
   e.g. $100, to illustrate the meaning of the percentages, i.e. 70% and 10%, in the context of the advertisement.

2. **Guiding Questions:**
   - What is the meaning of the term 'up to'?
   - An advertisement with the phrase 'Discount up to 80% on All Items' is displayed at the entrance of a shop. If an item in the shop is sold at a discount of 10%, does this mean that the shopkeeper is dishonest?
   - Is it true that all the items in the shop are sold at a discount? Could there be any exceptions?
   - Does it mean that the prices of all the items in the shop are very low?

   The term 'up to' in the phrase 'Discount up to 80% on All Items' suggests that the greatest percentage discount given on the items in the shop is 80%. This means that some items in the shop may be sold at a discount that is less than 80%. Hence, the shopkeeper is not dishonest.
   Most of the time, such advertisements come with terms and conditions that may state the items which are not subjected to a discount, such as 'New Arrivals'. These terms and conditions are normally shown in fine print.
   As some items may be sold at a discount of less than 80% and some items may not be subjected to a discount, the prices of the items in the shop may not be low. In addition, the original prices of some items may be very high, such that their prices are still high even after a discount.

   Teachers may wish to ask students to list other instances where such phrases are used. They may also want to take this opportunity to highlight to students the importance of being informed consumers. Students should not take information at face value. Instead, they should learn how to interpret information accurately.

3. **Guiding Questions:**
   The following shows examples of statements with percentages more than 100%:
   - In Singapore, the number of employers hiring ex-offenders has increased by more than 100% from 2004 to 2011.
   - The total number of registrants for a school during Phase 2B of the Primary 1 registration is 120% of the number of vacancies available.
   - The number of mobile subscriptions in Singapore in 2011 is about 150% of her population.
   - The population of Singapore in 2010 is about 240% of that in 1970.
   - The total fertility rate in Singapore in 1970 is about 270% of that in 2010.

   The phrase 'this year's sales is 200% of last year's sales' means that the sales this year is 2 times of that of last year.

   Teachers may ask students to refer to page 197 of the textbook for an example of how this phrase may be used. Teachers should also highlight to students that the use of percentages can be misleading, e.g. a salesman who sold a car in January and two cars in February can say that his sales in February is 200% of that in January.

Class Discussion (Expressing Two Quantities in Equivalent Forms)

1. **(a) (i)** Required percentage $= \frac{40}{50} \times 100\%$
   $= 80\%$
   - There are 80% as many male teachers as female teachers.
   - The number of male teachers is 80% of the number of female teachers.
   - The number of male teachers is $\frac{4}{5}$ of the number of female teachers.

(ii) Required percentage = $\dfrac{50}{40} \times 100\%$
$= 125\%$

- There are 125% as many female teachers as male teachers.
- The number of female teachers is 125% of the number of male teachers.
- The number of female teachers is $\dfrac{5}{4}$ of the number of male teachers.

(b)

In words	A is 80% of B.	B is 125% of A.
Percentage	$A = 80\% \times B$	$B = 125\% \times A$
Fraction	$A = \dfrac{4}{5}$ (fraction) $\times B$	$B = \dfrac{5}{4}$ (fraction) $\times A$
Decimal	$A = 0.8 \times B$	$B = 1.25 \times A$

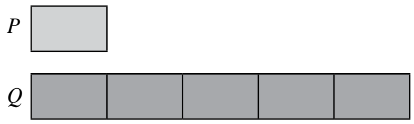
Table 8.1

2. (i)

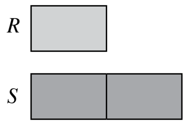
In words	P is 20% of Q.	R is 50% of S.	T is 125% of U.
Percentage	$P = 20\% \times Q$	$R = 50\% \times S$	$T = 125\% \times U$
Fraction	$P = \dfrac{1}{5}$ (fraction) $\times Q$	$R = \dfrac{1}{2}$ (fraction) $\times S$	$T = \dfrac{5}{4}$ (fraction) $\times U$
Decimal	$P = 0.2 \times Q$	$R = 0.5 \times S$	$T = 1.25 \times Q$

Table 8.2

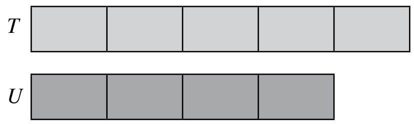
(ii) The relationship between P and Q can be illustrated as follows:



The relationship between R and S can be illustrated as follows:



The relationship between T and U can be illustrated as follows:



Thinking Time (Page 198)

1. No, it is not correct to say that $\dfrac{20\% + 80\%}{2}$, i.e. 50% of the total number of students in the two groups had done the survey. This is because there may be a different number of students in each of the two groups, e.g. if Ethan conducted the survey on 20% of a group of 100 students and on 80% of another group of 200 students, then $\dfrac{20\% \times 100 + 80\% \times 200}{100 + 200} \times 100\% = 60\%$ of the total number of students in the two groups had done the survey.

2. Mr Lee's monthly salary in 2011 $= 110\% \times \$x$
$= \dfrac{110}{100} \times \$x$
$= \$1.1x$

Mr Lee's monthly salary in 2012 $= 90\% \times \$1.1x$
$= \dfrac{90}{100} \times \$1.1x$
$= \$0.99x$

Hence, it is not correct to say that Mr Lee's monthly salary in 2012 was $\$x$.

Practise Now 1

(a) (i) $45\% = \dfrac{45}{100}$
$= \dfrac{9}{20}$

(ii) $305\% = \dfrac{305}{100}$
$= \dfrac{61}{20}$
$= 3\dfrac{1}{20}$

(iii) $5.5\% = \dfrac{5.5}{100}$
$= \dfrac{55}{1000}$
$= \dfrac{11}{200}$

(iv) $8\dfrac{5}{7}\% = \dfrac{61}{7}\%$
$= \dfrac{61}{7} \div 100$
$= \dfrac{61}{7} \times \dfrac{1}{100}$
$= \dfrac{61}{700}$

(b) (i) $\dfrac{17}{20} = \dfrac{17}{20} \times 100\%$
$= \dfrac{1700}{20}$
$= 85\%$

(ii) $23\dfrac{1}{5} = \dfrac{116}{5}$
$= \dfrac{116}{5} \times 100\%$
$= \dfrac{11\,600}{5}\%$
$= 2320\%$

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Practise Now 2

(a) **(i)** $12\% = \frac{12}{100}$

$= 0.12$

(ii) $413\% = \frac{413}{100}$

$= 4.13$

(iii) $23.6\% = \frac{23.6}{100}$

$= 0.236$

(iv) $6\frac{1}{4}\% = \frac{25}{4}\%$

$= \frac{25}{4} \div 100$

$= \frac{25}{4} \times \frac{1}{100}$

$= \frac{25}{400}$

$= 0.0625$

Alternatively,

$6\frac{1}{4}\% = 6.25\%$

$= \frac{6.25}{100}$

$= 0.0625$

(b) **(i)** $0.76 = 0.76 \times 100\%$

$= 76\%$

(ii) $2.789 = 2.789 \times 100\%$

$= 278.9\%$

Practise Now 3

1. **(i)** Total number of teachers in the school $= 45 + 75$

$= 120$

Percentage of male teachers in the school $= \frac{45}{120} \times 100\%$

$= 37.5\%$

(ii) **Method 1:**

Percentage of female teachers in the school $= \frac{75}{120} \times 100\%$

$= 62.5\%$

Method 2:

Percentage of female teachers in the school $= 100\% - 37.5\%$

$= 62.5\%$

2. Required percentage $= \frac{1400 \text{ m}l}{2.1\ l} \times 100\%$

$= \frac{1400 \text{ m}l}{2100 \text{ m}l} \times 100\%$

$= \frac{2}{3} \times 100\%$

$= 66\frac{2}{3}\%$

Practise Now (Page 190)

1. **(a)** 20% of $\$13.25 = 20\% \times \$13.25$

$= \frac{20}{100} \times \$13.25$

$= \$2.65$

(b) $15\frac{3}{4}\%$ of $\$640 = 15\frac{3}{4}\% \times \$640$

$= \frac{15\frac{3}{4}}{100} \times \$640$

$= \$100.80$

2. 2500% of $\$4.60 = 2500\% \times \$4.60$

$= \frac{2500}{100} \times \$4.60$

$= \$115$

Practise Now 4

1. **Method 1:**

Number of students who were late for school $= 3\% \times 1500$

$= \frac{3}{100} \times 1500$

$= 45$

Number of students who were punctual for school $= 1500 - 45$

$= 1455$

Method 2:

Percentage of students who were punctual for school $= 100\% - 3\%$

$= 97\%$

Number of students who were punctual for school $= 97\% \times 1500$

$= \frac{97}{100} \times 1500$

$= 1455$

2. Percentage of children who attended the dinner

$= 100\% - 35.5\% - 40\%$

$= 24.5\%$

Number of children who attended the dinner $= 24.5\% \times 1800$

$= \frac{24.5}{100} \times 1800$

$= 441$

Practise Now 5

Percentage of people who attended the New Year party in Village A

$= \frac{4000}{30\,000} \times 100\%$

$= 13\frac{1}{3}\%$

Percentage of people who attended the New Year party in Village B

$= \frac{2800}{25\,000} \times 100\%$

$= 11.2\%$

$\therefore$ Village A had a higher percentage of people who attended its New Year party.

Practise Now 6

1. **(a)** Value of award for a Secondary 1 student in 2009

$= 140\% \times \$250$

$= \frac{140}{100} \times \$250$

$= \$350$

(b) **(i)** Percentage increase in value of award from 2008 to 2009 for a Primary 1 student

$= \frac{\$250 - \$150}{\$150} \times 100\%$

$= \frac{\$100}{\$150} \times 100\%$

$= 66\frac{2}{3}\%$

(ii) Percentage increase in value of award from 2008 to 2009 for a Primary 6 student

$= \frac{\$300 - \$200}{\$200} \times 100\%$

$= \frac{\$100}{\$200} \times 100\%$

$= 50\%$

2. **(a)** Required result $= 75\% \times 32$

$= \frac{75}{100} \times 32$

$= 24$

(b) Percentage decrease in value of car

$= \frac{\$127\,000 - \$119\,380}{\$127\,000} \times 100\%$

$= \frac{\$7620}{\$127\,000} \times 100\%$

$= 6\%$

Practise Now 7

	Original Cost	**Percentage Change**	**New Cost**
Rental	\$2400	–5%	$\frac{95}{100} \times \$2400 = \$2280$
Wages	\$1800	–6%	$\frac{94}{100} \times \$1800 = \$1692$
Utilities	\$480	+7%	$\frac{107}{100} \times \$480 = \$513.60$
Business	\$4680		\$4485.60

Percentage decrease in monthly cost of running business

$= \frac{\$4680 - \$4485.60}{\$4680} \times 100\%$

$= \frac{\$194.40}{\$4680} \times 100\%$

$= 4\frac{2}{13}\%$

Practise Now 8

70% of the books $= 35$

1% of the books $= \frac{35}{70}$

100% of the books $= \frac{35}{70} \times 100$

$= 50$

There are 50 books on the bookshelf.

Practise Now 9

1. **Method 1:**

109% of original cost $= \$654$

1% of original cost $= \frac{\$654}{109}$

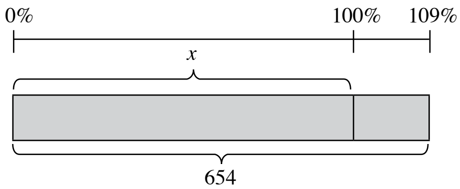
100% of original cost $= \frac{\$654}{109} \times 100$

$= \$600$

The original cost of the article is \$600.

Method 2:

Let the original cost of the article be $\$x$.



From the model, we form the equation:

$109\% \times x = 654$

$1.09x = 654$

$x = 600$

The original cost of the article is \$600.

2. 120% of value in 2011 $= \$180\,000$

1% of value in 2011 $= \frac{\$180\,000}{120}$

100% of value in 2011 $= \frac{\$180\,000}{120} \times 100$

$= \$150\,000$

The value of the vase was \$150 000 in 2011.

120% of value in 2010 $= \$150\,000$

1% of value in 2010 $= \frac{\$150\,000}{120}$

100% of value in 2010 $= \frac{\$150\,000}{120} \times 100$

$= \$125\,000$

The value of the vase was \$125 000 in 2010.

Practise Now 10

1. **Method 1:**

97% of original monthly salary $= \$3346.50$

1% of original monthly salary $= \dfrac{\$3346.50}{97}$

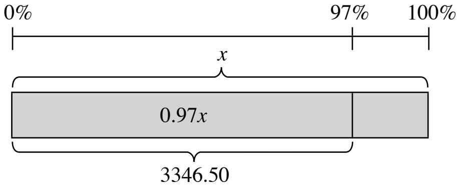
100% of original monthly salary $= \dfrac{\$3346.50}{97} \times 100$

$= \$3450$

Devi's original monthly salary is \$3450.

Method 2:

Let Devi's original monthly salary be \$$x$.



From the model, we form the equation:

$97\% \times x = 3346.50$

$0.97x = 3346.50$

$x = 3450$

Devi's original monthly salary is \$3450.

2. 85% of value in 2011 $= \$86\,700$

1% of value in 2011 $= \dfrac{\$86\,700}{85}$

100% of value in 2011 $= \dfrac{\$86\,700}{85} \times 100$

$= \$102\,000$

The value of the car was \$102 000 in 2011.

85% of value in 2010 $= \$102\,000$

1% of value in 2010 $= \dfrac{\$102\,000}{85}$

100% of value in 2010 $= \dfrac{\$102\,000}{85} \times 100$

$= \$120\,000$

The value of the car was \$120 000 in 2010.

Exercise 8A

1. **(a)** $28\% = \dfrac{28}{100} = \dfrac{7}{25}$

(b) $158\% = \dfrac{158}{100} = \dfrac{79}{50} = 1\dfrac{29}{50}$

(c) $12.4\% = \dfrac{12.4}{100} = \dfrac{124}{1000} = \dfrac{31}{250}$

(d) $6\dfrac{3}{5}\% = \dfrac{33}{5}\% = \dfrac{33}{5} \div 100 = \dfrac{33}{5} \times \dfrac{1}{100} = \dfrac{33}{500}$

2. **(a)** $4\% = \dfrac{4}{100} = 0.04$

(b) $633\% = \dfrac{633}{100} = 6.33$

(c) $0.02\% = \dfrac{0.02}{100} = 0.0002$

(d) $33\dfrac{2}{3}\% = \dfrac{101}{3}\% = \dfrac{101}{3} \div 100 = \dfrac{101}{3} \times \dfrac{1}{100} = \dfrac{101}{300} = 0.337$ (to 3 s.f.)

3. **(a)** $\dfrac{3}{5} = \dfrac{3}{5} \times 100\% = 60\%$

(b) $\dfrac{9}{10} = \dfrac{9}{10} \times 100\% = 90\%$

(c) $\dfrac{6}{125} = \dfrac{6}{125} \times 100\% = 4.8\%$

(d) $\dfrac{6}{5} = \dfrac{6}{5} \times 100\% = 120\%$

(e) $\dfrac{12}{25} = \dfrac{12}{25} \times 100\% = 48\%$

(f) $1\dfrac{6}{25} = \dfrac{31}{25} \times 100\% = 124\%$

4. **(a)** $0.78 = 0.78 \times 100\% = 78\%$

(b) $0.25 = 0.25 \times 100\% = 25\%$

(c) $0.07 = 0.07 \times 100\% = 7\%$

(d) $0.095 = 0.095 \times 100\% = 9.5\%$

(e) $1.35 = 1.35 \times 100\% = 135\%$

(f) $2.6 = 2.6 \times 100\%$
$= 260\%$

5. **(a)** 50% of $\$70 = 50\% \times \$70$
$= \dfrac{50}{100} \times \$70$
$= \$35$

(b) 80% of 4.5 m $= 80\% \times 4.5$ m
$= \dfrac{80}{100} \times 4.5$ m
$= 3.6$ m

6. **(i)** Total number of students in the class $= 20 + 18$
$= 38$

Percentage of boys in the class $= \dfrac{20}{38} \times 100\%$
$= 52\dfrac{12}{19}\%$

(ii) Percentage of girls in the class $= 100\% - 52\dfrac{12}{19}\%$
$= 47\dfrac{7}{19}\%$

7. Percentage of cars which are not blue $= 100\% - 30\%$
$= 70\%$

Number of cars which are not blue $= 70\% \times 120$
$= \dfrac{70}{100} \times 120$
$= 84$

8. Percentage of annual income Jun Wei donated to charitable organisations
$= \dfrac{\$1200}{12 \times \$1600} \times 100\%$
$= \dfrac{\$1200}{\$19\,200} \times 100\%$
$= 6.25\%$

Percentage of annual income Lixin donated to charitable organisations
$= \dfrac{\$4500}{12 \times \$6800} \times 100\%$
$= \dfrac{\$4500}{\$81\,600} \times 100\%$
$= 5\dfrac{35}{68}\%$

$\therefore$ Jun Wei donated a higher percentage of his annual income to charitable organisations.

9. **(a)** Required percentage $= \dfrac{25 \text{ seconds}}{3.5 \text{ minutes}} \times 100\%$
$= \dfrac{25 \text{ seconds}}{210 \text{ seconds}} \times 100\%$
$= \dfrac{5}{42} \times 100\%$
$= 11\dfrac{19}{21}\%$

(b) Required percentage $= \dfrac{45 \text{ minutes}}{1 \text{ hour}} \times 100\%$
$= \dfrac{45 \text{ minutes}}{60 \text{ minutes}} \times 100\%$
$= \dfrac{3}{4} \times 100\%$
$= 75\%$

(c) Required percentage $= \dfrac{1 \text{ year}}{4 \text{ months}} \times 100\%$
$= \dfrac{12 \text{ months}}{4 \text{ months}} \times 100\%$
$= 3 \times 100\%$
$= 300\%$

(d) Required percentage $= \dfrac{15 \text{ mm}}{1 \text{ m}} \times 100\%$
$= \dfrac{15 \text{ mm}}{1000 \text{ mm}} \times 100\%$
$= \dfrac{3}{200} \times 100\%$
$= 1.5\%$

(e) Required percentage $= \dfrac{335 \text{ cm}}{5 \text{ m}} \times 100\%$
$= \dfrac{335 \text{ cm}}{500 \text{ cm}} \times 100\%$
$= \dfrac{67}{100} \times 100\%$
$= 67\%$

(f) Required percentage $= \dfrac{1 \text{ kg}}{800 \text{ g}} \times 100\%$
$= \dfrac{1000 \text{ g}}{800 \text{ g}} \times 100\%$
$= \dfrac{5}{4} \times 100\%$
$= 125\%$

(g) Required percentage $= \dfrac{60°}{360°} \times 100\%$
$= \dfrac{1}{6} \times 100\%$
$= 16\dfrac{2}{3}\%$

(h) Required percentage $= \dfrac{63 \text{ cents}}{\$2.10} \times 100\%$
$= \dfrac{63 \text{ cents}}{210 \text{ cents}} \times 100\%$
$= \dfrac{3}{10} \times 100\%$
$= 30\%$

10. **(a)** $6\dfrac{1}{5}\%$ of 1.35 ml $= 6\dfrac{1}{5}\% \times 1.35$ ml
$= \dfrac{6\frac{1}{5}}{100} \times 1.35$ ml
$= \dfrac{837}{10\,000}$ ml

(b) $56\frac{7}{8}\%$ of 810 m $= 56\frac{7}{8}\% \times 810$ m

$$= \frac{56\frac{7}{8}}{100} \times 810 \text{ m}$$

$$= 460\frac{11}{16} \text{ m}$$

(c) 0.56% of 15 000 l = 0.56% × 15 000 l

$$= \frac{0.56}{100} \times 15\,000\ l$$

$$= 84\ l$$

(d) 2000% of 5¢ = 2000% × 5¢

$$= \frac{2000}{100} \times 5¢$$

= 100¢

= $1

11. Percentage of marks Kate obtains $= \frac{40}{60} \times 100\%$

$$= 66\frac{2}{3}\%$$

∴ Kate gets a bronze award.

Percentage of marks Priya obtains $= \frac{46}{60} \times 100\%$

$$= 76\frac{2}{3}\%$$

∴ Priya gets a silver award.

Percentage of marks Nora obtains $= \frac{49}{60} \times 100\%$

$$= 81\frac{2}{3}\%$$

∴ Nora gets a gold award.

12. Percentage of employees who were unaffected by the financial crisis

= 100% – 2.5% – 50.75%

= 46.75%

Number of employees who were unaffected by the financial crisis

= 46.75% × 12 000

$$= \frac{46.75}{100} \times 12\,000$$

= 5610

13. Amount Ethan spent on room rental = 20.5% × $1850

$$= \frac{20.5}{100} \times \$1850$$

= $379.25

Amount Ethan overspent = $379.25 + $690 + $940 – $1850

= $159.25

Required percentage $= \frac{\$159.25}{\$1850} \times 100\%$

= 8.61% (to 2 d.p.)

14. Number of remaining pages after Friday = 600 – 150

= 450

Number of pages that remains to be read = (100% – 40%) × 450

= 60% × 450

$$= \frac{60}{100} \times 450$$

= 270

Required percentage $= \frac{270}{600} \times 100\%$

= 45%

Exercise 8B

1. **(a)** Required value = 135% × 60

$$= \frac{135}{100} \times 60$$

= 81

(b) Required value = 225% × 28

$$= \frac{225}{100} \times 28$$

= 63

(c) Required value = 55% × 120

$$= \frac{55}{100} \times 120$$

= 66

(d) Required value $= 62\frac{1}{2}\% \times 216$

$$= \frac{62\frac{1}{2}}{100} \times 216$$

= 135

2. **(a)** 20% of number = 17

1% of number $= \frac{17}{20}$

100% of number $= \frac{17}{20} \times 100$

= 85

The number is 85.

(b) 175% of number = 49

1% of number $= \frac{49}{175}$

100% of number $= \frac{49}{175} \times 100$

= 28

The number is 28.

(c) 115% of number = 161

1% of number $= \frac{161}{115}$

100% of number $= \frac{161}{115} \times 100$

= 140

The number is 140.

(d) 80% of number $= 192$

1% of number $= \dfrac{192}{80}$

100% of number $= \dfrac{192}{80} \times 100$

$= 240$

The number is 240.

3. Percentage increase in length of elastic band $= \dfrac{90 - 72}{72} \times 100\%$

$= \dfrac{18}{72} \times 100\%$

$= 25\%$

4. **(i)** Value of award for a Secondary 1 student in the top 5% in 2009

$= 130\% \times \$500$

$= \dfrac{130}{100} \times \$500$

$= \$650$

(ii) Percentage increase in value of award from 2008 to 2009 for a Primary 4 student in the next 5%

$= \dfrac{\$350 - \$250}{\$250} \times 100\%$

$= \dfrac{\$100}{\$250} \times 100\%$

$= 40\%$

5. Percentage decrease in price of desktop computer

$= \dfrac{\$1360 - \$1020}{\$1360} \times 100\%$

$= \dfrac{\$340}{\$1360} \times 100\%$

$= 25\%$

6. Value of car at the end of 2010 $= 80\% \times \$120\ 000$

$= \dfrac{80}{100} \times \$120\ 000$

$= \$96\ 000$

Value of car at the end of 2011 $= 90\% \times \$96\ 000$

$= \dfrac{90}{100} \times \$96\ 000$

$= \$86\ 400$

7. 45% of the students $= 135$

1% of the students $= \dfrac{135}{45}$

100% of the students $= \dfrac{135}{45} \times 100$

$= 300$

There are 300 students who take part in the competition.

8. 136% of original cost $= \$333\ 200$

1% of original cost $= \dfrac{\$333\ 200}{136}$

100% of original cost $= \dfrac{\$333\ 200}{136} \times 100$

$= \$245\ 000$

The cost of the house when it was built is \$245 000.

9. 90% of original bill $= \$58.50$

1% of original bill $= \dfrac{\$58.50}{90}$

100% of original bill $= \dfrac{\$58.50}{90} \times 100$

$= \$65$

The original bill is \$65.

10. Value obtained after initial increase $= 130\% \times 2400$

$= \dfrac{130}{100} \times 2400$

$= 3120$

Final number $= 80\% \times 3120$

$= \dfrac{80}{100} \times 3120$

$= 2496$

11. Let the number of train passengers in 2010 be x.

Number of train passengers in 2011 $= 108\% \times x$

$= \dfrac{108}{100} \times x$

$= 1.08x$

Number of train passengers in 2012 $= 108\% \times 1.08x$

$= \dfrac{108}{100} \times 1.08x$

$= 1.1664x$

Percentage increase in number of train passengers from 2010 to 2012

$= \dfrac{1.1664x - x}{x} \times 100\%$

$= \dfrac{0.1664x}{x} \times 100\%$

$= 0.1664 \times 100\%$

$= 16.64\%$

12.

	Original Cost	Percentage Change	New Cost
Raw materials	\$100	+11%	$\frac{111}{100} \times \$100 = \$111$
Overheads	\$80	+20%	$\frac{120}{100} \times \$80 = \$96$
Wages	\$120	–15%	$\frac{85}{100} \times \$120 = \$102$
Printer	\$300		\$309

Percentage increase in production cost of printer

$= \dfrac{\$309 - \$300}{\$300} \times 100\%$

$= \dfrac{\$9}{\$300} \times 100\%$

$= 3\%$

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13. 115% of value in 2011 = $899 300

$$1\% \text{ of value in 2011} = \frac{\$899\ 300}{115}$$

$$100\% \text{ of value in 2011} = \frac{\$899\ 300}{115} \times 100$$

$$= \$782\ 000$$

The value of the condominium was $782 000 in 2011.

115% of value in 2010 = $782 000

$$1\% \text{ of value in 2010} = \frac{\$782\ 000}{115}$$

$$100\% \text{ of value in 2010} = \frac{\$782\ 000}{115} \times 100$$

$$= \$680\ 000$$

The value of the condominium was $680 000 in 2010.

14. 75% of value in 2011 = $11 250

$$1\% \text{ of value in 2011} = \frac{\$11\ 250}{75}$$

$$100\% \text{ of value in 2011} = \frac{\$11\ 250}{75} \times 100$$

$$= \$15\ 000$$

The value of the surveying machine was $15 000 in 2011.

75% of value in 2010 = $15 000

$$1\% \text{ of value in 2010} = \frac{\$15\ 000}{75}$$

$$100\% \text{ of value in 2010} = \frac{\$15\ 000}{75} \times 100$$

$$= \$20\ 000$$

The value of the surveying machine was $20 000 in 2010.

15. 105% of value at the end of 2010 = $61 824

$$1\% \text{ of value at the end of 2010} = \frac{\$61\ 824}{105}$$

$$100\% \text{ of value at the end of 2010} = \frac{\$61\ 824}{105} \times 100$$

$$= \$58\ 880$$

The value of the investment portfolio was $58 880 at the end of 2010.

92% of original value = $58 880

$$1\% \text{ of original value} = \frac{\$58\ 880}{92}$$

$$100\% \text{ of original value} = \frac{\$58\ 880}{92} \times 100$$

$$= \$64\ 000$$

The original value of the investment portfolio was $64 000.

16. Let Amirah's height be x m.

108% of Huixian's height = x m

$$1\% \text{ of Huixian's height} = \frac{x}{108} \text{ m}$$

$$100\% \text{ of Huixian's height} = \frac{x}{108} \times 100$$

$$= \frac{25}{27}x \text{ m}$$

Huixian's height is $\frac{25}{27}x$ m.

$$\text{Priya's height} = 90\% \times \frac{25}{27}x$$

$$= \frac{90}{100} \times \frac{25}{27}x$$

$$= \frac{5}{6}x \text{ m}$$

$$\text{Required percentage} = \frac{x}{\frac{5}{6}x} \times 100\%$$

$$= \frac{1}{\frac{5}{6}} \times 100\%$$

$$= 120\%$$

Review Exercise 8

1. $$\text{Required percentage} = \frac{1 \text{ m}}{56 \text{ mm}} \times 100\%$$

$$= \frac{1000 \text{ mm}}{56 \text{ mm}} \times 100\%$$

$$= \frac{125}{7} \times 100\%$$

$$= 1785\frac{5}{7}\%$$

2. **(i)** Pocket money Michael receives in a year = $52 \times \$28$
= $1456

$$\text{Savings in a year} = \frac{20}{100} \times \$1456$$

$$= \$291.20$$

(ii) Spending in a year = $1456 – $291.20
= $1164.80

3. $$\frac{a}{4b} = \frac{\frac{30}{100} \times b}{4b}$$

$$= \frac{\frac{30}{100}}{4}$$

$$= \frac{3}{40}$$

4. Huixian's percentage score $= \frac{68}{80} \times 100\%$
$= 85\%$

Priya's percentage score $= \frac{86}{120} \times 100\%$
$= 71\frac{2}{3}\%$

Rui Feng's percentage score $= \frac{120}{150} \times 100\%$
$= 80\%$

$\therefore$ Huixian performs the best in her Science test.

5. Number of apples the vendor has $= \frac{120}{100} \times 120$
$= 144$

60% of number of pears = 120

1% of number of pears $= \frac{120}{60}$

100% of number of pears $= \frac{120}{60} \times 100$
$= 200$

Number of pears the vendor has = 200

Total number of fruits the vendor has = 120 + 144 + 200
= 464

6. 120% of number of pages Kate reads on the second day = 60

1% of number of pages Kate reads on the second day $= \frac{60}{120}$

100% of number of pages Kate reads on the second day
$= \frac{60}{120} \times 100$
$= 50$

Number of pages Kate reads on the second day = 50

Number of pages in the book $= 6 \times 50$
= 300

7. Percentage of goats left $= \frac{94}{100} \times 86\%$
$= 80.84\%$

80.84% of original number of goats = 8084

1% of original number of goats $= \frac{8084}{80.84}$

100% of original number of goats $= \frac{8084}{80.84} \times 100$
= 10 000

The original number of goats in the village is 10 000.

8. Let Mr Neo's original salary be $\$x$.

Mr Neo's reduced salary $= \frac{85}{100} \times \$x$
$= \$0.85x$

Required percentage $= \frac{\$x - \$0.85x}{\$0.85x} \times 100\%$
$= \frac{\$0.15x}{\$0.85x} \times 100\%$
$= \frac{0.15}{0.85} \times 100\%$
$= 17\frac{11}{17}\%$

Challenge Yourself

1. Let the number of red jellybeans Amirah moves from Bottle A to Bottle B be x, the number of yellow jellybeans Amirah moves from Bottle A to Bottle B be y.

		Bottle A	Bottle B
Before	Red	300	150
	Yellow	100	150
After	Red	$300 - x$	$150 + x$
	Yellow	$100 - y$	$150 + y$

$\therefore \frac{300 - x}{100 - y} = \frac{80}{20}$

$\frac{300 - x}{100 - y} = \frac{4}{1}$

$300 - x = 4(100 - y)$

$300 - x = 400 - 4y$

$4y - x = 100$ — (1)

$\therefore \frac{150 + x}{150 + y} = \frac{60}{40}$

$\frac{150 + x}{150 + y} = \frac{3}{2}$

$2(150 + x) = 3(150 + y)$

$300 + 2x = 450 + 3y$

$2x - 3y = 150$ — (2)

$2 \times (1)$: $8y - 2x = 200$ — (3)

$(2) + (3)$: $5y = 350$

$y = 70$

Substitute $y = 70$ into (1): $4(70) - x = 100$

$280 - x = 100$

$x = 180$

Number of jellybeans Amirah moves from Bottle A to Bottle B
$= x + y$
$= 180 + 70$
$= 250$

2. Percentage of water which is poured from Cup B into Cup A
$= \frac{60}{100} \times 70\%$
$= 42\%$

Percentage of water in Cup A before 60% of solution in Cup A is poured into Cup B
$= 40\% + 42\%$
$= 82\%$

Percentage of water in Cup A after 60% of solution in Cup A is poured into Cup B
$= \frac{40}{100} \times 82\%$
$= 32.8\%$

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Chapter 9 Ratio, Rate, Time and Speed

TEACHING NOTES

Suggested Approach

Students have learnt how to solve problems involving ratios and speed in primary school. Teachers can bring in real-life examples for ratio, rate, time and speed to arouse students' interest in this topic. Students will also learn how to solve problems involving ratio, rate, time and speed through worked examples that involve situations in real-world contexts.

Section 9.1: Ratio

Teachers can build upon what students have learnt about ratio in primary school and introduce equivalent ratios through a recap of equivalent fractions. Teachers should emphasise that ratio does not indicate the actual size of quantities involved. Practical examples can be given to the students to let them recognise what equivalent ratios are (e.g. using 2 different kinds of fruits).

Teachers should highlight some common errors in ratio (i.e. the ratio of a part of a whole with the ratio of two parts, incorrect order of numbers expressed when writing ratio and incorrect numerator expressed when writing ratio as a fraction).

To make learning interesting, students can explore more about the Golden Ratio (see chapter opener and Investigation: Golden Ratio). Teachers can also get the students to find out what other man-made structures or natural occurrences have in common with the Golden Ratio (see Performance Task at page 210 of the textbook).

Section 9.2: Rate

Teachers should explain that rate is a relationship between two quantities with different units of measure (which is different from ratio). Teachers can give real life examples (e.g. rate of flow, consumption) for students to understand the concept of rate. Teachers can also get students to interpret using tables which show different kinds of rates (e.g. interest rate, postage rate, parking rate etc.).

Students can get more practice by learning to calculate rates they are familiar with (see Investigation: Average Pulse Rate). Teachers should impress upon them to distinguish between constant and average rates.

Section 9.3: Time

Teachers should emphasise that the addition and subtraction of times are not simply the same as adding and subtracting the numbers. For example, teachers can ask students why 30 + 40 = 70 = 110 (where 110 refers to 1 h 10 min). To prevent students from making careless mistakes, teachers should help students understand that:
6 hours 45 minutes is not the same as 6.45 hours, 1 hour is not the same as 100 minutes, 1 minute is not the same as 100 seconds.
Another important learning point would be dealing with time during the period before midnight and early morning. Teachers may also compare the time displayed on a digital clock with that on an analogue clock, and show students how the time is read.

Section 9.4: Speed

Teachers should inform students that speed is a special type of rate, i.e. speed is the distance covered per unit time. Teachers can get students to match appropriate speed to examples given (e.g. speed of a moving bicycle, lorry, car and aeroplane) to bring across the notion of speed.

Teachers can build upon what students have learnt about distance, time and speed in primary school. Students need to know that average speed is defined as the total distance travelled by the object per unit time and not the average of the speeds of the object. Teachers should also impress upon students that there are differences between average speed and constant speed.

Teachers should teach students the conversion of units and highlight to them to use appropriate units when solving problems.

Challenge Yourself

Questions 1 and 2: Teachers can guide the students by getting them to use appropriate algebraic variables to represent the rates involved in the question. Students have to read the question carefully and form the linear equations which then can be solved to get the answers.

WORKED SOLUTIONS

Class Discussion (Making Sense of the Relationship between Ratios and Fractions)

There are 40 green balls and 60 red balls in a bag.
Let A and B represent the number of green balls and red balls respectively.

1. Find the ratio of A to B.

$A : B = 40 : 60$
$= \underline{2} : \underline{3}$

We can conclude that:
The ratio of A to B is $\underline{2} : \underline{3}$.
The following statement is equivalent to the above statement.
A is $\underline{\frac{2}{3}}$ (fraction) of B, i.e. $\frac{A}{B} = \underline{\frac{2}{3}}$ (fraction).

2. Find the ratio of B to A.

$B : A = 60 : 40$
$= \underline{3} : \underline{2}$

We can conclude that:
The ratio of B to A is $\underline{3} : \underline{2}$.
The following statement is equivalent to the above statement.
B is $\underline{\frac{3}{2}}$ (fraction) of A, i.e. $\frac{B}{A} = \underline{\frac{3}{2}}$ (fraction).

3.

A	20	20	
B	20	20	20

4. Example:

There are 30 girls and 10 boys in a class.
Let G and B represent the number of girls and boys respectively.

$G : B = 30 : 10$
$= 3 : 1$

We can conclude that:
The ratio of G to B is 3 : 1.
The following statement is equivalent to the above statement.
G is $\frac{3}{1}$ (fraction) of B, i.e. $\frac{G}{B} = \frac{3}{1}$ (fraction).

OR

$B : G = 10 : 30$
$= 1 : 3$

We can conclude that:
The ratio of B to G is 1 : 3.
The following statement is equivalent to the above statement.
B is $\frac{1}{3}$ (fraction) of G, i.e. $\frac{B}{G} = \frac{1}{3}$ (fraction).

G	10	10	10
B	10		

Journal Writing (Page 208)

1. Aspect ratio is used to describe the relationship between the width and height of an image. It does not represent the actual length and height, but instead represents the proportion of its width and height. This is usually represented by two numbers separated by a colon, for example, 4 : 3 and 16 : 9.

The standard size of televisions has an aspect ratio of 4 : 3 which means the image is 4 units wide for every 3 units of height. Meanwhile, the latest size of televisions for the aspect ratio is 16 : 9 which is 16 units of width for every 9 units of height.

The following are some examples of aspect ratio used in our daily lives:

- 16 : 10 is used mainly in widescreen computer monitors.
- 16 : 9 is the aspect ratio used in cinema halls as well as High Definition TV.
- 14 : 9 is a compromise aspect ratio used to create an image that is viewable to both 4 : 3 and 16 : 9 televisions.
- 5 : 4 is a computer monitor resolution and also in mobile phones.
- 4 : 3 is used in the older TVs (mainly non-widescreen) and computer monitors.
- 1 : 1 is an uncommon aspect ratio that is used mainly in photography.

2. Example 1:

Scale drawings of maps and buildings are often represented by ratios. This is because it is impossible for a map to be exactly of the same size as the area it represents. Therefore, the measurements are scaled down in a fixed proportion so that the map can be used easily. Similarly, a scale drawing of a building will have the same shape as the actual building except that is scaled down.

Example 2:

In Chemistry and Biology, ratios are used for simple dilution of chemicals. A fixed unit volume of a chemical is added to an appropriate volume of solvent in order to dilute the chemical. For example, a 1 : 5 dilution (verbalize as “1 to 5” dilution) entails combining 1 unit volume of solute (the material to be diluted) + 4 unit volumes (approximately) of the solvent to give 5 units of the total volume.

Investigation (Golden Ratio)

1. $AB = 1.7$ cm
$BC = 1.05$ cm

$$\frac{AC}{AB} = \frac{2.75}{1.7} = 1.62 \text{ (to 2 s.f.)}$$

$$\frac{AB}{BC} = \frac{1.7}{1.05} = 1.62 \text{ (to 2 s.f.)}$$

2. $XY = 2.75$ cm
$YZ = 1.7$ cm

$$\frac{XY}{YZ} = \frac{2.75}{1.7} = 1.62 \text{ (to 2 s.f.)}$$

3. $$\frac{1+\sqrt{5}}{2} = 1.62 \text{ (to 2 s.f.)}$$

4. All the values in the previous questions are all equal.
5. –
6. (a) $\varphi^2 = \dfrac{3+\sqrt{5}}{2}$

$\varphi + 1 = \dfrac{3+\sqrt{5}}{2}$

Both answers are the same.

(b) $\dfrac{1}{\varphi} = \dfrac{-1+\sqrt{5}}{2}$

$\dfrac{1}{\varphi} = \varphi - \_\_\_\_$

$\varphi - \dfrac{1}{\varphi} = \dfrac{1+\sqrt{5}}{2} - \dfrac{-1+\sqrt{5}}{2}$

$= 1$

It is equal to 1.

Performance Task (Page 210)

Teachers may wish to give some examples of

- man-made structures such as the
  - **a)** Acropolis of Athens (468–430 BC), including the Parthenon;
  - **b)** Great Mosque of Kairouan (built by Uqba ibn Nafi c. 670 A.D);
  - **c)** Cathedral of Chartres (begun in the 12th century), Notre-Dame of Laon (1157–1205), and Notre Dame de Paris (1160);
  - **d)** Mexico City Metropolitan Cathedral (1667–1813).
- natural occurrences
  - **a)** spiral growth of sea shells;
  - **b)** spiral of a pinecone;
  - **c)** petals of sunflower;
  - **d)** horns of antelopes, goats and rams;
  - **e)** tusks of elephants;
  - **f)** body dimensions of penguins.

Investigation (Average Pulse Rate)

	First reading	Second reading	Third reading
Pulse rate (per minute)			

Thinking Time (Page 216)

1. The parking charges per minute are \$0.40 is a constant rate as the rate of charges per minute is the same throughout.
   The rate of petrol consumption is 13.5 km per litre is an average rate as the rate of consumption is not the same per minute.
2. The following are 3 examples of average rate that can be found in daily life:
   - Average speed
   - Downloading rate of a file
   - Average daily population growth

   The following are 3 example of constant rate that can be found in daily life:
   - Simple interest rate
   - Income Tax rate
   - Currency exchange rate

Thinking Time (Page 224)

Do a recap on the definition of average speed which is defined as the total distance travelled by the total time taken. Average speed is different from the general meaning of 'average' in statistics. The word 'average' here does not refer to the sum of all individual speeds divided by the total number of individual speeds.

Performance Task (Page 225)

1. **(a)** *Teachers may wish to assign this activity as a pair work for the students to do.*
   Students can help to record each other's walking speed.
   Average walking speed of a human
   = 5 km/h

   (b) Minimum average speed

   $= \dfrac{2.4}{\left(\dfrac{11 \times 60 + 30}{3600}\right)}$

   $= \dfrac{2.4}{\left(\dfrac{690}{3600}\right)}$

   = 12.5 km/h (to 3.s.f.)

   (c) Average speed of a bicycle
   = 22.5 km/h
   Average speed of a sports car
   = 280 km/h

   (d) Average speed of an MRT train
   = 45 km/h

   (e) Average speed of an aeroplane
   = 805 km/h

   (f) Average speed of the spaceship
   = 28 000 km/h

		Average speed (km/h)
(a)	Walking	5 km/h
(b)	Running	12.5 km/h
(c)	Bicycle	22.5 km/h
(d)	Sports car	280 km/h
(e)	MRT train	45 km/h
(f)	Aeroplane	805 km/h
(g)	Spaceship	28 000 km/h

2. Priya cycles to school while Devi walks to school.
3. Number of times a spaceship is as fast as an aeroplane

   $= \dfrac{28\,000}{805}$

   = 34.8
4. Other examples of speeds which can be encountered in real life:
   - Speed of a bus
   - Speed of a cheetah
   - Speed of the Singapore Flyer capsule

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5. Teachers may wish to ask the students to present their findings to the class.

Practise Now 1

(i) Ratio of the number of lemons to the number of pears
= 33 : 20

(ii) Ratio of the number of pears to the number of fruits in the basket
= 20 : (33 + 20)
= 20 : 53

Practise Now 2

(a) 240 g : 1.8 kg = 240 g : 1800 g
= 2 : 15

Alternatively,

$$\frac{2.40\text{ g}}{1.8\text{ kg}} = \frac{240\text{ g}}{1800\text{ g}} = \frac{2}{15}$$

$\therefore$ 240 g : 1.8 kg = 2 : 15

(b) $\frac{3}{5} : \frac{8}{9} = \frac{3}{5} \times 45 : \frac{8}{9} \times 45$
$= 27 : 40$

(c) $0.36 : 1.2 = 0.36 \times 100 : 1.2 \times 100$
$= 36 : 120$
$= 3 : 10$

Practise Now 3

$3a : 7 = 8 : 5$

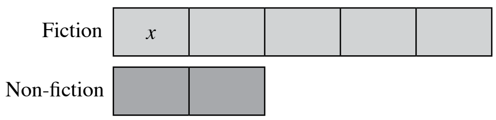
$$\frac{3a}{7} = \frac{8}{5}$$

$15a = 56$

$a = 3\frac{11}{15}$

Practise Now 4

1. Let the number of fiction books = $5x$.
Then the number of non-fiction book = $2x$.



From the model, we form the equation:

$5x + 2x = 1421$
$7x = 1421$
$x = 203$

There are $3 \times 203 = 609$ more fiction than non-fiction books in the library.

2. Let the amount of money Kate had initially be $\$3x$.
Then the amount of money Nora had initially is $\$5x$.

	Kate	Nora
Before	$\$3x$	$\$5x$
After	$\$(3x + 150)$	$\$(5x - 150)$

$$\therefore \frac{3x + 150}{5x - 150} = \frac{7}{9}$$

$9(3x + 150) = 7(5x - 150)$
$27x + 1350 = 35x - 1050$
$27x - 35x = -1050 - 1350$
$-8x = -2400$
$x = 300$

$\therefore$ Amount of money Kate had initially
= $[3(300)]
= $900

Practise Now 5

$x : y = 5 : 6$ $\downarrow\times 2$ $= 10 : 12$

$y : z = 4 : 9$ $\downarrow\times 3$ $= 12 : 27$

(i) $x : y : z = 10 : 12 : 27$

(ii) $x : z = 10 : 27$

Practise Now 6

Let the amount of money Khairul had initially be $\$6x$.
Then the amount of money Michael and Ethan had initially is $\$4x$ and $\$5x$ respectively.

	Khairul	Michael	Ethan
Before	$\$6x$	$\$4x$	$\$5x$
After	$\$(6x - 45)$	$\$(4x + 30)$	$\$(5x + 15)$

$$\therefore \frac{6x - 45}{4x + 30} = \frac{7}{6}$$

$6(6x - 45) = 7(4x + 30)$
$36x - 270 = 28x + 210$
$36x - 28x = 210 + 270$
$8x = 480$
$x = 60$

$\therefore$ Amount of money Khairul had initially
= $[6(60)]
= $360

Practise Now 7

Number of words per minute that Amirah can type

$= \frac{720}{16}$

$= 45$

Number of words per minute that Lixin can type

$= \frac{828}{18}$

$= 46$

Number of words per minute that Shirley can type

$= \frac{798}{19}$

$= 42$

Thus, Lixin is the fastest typist.

Practise Now 8

1. **(a)** Amount each child have to pay

$= \frac{\$2.70 \times 32.5}{36}$

$= \$2.44$

(b) **(i)** Distance travelled on 1 litre of petrol

$= \frac{265}{25}$

$= \frac{265}{25}$

$= 10.6$ km

Distance travelled on 58 litres of petrol

$= 10.6 \times 58$

$= 614.8$ km

(ii) Amount of petrol required to travel a distance of 1007 km

$= \frac{1007}{10.6}$

$= 95$ litres

Amount that the car owner has to pay

$= 95 \times \$1.95$

$= \$185.25$

2. In 1 minute, 5 people can finish

$= 20 \div 3\frac{20}{60}$

$= 6$ buns

In 5 minutes, 5 people can finish

$= 6 \times 5$

$= 30$ buns

In 5 minutes, 10 people can finish

$= 30 \times 2$

$= 60$ buns

Practice Now 9

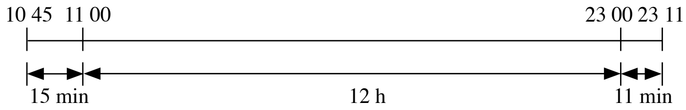
$7\frac{1}{4}$ h = 7 h 15 min

$22\ 45 \xrightarrow{+\,7\text{ h}} 29\ 45 \xrightarrow{+\,15\text{ min}} 06\ 00$

(05 45)

$\therefore$ The ship arrived at Port Y at 06 00 or 6 a.m. on Saturday.

Practise Now 10



15 min + 11 min = 26 min

$\therefore$ The bus journey was 12 h 26 min long.

Practise Now 11

1. **(i)** 25 minutes = $\frac{25}{60}$ hours

Speed of the train = $\frac{16.8}{\left(\frac{25}{60}\right)} = 40.32$ km/h

(ii) 16.8 km = 16 800 m

25 minutes = $25 \times 60 = 1500$ seconds

Speed of the train = $\frac{16\ 800}{1500} = 11.2$ m/s

2. $55 \text{ km/h} = \frac{55\text{ km}}{1\text{ h}} = \frac{(55 \times 1000)\text{ m}}{3600\text{s}} = 15\frac{5}{18}$ m/s

12 minutes 30 seconds = $(12 \times 60) + 30 = 750$ seconds

Distance travelled = $15\frac{5}{18} \times 750 = 11\ 458\frac{1}{3}$ m

3. Let the speed of the bus be x km/h.

$13\ 20 \text{ hours} \xrightarrow{3\text{ hours}} 16\ 20 \text{ hours}$

Distance the car travelled in 3 hours = $90 \times 3 = 270$ km

$270 + (3 \times x) = 510$

$3x = 510 - 270$

$3x = 240$

$x = 80$

The speed of the bus is 80 km/h.

Practise Now 12

1. **(i)** Speed of the train

$= 48.6$ km/h

$= \frac{48.6\text{ km}}{1\text{ h}}$

$= \frac{48\ 600\text{ m}}{3600\text{ s}}$ (convert 48.6 km to m and 1 h into s)

$= 13.5$ m/s

(ii) Speed of the train

$= 48.6$ km/h

$= \frac{48.6\text{ km}}{1\text{ h}}$

$= \frac{4\ 860\ 000\text{ cm}}{60\text{ min}}$ (convert 48.6 km to cm and 1 h into min)

$= 81\ 000$ cm/min

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2. Speed of the fastest human sprinter

$= \dfrac{100\text{ m}}{9.58\text{ s}}$

$= \dfrac{(100 \div 1000)\text{ km}}{(9.58 \div 3600)\text{ h}}$

$= 37\dfrac{277}{479}$ km/h

No. of times a cheetah is as fast as the fastest human sprinter

$= \dfrac{110}{37\frac{277}{479}}$

$= 2.93$

Practise Now 13

Time taken for Farhan to swim a distance of 1.5 km

$= \dfrac{1.5}{2.5}$ h

$= \dfrac{3}{5}$ h

Total time taken

$= \dfrac{3}{5} + 1\dfrac{1}{2} + 1\dfrac{1}{9}$

$= \dfrac{3}{5} + \dfrac{3}{2} + \dfrac{10}{9}$

$= \dfrac{54}{90} + \dfrac{135}{90} + \dfrac{100}{90}$

$= \dfrac{289}{90}$ hours

Distance that Farhan runs

$= 9 \times 1\dfrac{1}{9}$

$= 9 \times \dfrac{10}{9}$

$= 10$ km

Total distance travelled

$= 1.5 + 40 + 10$

$= 51.5$ km

Average speed for the entire competition

$= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{51.5}{\left(\frac{289}{90}\right)}$

$= 16\dfrac{11}{289}$ km/h

Practise Now 14

Let the distance for the car to travel from Town A to Town B to meet the truck $= x$ km.

Then the time taken for the car to travel from Town A to Town B to meet the truck at an average speed of 72 km/h

$= \dfrac{x}{72}$ hour, and

the time taken for the truck to travel from Town B to Town A to meet the car at an average speed of 38 km/h

$= \dfrac{550 - x}{38}$ hour

$$\therefore \frac{x}{72} = \frac{550 - x}{38}$$

$$38x = 39\,600 - 72x$$

$$38x + 72x = 39\,600$$

$$110x = 39\,600$$

$$x = 360 \text{ km}$$

Hence, the time taken for the two vehicles to meet

$= \dfrac{360}{72}$

$= 5$ hours

Practise Now 15

Radius of the wheel of the car

$= \dfrac{0.75}{2}$

$= 0.375$ m

Circumference of the wheel of the car

$= 2 \times \pi \times 0.375$

$= 2 \times 3.142 \times 0.375$

$= 2.3565$ m

Distance travelled by a car in 1 minute

$= 14 \times 60$

$= 840$ m

Number of revolutions made by the wheel per minute

$= \dfrac{840}{2.3565}$

$= 356$ (to the nearest whole number)

Exercise 9A

1. **(a)** 1.5 kg : 350 g = 1500 g : 350 g

= 30 : 7

Alternatively,

$\dfrac{1.5\text{ kg}}{350\text{ g}} = \dfrac{1500\text{ g}}{350\text{ g}}$

$= \dfrac{30}{7}$

$\therefore$ 1.5 kg : 350 g = 30 : 7

(b) $\dfrac{15}{24} : \dfrac{9}{7} = \dfrac{15}{24} \times 168 : \dfrac{9}{7} \times 168$

$= 105 : 216$

$= 35 : 72$

(c) $0.45 : 0.85 = 0.45 \times 100 : 0.85 \times 100$
$= 45 : 85$
$= 9 : 17$

(d) $580\text{ m}l : 1.12\ l : 104\text{ m}l = 580 : 1120 : 104$
$= 145 : 280 : 26$

(e) $\frac{2}{3} : \frac{3}{2} : \frac{5}{8} = \frac{2}{3} \times 24 : \frac{3}{2} \times 24 : \frac{5}{8} \times 24$
$= 16 : 36 : 15$

(f) $0.33 : 0.63 : 1.8 = 0.33 \times 100 : 0.63 \times 100 : 1.8 \times 100$
$= 33 : 63 : 180$
$= 11 : 21 : 60$

2. **(a)** $a : 400 = 6 : 25$

$\frac{a}{400} = \frac{6}{25}$

$25a = 2400$

$a = 96$

(b) $5b : 8 = 2 : 5$

$\frac{5b}{8} = \frac{2}{5}$

$25b = 16$

$b = \frac{16}{25}$

3. $\frac{2x}{5} = \frac{3y}{8}$

$16x = 15y$

$\frac{x}{y} = \frac{15}{16}$

$x : y = 15 : 16$

4. $a : b : c = 75 : 120 : 132$

(i) $a : b : c = 25 : 40 : 44$

(ii) $b : a = 40 : 25$
$= 8 : 5$

(iii) $b : c = 40 : 44$
$= 10 : 11$

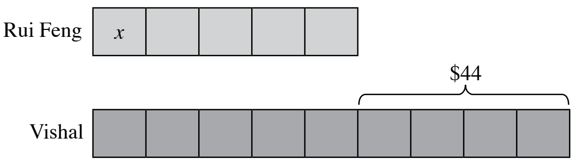
5. **(i)** Ratio of the number of boys to the number of girls = 14 : 25

(ii) Ratio of the number of girls to the total number of players in the team = 25 : 39

6. **(i)** Ratio of the number of athletes to the number of volunteers
= 3600 : 20 000
= 9 : 50

(ii) Ratio of the number of media representatives to the number of athletes to the number of spectators
= 1200 : 3600 : 370 000
= 3 : 9 : 925

7. Let the amount of money that Rui Feng gets = $5x$.
Then the amount of money that Vishal gets = $9x$.



From the model, we form the equation:

$9x - 5x = 44$

$4x = 44$

$x = 11$

Total amount of money that is shared between the two boys
$= (5 + 9) \times 11$
$= \$154$

8. **(i)** Number of toys Huixian makes $= \frac{1530}{12 + 16 + 17} \times 16$
$= \frac{1530}{45} \times 16$
$= 544$

(ii) Number of toys Priya makes $= \frac{1530}{12 + 16 + 17} \times 17$
$= \frac{1530}{45} \times 17$
$= 578$

Amount of money Priya earns = $578 \times \$1.65$
$= \$953.70$

9. **(a)** $4\frac{1}{5}\text{ kg} : 630\text{ g} = 4200\text{ g} : 630\text{ g}$
$= 20 : 3$

(b) $0.75 : 3\frac{5}{16} = \frac{75}{100} : 3\frac{5}{16}$
$= \frac{3}{4} : \frac{53}{16}$
$= \frac{3}{4} \times 16 : \frac{53}{16} \times 16$
$= 12 : 53$

(c) $0.6\text{ kg} : \frac{3}{4}\text{ kg} : 400\text{ g} = 600\text{ g} : 750\text{ g} : 400\text{ g}$
$= 12 : 15 : 8$

(d) $\frac{1}{3} : 2.5 : 3\frac{3}{4} = \frac{1}{3} \times 12 : 2.5 \times 12 : \frac{15}{4} \times 12$
$= 4 : 30 : 45$

(e) $1.2 : 3\frac{3}{10} : 5.5 = 1.2 \times 10 : \frac{33}{10} \times 10 : 5.5 \times 10$
$= 12 : 33 : 55$

10. **(a)** $2\frac{1}{4} : 6 = m : 1\frac{1}{5}$

$\frac{9}{4} : 6 = m : \frac{6}{5}$

$\frac{9}{4} \times 20 : 6 \times 20 = m \times 20 : \frac{6}{5} \times 20$

$45 : 120 = 20m : 24$

$9 : 24 = 20m : 24$

$9 = 20m$

$20m = 9$

$m = \frac{9}{20}$

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(b) $x : 3 : \frac{9}{2} = \frac{15}{4} : 4\frac{1}{2} : y$

$x \times 4 : 3 \times 4 : \frac{9}{2} \times 4 = \frac{15}{4} \times 4 : \frac{9}{2} \times 4 : y \times 4$

$4x : 12 : 18 = 15 : 18 : 4y$

$\frac{4x}{12} = \frac{15}{18}$ $\qquad$ $\frac{12}{18} = \frac{18}{4y}$

$\frac{x}{3} = \frac{5}{6}$ $\qquad$ $\frac{2}{3} = \frac{9}{2y}$

$6x = 15$ $\qquad$ $4y = 27$

$x = \frac{15}{6}$ $\qquad$ $y = \frac{27}{4}$

$x = \frac{5}{2}$ $\qquad$ $y = 6\frac{3}{4}$

$x = 2\frac{1}{2}$

11. $p : q = \frac{3}{4} : 2$ $\qquad$ $p : r = \frac{1}{3} : \frac{1}{2}$

$\downarrow \times 4$ $\qquad$ $\downarrow \times 6$

$= 3 : 8$ $\qquad$ $= 2 : 3$

$\downarrow$ $\qquad$ $\downarrow$

$= 6 : 16$ $\qquad$ $= 6 : 9$

(i) $p : q : r = 6 : 16 : 9$

(ii) $q : r = 16 : 9$

12. (i) Let the initial number of teachers in the school be x.

Then the number of students in the school is $15x$.

$15x = 1200$

$x = 80$

The initial number of teachers in the school is 80.

(ii) Let the number of teachers who join the school be y.

$\frac{80 + y}{1200} = \frac{3}{40}$

$40(80 + y) = 3(1200)$

$3200 + 40y = 3600$

$40y = 3600 - 3200$

$40y = 400$

$y = 10$

The number of teachers who join the school is 10.

13. Ratio of Ethan, Farhan and Michael's property investment

= \$427 000 : \$671 000 : \$305 000

= 427 : 671 : 305

= 7 : 11 : 5

Total amount of profit earned

= \$1 897 500 – (\$427 000 + \$671 00 + \$305 000)

= \$494 500

Amount of profit Ethan received

$= \frac{7}{7 + 11 + 5} \times \$494\,500$

= \$150 000

Amount of profit Farhan received

$= \frac{11}{7 + 11 + 5} \times \$494\,500$

= \$236 500

Amount of profit Michael received

$= \frac{5}{7 + 11 + 5} \times \$494\,500$

= \$107 500

14. Let the number that must be added be x.

$\frac{3 + x}{8 + x} = \frac{2}{3}$

$3(3 + x) = 2(8 + x)$

$9 + 3x = 16 + 2x$

$3x - 2x = 16 - 9$

$x = 7$

The number is 7.

15. Let the amount of money Ethan had initially be \$$5x$.

Then the amount of money Jun Wei and Raj had initially is \$$6x$ and \$$9x$ respectively.

	Ethan	Jun Wei	Raj
Before	\$$5x$	\$$6x$	\$$9x$
After	\$$(5x - 50)$	\$$6x$	\$$9x$

$\therefore \frac{5x - 50}{6x} = \frac{3}{4}$

$4(5x - 50) = 3(6x)$

$20x - 200 = 18x$

$20x - 18x = 200$

$2x = 200$

$x = 100$

∴ Amount of money Ethan has after giving \$50 to his mother

= \$[5(100) – 50]

= \$450

16. $\frac{x}{y} = \frac{3}{4}$ $\qquad$ $\frac{y}{z} = \frac{5}{8}$

$4x = 3y$ $\qquad$ $8y = 5z$

$x = \frac{3}{4}y$ $\qquad$ $z = \frac{8}{5}y$

$$\frac{2y}{3x - y + 2z} = \frac{2y}{3\left(\frac{3}{4}y\right) - y + 2\left(\frac{8}{5}y\right)}$$

$$= \frac{2y}{\frac{9}{4}y - y + \frac{16}{5}y}$$

$$= \frac{2y}{\frac{45}{20}y - \frac{20}{20}y + \frac{64}{20}y}$$

$$= \frac{2y}{\frac{89}{20}y}$$

$$= \frac{40}{89}$$

Exercise 9B

1. **(a)** Number of words that she can type per minute

$= \frac{1800}{60}$

(1 hour = 60 minutes)

= 30

(b) Cost of one unit of electricity

$= \$\frac{120.99}{654}$

= $0.19

(c) His monthly rental rate

$= \$\frac{4800}{3}$

= $1600

(d) Its mass per metre

$= \frac{15}{3.25}$

$= 4\frac{8}{13}$ kg/m

2. Time taken for Ethan to blow 1 balloon

$= \frac{20}{15}$

$= 1.\dot{3}$ minutes

Time taken for Jun Wei to blow 1 balloon

$= \frac{25}{18}$

$= 1.3\dot{8}$ minutes

Time taken for Vishal to blow 1 balloon

$= \frac{21}{16}$

= 1.3125 minutes

Thus, Vishal can blow balloons at the fastest rate.

3. 3 hours = 180 minutes

Number of ornaments made in 3 hours

$= \frac{180}{15} \times 4$

= 48

Amount earned by the worker

$= 48 \times \$1.15$

= $55.20

4. **(i)** Amount he is charged for each minute of outgoing calls

$= \$\frac{39}{650}$

= $0.06

(ii) Amount he has to pay

$= \$0.06 \times 460$

= $27.60

5. **(i)** Distance travelled on 1 litre of petrol

$= \frac{259.6}{22}$

= 11.8 km

Distance travelled on 63 litres of petrol

$= 11.8 \times 63$

= 743.4 km

(ii) Amount of petrol required to travel a distance of 2013.2 km

$= \frac{2013.2}{11.8}$

$= 170\frac{36}{59}$ litres

Amount that the car owner has to pay

$= 170\frac{36}{59} \times \$1.99$

= $339.51

6. **(i)** Amount of fertiliser needed for a plot of land that has an area of 1 m^2

$= \frac{200}{8}$

= 25 g

Amount of fertiliser needed for a plot of land that has an area of 14 m^2

$= 25 \times 14$

= 350 g

(ii) Area of land that can be fertilised by 450 g of fertiliser

$= \frac{450}{25}$

= 18 m^2

7. **(i)** Temperature of the metal after 9 minutes

$= 428\ °C - [(23\ °C \times 3) + (15\ °C \times 6)]$

= 269 °C

(ii) Temperature of the metal after 18 minutes

$= 428\ °C - [(23\ °C \times 3) + (15\ °C \times 15)]$

= 134 °C

Amount of temperature needed for the metal to fall so that it will reach a temperature of 25 °C

= 134 °C – 25 °C

= 109 °C

Time needed for the metal to reach a temperature of 25 °C

$= \frac{109}{8}$

$= 13\frac{5}{8}$ minutes

8. 4 weeks ⇒ fifteen 2-litre bottles of cooking oil

1 week ⇒ $\frac{15 \times 2}{4}$ = 7.5 litres of cooking oil

10 weeks ⇒ 10×7.5 = 75 litres of cooking oil

Number of 5-litre tins of cooking oil needed for a 10-week period

$= \frac{75}{5}$

= 15

9. **(i)** Total amount to be paid to the man

$= 224 \times \$7.50$

= $1680

(ii) Number of normal working hours from 9 a.m. to 6 p.m. excluding lunch time
= 8 hours
Let the number of overtime hours needed to complete the project in 4 days by each worker be x.
$4[4(8 + x)] = 224$
$16(8 + x) = 224$
$128 + 16x = 224$
$16x = 224 - 128$
$16x = 96$
$x = 6$
Overtime hourly rate
$= 1.5 \times \$7.5$
$= \$11.25$
Total amount to be paid to the 4 men if the project is to be completed in 4 days
$= 4\{4[(8 \times \$7.5) + (6 \times \$11.25)]\}$
$= \$2040$

10. 10 chefs can prepare a meal for 536 people in 8 hours and so 1 chef can prepare a meal for 536 people in $8 \times 10 = 80$ hours.
Hence, 22 chefs can prepare a meal for $536 \times 22 = 11\,792$ people in 80 hours and so 22 chefs can prepare a meal for $\frac{536 \times 22}{80}$ people in $\frac{80}{80} = 1$ hour.
Thus 22 chefs can prepare a meal for $\frac{536 \times 22}{80} \times 5$ people in $1 \times 5 = 5$ hours.
$\frac{536 \times 22}{80} \times 5$
$= 737$

Exercise 9C

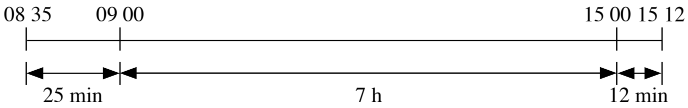
1. **(a)** 08 00
(b) 21 42
(c) 00 00
(d) 02 42

2. **(a)** 3.30 a.m.
(b) 11.12 p.m.
(c) 7.15 p.m.
(d) 12.00 a.m.

3.

	Departure Time	Journey Time	Arrival Time
(a)	02 40	55 minutes	03 35
(b)	22 35	8 hours	06 35 (next day)
(c)	15 45	$2\frac{1}{4}$ h or 2 h 15 min	17 50
(d)	09 48	$12\frac{7}{15}$ h or 12 h 28 min	22 16
(e)	20 35 (Tuesday)	$10\frac{2}{3}$ h or 10 h 40 min	07 15 (Wednesday)
(f)	22 35	$1\frac{1}{4}$ h	23 50

4. 8.35 a.m. ⇒ 08 35
3.12 p.m. ⇒ 15 12



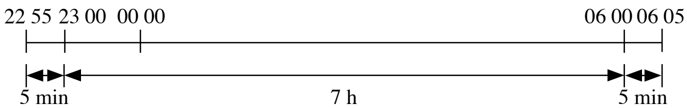
25 min + 12 min = 37 min
∴ The journey took 6 h 37 min.

5. $21\ 55 \xrightarrow{+9\text{ h}} 30\ 55 \xrightarrow{+5\text{ min}} 07\ 00 \xrightarrow{+13\text{ min}} 07\ 13$
(06 55)
∴ The train arrived at its destination at 07 13 or 7.13 a.m. on Tuesday.

6. $? \xrightarrow{+4\text{ h}} ? \xrightarrow{+15\text{ min}} 15\ 06$
$10\ 51 \xleftarrow[-4\text{ h}]{} 14\ 51 \xleftarrow[-9\text{ min}]{} 15\ 00 \xleftarrow[-6\text{ min}]{} 15\ 06$

Working backwards, the car started the journey at 10 51.

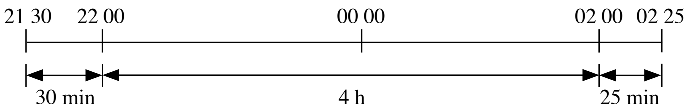
7. **(i)**



5 min + 5 min = 10 min
∴The journey took 7 h 10 min.

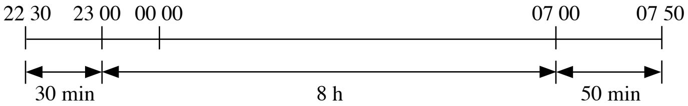
(ii) 35 min = 30 min + 5 min
5 min before 06 05 is 06 00
30 min before 06 00 is 05 30
∴ The coach reached its destination at 05 30.

8. Assume time taken includes breaks in between stations.
(a) Depart from A: 21 30; Arrive at C: 02 25



30 min + 25 min = 55 min
∴ The time taken from A to C was 4 h 55 min.

(b) Depart from B: 22 30; Arrive at E: 07 50

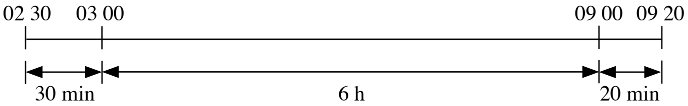


30 min + 50 min = 80 min

= 1 h 20 min

$\therefore$ The time taken from B to E was 9 h 20 min.

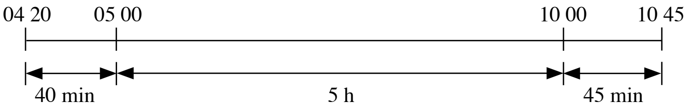
(c) Depart from C: 02 30; Arrive at F: 09 20



30 min + 20 min = 50 min

$\therefore$ The time taken from C to F was 6 h 50 min.

(d) Depart from D: 04 20; Arrive at G: 10 45

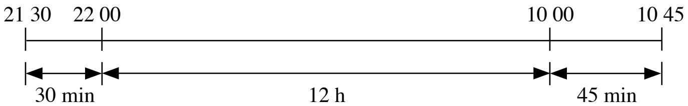


40 min + 45 min = 85 min

= 1 h 25 min

$\therefore$ The time taken from D to G was 6 h 25 min.

(e) Depart from A: 21 30; Arrive at G: 10 45



30 min + 45 min = 75 min

= 1 h 15 min

$\therefore$ The time taken from A to G was 13 h 15 min.

Exercise 9D

1. (i) 30 minutes = $\frac{30}{60} = \frac{1}{2}$ hour

Speed of the particle

$= \dfrac{24.6 \text{ km}}{\left(\frac{1}{2}\right)\text{hour}}$

= 49.2 km/h

(ii) 24.6 km = 24.6×1000 = 24 600 m

30 minutes = 30×60 = 1800 s

Speed of the particle

$= \dfrac{24\ 600 \text{ m}}{1800 \text{ s}}$

$= 13\frac{2}{3}$ m/s

2. 12 24 hours $\xrightarrow{\text{1 hour 48 minutes}}$ 14 12 hours

1 hour 48 minutes = $1\frac{48}{60} = 1\frac{4}{5}$ hours

Distance between the two stations

$= 200 \times 1\frac{4}{5}$

= 360 km

= 360×1000 m

= 360 000 m

3. (a) 8.4 km/min

$= \dfrac{8.4 \text{ km}}{1 \text{ min}}$

$= \dfrac{8.4 \text{ km}}{\left(\frac{1}{60}\right)\text{h}}$

= 504 km/h

(b) 315 m/s

$= \dfrac{315 \text{ m}}{1 \text{ s}}$

$= \dfrac{\left(\frac{315}{1000}\right)\text{km}}{\left(\frac{1}{3600}\right)\text{h}}$

= 1134 km/h

(c) 242 m/min

$= \dfrac{242 \text{ m}}{1 \text{ min}}$

$= \dfrac{\left(\frac{242}{1000}\right)\text{km}}{\left(\frac{1}{60}\right)\text{h}}$

$= 14\frac{13}{25}$ km/h

(d) 125 cm/s

$= \dfrac{125 \text{ cm}}{1 \text{ s}}$

$= \dfrac{\left(\frac{125}{100\ 000}\right)\text{km}}{\left(\frac{1}{3600}\right)\text{h}}$

= 4.5 km/h

4. (a) 65 cm/s

$= \dfrac{65 \text{ cm}}{1 \text{ s}}$

$= \dfrac{\left(\frac{65}{100}\right)\text{km}}{1 \text{ s}}$

$= \frac{13}{20}$ m/s

(b) 367 km/h

$= \dfrac{367 \text{ km}}{1 \text{ h}}$

$= \dfrac{367 \times 1000 \text{ m}}{3600 \text{ s}}$

$= \dfrac{367\ 000 \text{ m}}{3600 \text{ s}}$

$= 107\frac{17}{18}$ m/s

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(c) 1000 cm/min

$= \frac{1000 \text{ cm}}{1 \text{ min}}$

$= \frac{\left(\frac{1000}{100}\right) \text{m}}{60 \text{ s}}$

$= \frac{1}{6}$ m/s

(d) 86 km/min

$= \frac{86 \text{ km}}{1 \text{ min}}$

$= \frac{86 \times 1000 \text{ m}}{60 \text{ s}}$

$= 1433\frac{1}{3}$ m/s

5. Speed of the fastest Singaporean sprinter

$= \frac{100 \text{ m}}{10.37 \text{ s}}$

$= \frac{\left(\frac{100}{1000}\right) \text{km}}{\left(\frac{10.37}{3600}\right) \text{h}}$

$= 34\frac{742}{1037}$ km/h

Number of times the bullet train is as fast as the fastest Singaporean sprinter

$= \frac{365}{\left(34\frac{742}{1037}\right)}$

$= 10\frac{3701}{7200}$

6. Time taken to travel the first part of the journey

$= \frac{19}{57}$

$= \frac{1}{3}$ h

Time taken to travel the remaining part of the journey

$= \frac{55}{110}$

$= \frac{1}{2}$ h

Average speed of the car for its entire journey

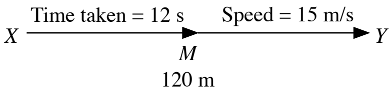
$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{19 + 55}{\frac{1}{3} + \frac{1}{2}}$

$= \frac{74}{\frac{5}{6}}$

$= 88\frac{4}{5}$ km/h

7.



(i) Time taken to travel from M to Y

$= \frac{60}{15}$

$= 4$ s

(ii) Average speed of the object for its entire journey from X to Y

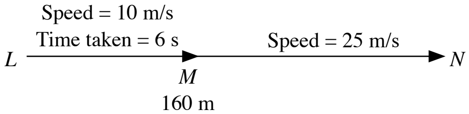
$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{120}{12 + 4}$

$= \frac{120}{16}$

$= 7\frac{1}{2}$ m/s

8.



Distance travelled from L to M

$= 10 \times 6$

$= 60$ m

Thus, distance travelled from M to N

$= 160 - 60$

$= 100$ m

Time taken to travel from M to N

$= \frac{100}{25}$

$= 4$ s

Average speed of the object for its entire journey from L to N

$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{160}{6 + 4}$

$= \frac{160}{10}$

$= 16$ m/s

9. Time taken to travel the first 50 km of its journey

$= \frac{50 \times 1000 \text{ m}}{25 \text{ m/s}}$

$= 2000$ s

$= \frac{2000}{3600}$ h

$= \frac{5}{9}$ h

Time taken to travel the next 120 km of its journey

$= \frac{120}{80}$

$= 1\frac{1}{2}$ h

Distance travelled for the last part of its journey

$= 90 \times \dfrac{35}{60}$

$= 52.5$ km

Average speed of the object for its entire journey

$= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{50 + 120 + 52.5}{\frac{5}{9} + 1\frac{1}{2} + \frac{35}{60}}$

$= 84\dfrac{6}{19}$ km/h

10. Radius of the wheel of the car

$= \dfrac{60}{2}$

$= 30$ cm

$= 0.3$ m

Circumference of the wheel of the car

$= 2 \times \pi \times 0.3$

$= 2 \times 3.142 \times 0.3$

$= 1.8852$ m

Distance travelled by the car in 1 hour

$= 13.2 \times 60 \times 60$

$= 47\ 520$ m

Number of revolutions made by the wheel per minute

$= \dfrac{47\ 520}{1.8852}$

$= 25\ 207$ (to the nearest whole number)

11. Length of the goods train

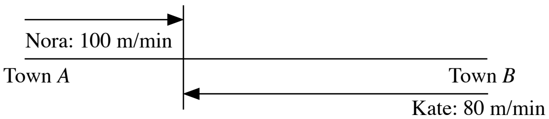
$= \left(72 \times \dfrac{8}{3600}\right) + \left(54 \times \dfrac{8}{3600}\right)$

$= \dfrac{7}{25}$ km

$= \dfrac{7}{25} \times 1000$

$= 280$ m

12.

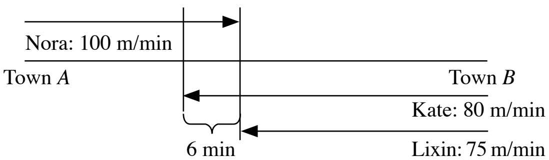


Let the time taken for Nora to meet Kate be x min.

So distance between Town A and Town B

$= 100 \times x + 80 \times x$

$= 180x$ m



Then the time taken for Nora to meet Lixin will be $(x + 6)$ min.

Distance between Town A and Town B

$= 100 \times (x + 6) + 75 \times (x + 6)$

$= 100x + 600 + 75x + 450$

$= (175x + 1050)$ m

Thus,

$$\begin{aligned} 180x &= 175x + 1050 \\ 10x - 175x &= 1050 \\ 5x &= 1050 \\ x &= 210 \end{aligned}$$

Distance between Town A and Town B

$= 180 \times 210$

$= 37\ 800$ m

Review Exercise 9

1. $a : b = \dfrac{1}{2} : \dfrac{1}{3}$ $\quad\downarrow\times 6$ $\quad = 3 : 2$ $\quad\downarrow\times 3$ $\quad = 9 : 6$

$b : c = 3 : 4$ $\quad\downarrow\times 2$ $\quad = 6 : 8$

$\therefore a : c = 9 : 8.$

2. **(i)** Let the mass of type A coffee beans in the mixture be $3x$ kg.

Then the mass of type B and C coffee beans in the mixture be $5x$ kg and $7x$ kg respectively.

$$\begin{aligned} \therefore 3x + 5x + 7x &= 35 \\ 15x &= 45 \\ x &= 3 \end{aligned}$$

Mass of type A coffee beans in the mixture $= 3 \times 3 = 9$ kg

Mass of type B coffee beans in the mixture $= 5 \times 3 = 15$ kg

Mass of type C coffee beans in the mixture $= 7 \times 3 = 21$ kg

(ii) Cost of the mixture per kg

$= \dfrac{(9 \times \$7) + (15 \times \$10) + (21 \times \$13)}{45}$

$= \$10.80$

3. **(i)** Let the number of books in the box be $4x$.

Then the initial number of toys in the box be $5x$.

$\therefore 4x = 36$

$x = 9$

So the initial number of toys in the box is $5 \times 9 = 45$.

(ii) Let the number of toys that are given away be y.

$$\begin{aligned} \therefore \frac{36}{45 - y} &= \frac{12}{11} \\ 11(36) &= 12(45 - y) \\ 396 &= 540 - 12y \\ 12y &= 540 - 396 \\ 12y &= 144 \\ y &= 12 \end{aligned}$$

The number of toys that are given away is 12.

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4. **(i)** Total cost of placing an advertisement containing 22 words
$= 350 + (22 \times 25)$
$= 900$ cents
$= \$9$

(ii) Let the number of words he can use be x.

Then $\quad 350 + 25x \leqslant 1500$

$$25x \leqslant 1500 - 350$$
$$25x \leqslant 1150$$
$$x \leqslant 46$$

The greatest number of words he can use is 46.

5. **(i)**

$$? \xrightarrow{+2\text{ h}} ? \xrightarrow{+15\text{ min}} 12\ 06$$
$$09\ 51 \xleftarrow[-2\text{ h}]{} 11\ 51 \xleftarrow[-9\text{ min}]{} 12\ 00 \xleftarrow[-6\text{ min}]{} 12\ 06$$

Working backwards, the time the journey started was 09 51.

(ii) Total distance = 198 km

Total time = 2 h 15 min = $2\frac{1}{4}$ h = $\frac{9}{4}$ h

Average speed = $\dfrac{198}{\frac{9}{4}}$

$= 88$ km/h

6. **(i)** Time taken = $\dfrac{195}{52}$

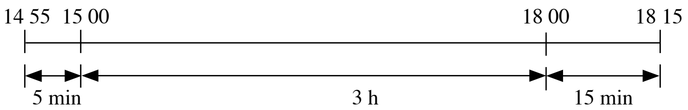
$= 3\frac{3}{4}$ h

$= 3$ h 45 min

$$08\ 45 \xrightarrow{+3\text{ h}} 11\ 45 \xrightarrow{+15\text{ min}} 12\ 00 \xrightarrow{+30\text{ min}} 12\ 30$$

$\therefore$ The time at which the lorry arrives at its destination is 12 30.

(ii)



5 min + 15 min = 20 min

$\therefore$ The time taken was 3 h 20 min.

3 h 20 min = $3\frac{1}{3}$ h

$= \frac{10}{3}$ h

Average speed = $\dfrac{195}{\frac{10}{3}}$

$= 58.5$ km/h

7. Distance that the athlete cycles

$= 40 \times \dfrac{30}{60}$

$= 20$ km

Time taken for the athlete to run

$= \dfrac{5 \times 1000}{3}$

$= 1666\frac{2}{3}$ s

$= 1666\frac{2}{3} \div 3600$

$= \dfrac{25}{54}$ h

His average speed for the entire competition

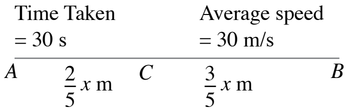
$= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{\frac{750}{1000} + 20 + 5}{\frac{15}{60} + \frac{30}{60} + \frac{25}{54}}$

$= 21\frac{30}{131}$ km/h

8. Let the distance from A to C be $\frac{2}{5}x$ m.

Then the distance from C to B be $\frac{3}{5}x$ m.



(i) Time taken for the object to travel from C to B

$= \dfrac{\frac{3}{5}x}{30}$

$= \frac{3}{5}x \div 30$

$= \frac{3}{5}x \times \frac{1}{30}$

$= \frac{1}{50}x$ s

(ii) Average speed of the object for its entire journey from A to B

$= \dfrac{x\text{ m}}{\left(30 + \frac{1}{50}x\right)\text{s}}$

$= \dfrac{50x}{1500 + x}$ m/s

9. Let the first part of the journey be x km.

Then the remaining part of the journey be $(150 - x)$ km.

Time taken for the entire journey = 4.5 h

$$\therefore \frac{x}{35} + \frac{150 - x}{5} = 4.5$$
$$35 \times \frac{x}{35} + 35 \times \frac{150 - x}{5} = 35 \times 4.5$$
$$x + 7(150 - x) = 157.5$$
$$x + 1050 - 7x = 157.5$$
$$x - 7x = 157.5 - 1050$$
$$-6x = -892.5$$
$$x = 148.75$$

$\therefore$ Distance = 148.75 km

10. Radius of the wheel of the car

$= \frac{48}{2}$

$= 24$ cm

Circumference of the wheel of the car

$= 2 \times \pi \times 24$

$= 2 \times 3.142 \times 24$

$= 150.816$ cm

Distance travelled by a car in 1 minute

$= 3.5 \div 60$

$= \frac{7}{120}$ km

$= \frac{7}{120} \times 100\,000$ cm

$= 5833\frac{1}{3}$ cm

Number of revolutions made by the wheel per minute

$= \frac{5833\frac{1}{3}}{150.816}$

$= 39$ (to the nearest whole number)

Challenge Yourself

1. Let the rate of the moving escalator be x steps per second.

When she is walking down at a rate of 2 steps per second, then the total steps (including the steps covered by the moving escalator) covered in 1 second is $(x + 2)$. Since she use 18 steps to reach the bottom from the top, therefore, the time taken is $(18 \div 2) = 9$ seconds. When she is exhausted, then the total steps (including the steps covered by the moving escalator) covered in 1 second is $(x + 1)$. Since she use 12 steps to reach the bottom from the top, therefore, the time taken is $(12 \div 1) = 12$ seconds.

Hence,

$$9(x + 2) = 12(x + 1)$$
$$9x + 18 = 12x + 12$$
$$9x - 12x = 12 - 18$$
$$-3x = -6$$
$$x = 2$$

$\therefore$ Total steps covered by the moving escalator $= 9(2 + 2) = 36$.

Hence the time taken for her to reach the bottom from the top if she stands on the escalator

$= \frac{36}{2}$

$= 18$ s

2. Let Vishal's speed be x m/s and Jun Wei's speed be y m/s.

Then in the first race, when Vishal ran pass the end point 100 m, Jun Wei is only at 90 m of the race. Hence, at the same time,

$$\frac{100}{x} = \frac{90}{y}$$
$$100y = 90x$$
$$x = \frac{100y}{90}$$

For the second race,

Let the time for the first person to pass the end point be t s.

Time taken for Vishal to finish the 100 m race

$= \frac{110}{x}$

$= \frac{110}{\left(\frac{100y}{90}\right)}$ (Substitute $x = \frac{100y}{90}$)

$= \frac{99}{y}$ s

Time taken for Jun Wei to finish the 100 m race

$= \frac{100}{y}$ s

At time t s, distance that Vishal covered

$t = \frac{99}{y}$

$ty = 99$ m

At time t s, distance that Jun Wei covered

$t = \frac{100}{y}$

$ty = 100$ m

$\therefore$ Vishal win the race by $100 - 99 = 1$ m.

Chapter 10 Basic Geometry

TEACHING NOTES

Suggested Approach

Students have learnt angle measurement in primary school. They have learnt the properties, namely, angles on a straight line, angles at a point and vertically opposite angles. However, students are unfamiliar with the types of angles and using algebraic terms in basic geometry. There is a need to guide students to apply basic algebra and linear equations in this topic. Students will learn how to do this through the worked examples in this topic. Teachers can introduce basic geometry by showing real-life applications (see chapter opener on page 231).

Section 10.1: Points, Lines and Planes

Teachers should illustrate what a point, a line, intersecting lines and planes look like. Teachers can impress upon the students that there is a difference between a line and a ray. A ray has a direction while a line has no direction. Teachers can highlight to the students that for a ray, the arrowhead indicates the direction in which the ray extends while for a line, its arrowhead is to indicate that the line continues indefinitely.

The thinking time on page 234 of the textbook requires students to think and determine whether each of the statements is true or false. Teachers should make use of this opportunity to highlight and clear some common misconceptions about points, lines and planes.

Section 10.2: Angles

Teachers can build upon prerequisites, namely angle measurement, to introduce the types of angles by classifying angle measurements according to their sizes.

To make practice more interesting, teachers can get the students to work in groups to measure and classify the various types of angles of different objects (i.e. scissors, set square, compass and the hands of a clock).

Teachers should recap with students on what they have learnt in primary school, i.e. angles on a straight line, angles at a point and vertically opposite angles. After going through Worked Examples 1 to 4, students should be able to identify the properties of angles and use algebraic terms to form and solve a linear equation to find the value of the unknowns. Students are expected to state reasons in their working.

Section 10.3: Angles Formed by Two Parallel Lines and a Transversal

Teachers can get students to discuss examples where they encounter parallel lines in their daily lives and ask them what happens when a line or multiple lines cut the parallel lines.

To make learning more interactive, students are given the opportunity to explore the three angle properties observed when a pair of parallel lines is cut by a transversal (see Investigation: Corresponding Angles, Alternate Angles and Interior Angles). Through this investigation, students should be able to observe the properties of angles associated with parallel lines. The investigation also helps students to learn how to solve problems involving angles formed by two parallel lines and a transversal. Students are expected to use appropriate algebraic variables to form and solve linear equations to find the value of the unknowns. Teachers should emphasise the importance of stating the properties when the students are solving questions on basic geometry.

Challenge Yourself

Question 1: Teachers can guide the students by hinting to them that this question is similar to a problem involving number patterns. Students have to draw a table and write down the first few numbers of rays between OA and OB, and their respective number of different angles. The students will then have to observe carefully and find an expression that represents rays between OA and OB.

Question 2: Teachers can guide the students by telling them to find the different angles that both the hour hand and minute hand makes from one specific position to another.

Question 3: Teachers can guide the students by telling them to find the number of times the bell will sound between certain times of the day.

WORKED SOLUTIONS

Thinking Time (Page 234)

(a) False. There are an infinite number of points lying on a line segment.
(b) False. There is exactly one line that passes through any three distinct points which are collinear; there is no line that passes through any three distinct points which are non-collinear.
(c) False. There is exactly one line that passes through any two distinct points.
(d) False. Two distinct lines intersect at one point; two coincident lines intersect at an infinite number of points; two parallel lines do not intersect at any point.
(e) True.

Investigation (Corresponding Angles, Alternate Angles and Interior Angles)

1. $\angle a = \angle b$
2. $\angle a = \angle c$
3. $\angle a + \angle d = 180°$
(a) $\angle a = \angle b$ (corr. $\angle$s)
(b) $\angle a = \angle c$ (alt. $\angle$s)
(c) $\angle a + \angle d = 180°$ (int. $\angle$s)
5. $\angle b = \angle a$ (corr. $\angle$s)
$\angle c = \angle b$ (vert. opp. $\angle$s)
$= \angle a$
$\therefore \angle a = \angle c$ (proven)
6. **Method 1:**
$\angle b = \angle a$ (corr. $\angle$s)
$\angle b + \angle d = 180°$ (adj. $\angle$s on a str. line)
$\therefore \angle a + \angle d = 180°$ (proven)
Method 2:
$\angle c = \angle a$ (alt. $\angle$s)
$\angle c + \angle d = 180°$ (adj. $\angle$s on a str. line)
$\therefore \angle a + \angle d = 180°$ (proven)

Practise Now (Page 236)

(a) Acute
(b) Reflex
(c) Obtuse
(d) Obtuse
(e) Reflex
(f) Acute

Practise Now 1

1. **(a)** $122° + a° = 180°$ (adj. $\angle$s on a str. line)
$a° = 180° - 122°$
$= 58°$
$\therefore a = 58$
(b) $95° + 65° + b° = 180°$ (adj. $\angle$s on a str. line)
$b° = 180° - 95° - 65°$
$= 20°$
$\therefore b = 20$

2. $2c° + 100° + 3c° = 180°$ (adj. $\angle$s on a str. line)
$2c° + 3c° = 180° - 100°$
$5c° = 80°$
$c° = 16°$
$\therefore c = 16$

Practise Now 2

1. $58° + 148° + 7a° = 360°$ ($\angle$s at a point)
$7a° = 360° - 58° - 148°$
$= 154°$
$a° = 22°$
$\therefore a = 22$
2. $b° + 90° + b° + 4b° = 360°$ ($\angle$s at a point)
$b° + b° + 4b° = 360° - 90°$
$6b° = 270°$
$b° = 45°$
$\therefore b = 45$

Practise Now 3

(i) $A\hat{O}C + 90° + 53° = 180°$ (adj. $\angle$s on a str. line)
$A\hat{O}C = 180° - 90° - 53°$
$= 37°$
(ii) $B\hat{O}D = A\hat{O}C$
$= 37°$ (vert. opp. $\angle$s)

Practise Now 4

$3a° + 40° = a° + 60°$ (vert. opp. $\angle$s)
$3a° - a° = 60° - 40°$
$2a° = 20°$
$a° = 10°$
$\therefore a = 10$
$a° + 60° + 4b° + 10° = 180°$ (adj. $\angle$s on a str. line)
$10° + 60° + 4b° + 10° = 180°$
$4b° = 180° - 10° - 60° - 10°$
$= 100°$
$b° = 25°$
$\therefore b = 25$

Practise Now (Page 246)

(a) **(i)** $\angle a$ and $\angle m$, $\angle b$ and $\angle n$, $\angle c$ and $\angle o$, $\angle d$ and $\angle p$, $\angle e$ and $\angle i$, $\angle f$ and $\angle j$, $\angle g$ and $\angle k$, $\angle h$ and $\angle l$
(ii) $\angle c$ and $\angle m$, $\angle d$ and $\angle n$, $\angle g$ and $\angle i$, $\angle h$ and $\angle j$
(iii) $\angle c$ and $\angle n$, $\angle d$ and $\angle m$, $\angle g$ and $\angle j$, $\angle h$ and $\angle i$
(b) No, $\angle c \neq \angle g$ as PQ is not parallel to RS.

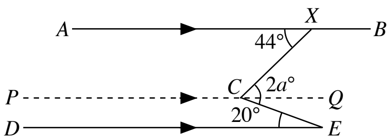
Practise Now 5

1. $a° = 54°$ (corr. $\angle$s, $AB // CD$)
$\therefore a = 54$
$c° + 106° = 180°$ (int. $\angle$s, $AB // CD$)
$c° = 180° - 106°$
$= 74°$
$\therefore c = 74$
$b° = c°$ (vert. opp. $\angle$s)
$= 74°$
$\therefore b = 74$
$d° = c°$ (corr. $\angle$s, $AB // CD$)
$= 74°$
$\therefore d = 74$

2. $2e° + 30° = 69°$ (corr. $\angle$s, $AB // CD$)
$2e° = 69° - 30°$
$= 39°$
$e° = 19.5°$
$\therefore e = 19.5$
$f° = 2e°$ (corr. $\angle$s, $AB // CD$)
$= 39°$
$\therefore f = 39$

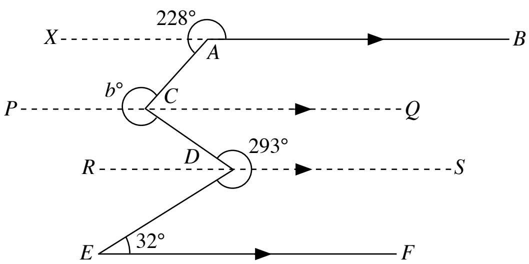
Practise Now 6

1.



$X\hat{C}Q = 44°$ (alt. $\angle$s, $AB // PQ$)
$E\hat{C}Q = 20°$ (alt. $\angle$s, $PQ // DE$)
$2a° = X\hat{C}Q + E\hat{C}Q$
$= 44° + 20°$
$= 64°$
$a° = 32°$
$\therefore a = 32$

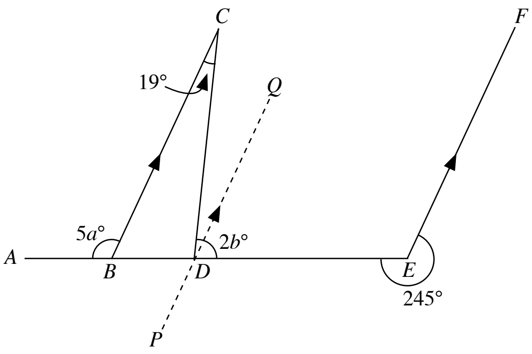
2.



$X\hat{A}C = 228° - 180°$
$= 48°$
$A\hat{C}Q = X\hat{A}C$ (alt. $\angle$s, $XB // PQ$)
$= 48°$
$E\hat{D}R = 32°$ (alt. $\angle$s, $RS // EF$)
$E\hat{D}R + C\hat{D}R + 293° = 360°$ ($\angle$s at a point)
$32° + C\hat{D}R + 293° = 360°$
$C\hat{D}R = 360° - 32° - 293°$
$= 35°$

$D\hat{C}Q = C\hat{D}R = 35°$ (alt. $\angle$s, $PQ // RS$)
$b° + A\hat{C}Q + D\hat{C}Q = 360°$ ($\angle$s at a point)
$b° + 48° + 35° = 360°$
$b° = 360° - 35° - 48°$
$= 277°$
$\therefore b = 277$

Practise Now 7



$D\hat{E}F + 245° = 360°$ ($\angle$s at a point)
$D\hat{E}F = 360° - 245°$
$= 115°$
$5a° = 115°$ (corr. $\angle$s, $BC // EF$)
$a° = 23°$
$\therefore a = 23$
$C\hat{D}Q = 19°$ (alt. $\angle$s, $BC // PQ$)
$E\hat{D}Q + D\hat{E}F = 180°$ (int. $\angle$s, $PQ // EF$)
$E\hat{D}Q + 115° = 180°$
$E\hat{D}Q = 180° - 115°$
$= 65°$
$2b° = C\hat{D}Q + E\hat{D}Q$
$= 19° + 65°$
$= 84°$
$b° = 42°$
$\therefore b = 42$

Practise Now 8

Since $B\hat{W}Q = D\hat{Y}Q$ (= 122°), then $AB // CD$ (converse of corr. $\angle$s).
$\therefore B\hat{X}S = C\hat{Z}R = 65°$ (alt. $\angle$s, $AB // CD$)

Exercise 10A

1. **(a)** $a = 79$, $b = 106$, $c = 98$
(b) $d = 50$, $e = 228$
(c) $f = 117$, $g = 45$
(d) $h = 243$, $i = 94$, $j = 56$

2. **(a)** Obtuse
(b) Reflex
(c) Acute
(d) Reflex
(e) Acute
(f) Obtuse

3. **(a)** Complementary angle of 18° = 90° – 18°
= 72°

(b) Complementary angle of 46° = 90° – 46°
= 44°

(c) Complementary angle of 53° = 90° – 53°
= 37°

(d) Complementary angle of 64° = 90° – 64°
= 26°

4. **(a)** Supplementary angle of 36° = 180° – 36°
= 144°

(b) Supplementary angle of 12° = 180° – 12°
= 168°

(c) Supplementary angle of 102° = 180° – 102°
= 78°

(d) Supplementary angle of 171° = 180° – 171°
= 9°

5. **(a)** $a° + 33° = 180°$ (adj. ∠s on a str. line)

$a° = 180° - 33°$

$= 147°$

$\therefore a = 147$

(b) $b° + 42° + 73° = 180°$ (adj. ∠s on a str. line)

$b° = 180° - 42° - 73°$

$= 65°$

$\therefore b = 65$

(c) $4c° + 80° + c° = 180°$ (adj. ∠s on a str. line)

$4c° + c° = 180° - 80°$

$5c° = 100°$

$c° = 20°$

$\therefore c = 20$

(d) $4d° + 16° + 2d° + 14° = 180°$ (adj. ∠s on a str. line)

$4d° + 2d° = 180° - 16° - 14°$

$6d° = 150°$

$d° = 25°$

$\therefore d = 25$

6. **(a)** $x° + y° + z° = 180°$ (adj. ∠s on a str. line)

When $y° = 45°$, $z° = 86°$,

$x° + 45° + 86° = 180°$

$x° = 180° - 45° - 86°$

$= 49°$

$\therefore x = 49$

(b) $x° + y° + z° = 180°$ (adj. ∠s on a str. line)

When $x° = 2y°$, $z° = 3y°$,

$2y° + y° + 3y° = 180°$

$6y° = 180°$

$y° = 30°$

$\therefore y = 30$

7. **(a)** $a° + 67° + 52° + 135° = 360°$ (∠s at a point)

$a° = 360° - 67° - 52° - 135°$

$= 106°$

$\therefore a = 106$

(b) $5b° + 4b° + 3b° = 360°$ (∠s at a point)

$12b° = 360°$

$b° = 30°$

$\therefore b = 30$

(c) $16c° + 4c° + 90° + 4c° = 360°$ (∠s at a point)

$16c° + 4c° + 4c° = 360° - 90°$

$24c° = 270°$

$c° = 11.25°$

$\therefore c = 11.25$

(d) $(7d + 23)° + 6d° + 139° + 5d° = 360°$ (∠s at a point)

$7d° + 23° + 6d° + 139° + 5d° = 360°$

$7d° + 6d° + 5d° = 360° - 23° - 139°$

$18d° = 198°$

$d° = 11°$

$\therefore d = 11$

8. **(i)** $A\hat{O}C = 48°$ (vert. opp. ∠s)

(ii) $90° + D\hat{O}E + 48° = 180°$ (adj. ∠s on a str. line)

$D\hat{O}E = 180° - 90° - 48°$

$= 42°$

9. **(a)** $40° + 30° + a° = 117°$ (vert. opp. ∠s)

$a° = 117° - 40° - 30°$

$= 47°$

$\therefore a = 47$

(b) $7b° + 3b° = 180°$ (adj. ∠s on a str. line)

$10b° = 180°$

$b° = 18°$

$\therefore b = 18$

$c° = 7b°$ (vert. opp. ∠s)

$= 7(18°)$

$= 126°$

$\therefore c = 126$

10. **(a)** $x° + y° + z° = 180°$ (adj. ∠s on a str. line)

$y° + x° + z° = 180°$

When $y° = x° + z°$,

$y° + y° = 180°$

$2y° = 180°$

$y° = 90°$

$\therefore y = 90$

(b) $x° + y° + z° = 180°$ (adj. ∠s on a str. line)

When $x° = y° = z°$,

$z° + z° + z° = 180°$

$3z° = 180°$

$z° = 60°$

$\therefore z = 60$

11. $A\hat{O}B + D\hat{O}A = 180°$ (adj. ∠s on a str. line)

$A\hat{O}B + 5A\hat{O}B = 180°$

$6A\hat{O}B = 180°$

$\therefore A\hat{O}B = 30°$

$B\hat{O}C = 2A\hat{O}B$

$= 2 \times 30°$

$= 60°$

$C\hat{O}D = 4A\hat{O}B$

$= 4 \times 30°$

$= 120°$

$D\hat{O}A = 5A\hat{O}B$

$= 5 \times 30°$

$= 150°$

12. (a) $7a° + 103° = 180°$ (adj. $\angle$s on a str. line)

$7a° = 180° - 103°$

$= 77°$

$a° = 11°$

$\therefore a = -11$

$2b° + 13° = 103°$ (vert. opp. $\angle$s)

$2b° = 103° - 13°$

$= 90°$

$b° = 45°$

$\therefore b = 45$

(b) $62° + 49° + 3c° = 180°$ (adj. $\angle$s on a str. line)

$3c° = 180° - 62° - 49°$

$= 69°$

$c° = 23°$

$\therefore c = 23$

$d° = 3c°$ (vert. opp. $\angle$s)

$= 69°$

$\therefore d = 69$

$e° = 62° + 49°$ (vert. opp. $\angle$s)

$= 111°$

$\therefore e = 111$

(c) $7f° + 5° = 2f° + 35°$ (vert. opp. $\angle$s)

$7f° - 2f° = 35° - 5°$

$5f° = 30°$

$f° = 6°$

$\therefore f = 6$

$2f° + 35° + 5g° + 18° = 180°$ (adj. $\angle$s on a str. line)

$2(6°) + 35° + 5g° + 18° = 180°$

$12° + 35° + 5g° + 18° = 180°$

$5g° = 180° - 12° - 35° - 18°$

$= 115°$

$g° = 23°$

$\therefore g = 23$

(d) $24° + 90° + h° = 104° + 32°$ (vert. opp. $\angle$s)

$h° = 104° + 32° - 24° - 90°$

$= 22°$

$\therefore h = 22$

$24° + 90° + h° + 2i° = 180°$ (adj. $\angle$s on a str. line)

$24° + 90° + 22° + 2i° = 180°$

$2i° = 180° - 24° - 90° - 22°$

$= 44°$

$i° = 22°$

$\therefore i = 22$

$j° = 2i°$ (vert. opp. $\angle$s)

$= 44°$

$\therefore j = 44$

13. (i) $(186 - 4x)° + 34° = 6x°$ (vert. opp. $\angle$s)

$186° - 4x° + 34° = 6x°$

$6x° + 4x° = 186° + 34°$

$10x° = 220°$

$x° = 22°$

$\therefore x = 22$

$6x° + 3y° = 180°$ (adj. $\angle$s on a str. line)

$6(22°) + 3y° = 180°$

$132° + 3y° = 180°$

$3y° = 180° - 132°$

$= 48°$

$y° = 16°$

$\therefore y = 16$

(ii) Obtuse $A\hat{O}D = (186 - 4x)° + 34°$

$= [186 - 4(22)]° + 34°$

$= 98° + 34°$

$= 132°$

Reflex $C\hat{O}E = 180° + (186 - 4x)°$

$= 180° + 98°$

$= 278°$

Exercise 10B

1. (a) (i) $B\hat{X}R$ and $D\hat{Z}R$, $A\hat{X}R$ and $C\hat{Z}R$, $A\hat{X}S$ and $C\hat{Z}S$, $B\hat{X}S$ and $D\hat{Z}S$, $B\hat{W}P$ and $D\hat{Y}P$, $A\hat{W}P$ and $C\hat{Y}P$, $A\hat{W}Q$ and $C\hat{Y}Q$, $B\hat{W}Q$ and $D\hat{Y}Q$

(ii) $A\hat{X}S$ and $D\hat{Z}R$, $B\hat{X}S$ and $C\hat{Z}R$, $A\hat{W}Q$ and $D\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(iii) $A\hat{X}S$ and $C\hat{Z}R$, $B\hat{X}S$ and $D\hat{Z}R$, $A\hat{W}Q$ and $C\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(b) No, $B\hat{W}Q \neq A\hat{X}R$ as PQ is not parallel to RS.

(c) No, the sum of $D\hat{Y}P$ and $C\hat{Z}R$ is not equal to 180° as PQ is not parallel to RS.

2. (a) $a° = 117°$ (vert. opp. $\angle$s)

$\therefore a = 117$

$b° = 117°$ (corr. $\angle$s, AB // CD)

$\therefore b = 117$

$c° + a° = 180°$ (int. $\angle$s, AB // CD)

$c° + 117° = 180°$

$c° = 180° - 117°$

$= 63°$

$\therefore c = 63$

$d° = 78°$ (corr. $\angle$s, AB // CD)

$\therefore d = 78$

(b) $e° = 31°$ (alt. $\angle$s, AB // CD)

$\therefore e = 31$

$f° = 35° + 31°$ (alt. $\angle$s, AB // CD)

$= 66°$

$\therefore f = 66$

(c) $g° = 83°$ (alt. $\angle$s, AB // CD)

$\therefore g = 83$

$h° = 69°$ (corr. $\angle$s, AB // CD)

$\therefore h = 69$

(d) $i° + 75° + 60° = 180°$ (int. $\angle$s, AB // CD)

$i° = 180° - 75° - 60°$

$= 45°$

$\therefore i = 45$

$j° = 60°$ (alt. $\angle$s, AB // CD)

$\therefore j = 60$

3. **(a)** $a° = 38°$ (corr. ∠s, AB // CD)

$\therefore a = 38$

$a° + 30° = 2b°$ (corr. ∠s, AB // CD)

$38° + 30° = 2b°$

$2b° = 68°$

$b° = 34°$

$\therefore b = 34$

(b) $7c° = 140°$ (corr. ∠s, AB // CD)

$c° = 20°$

$\therefore c = 20$

$2d° = 7c°$ (vert. opp. ∠s)

$= 140°$

$d° = 70°$

$\therefore d = 70$

(c) $7e° + 3e° = 180°$ (int. ∠s, AB // CD)

$10e° = 180°$

$e° = 18°$

$\therefore e = 18$

(d) $(2f + 6)° = (3f – 23)°$ (alt. ∠s, AB // CD)

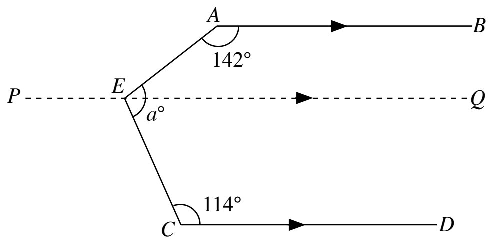
$2f° + 6° = 3f° – 23°$

$3f° – 2f° = 6° + 23°$

$f° = 29°$

$\therefore f = 29$

4. **(a)**



$A\hat{E}Q + 142° = 180°$ (int. ∠s, AB // PQ)

$A\hat{E}Q = 180° – 142°$

$= 38°$

$C\hat{E}Q + 114° = 180°$ (int. ∠s, PQ // CD)

$C\hat{E}Q = 180° – 114°$

$= 66°$

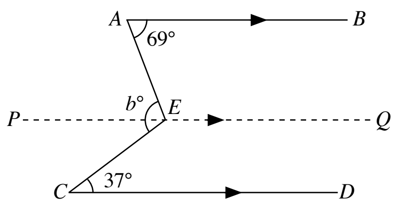
$a° = A\hat{E}Q + C\hat{E}Q$

$= 38° + 66°$

$= 104°$

$\therefore a = 104$

(b)



$A\hat{E}P = 69°$ (alt. ∠s, AB // PQ)

$C\hat{E}P = 37°$ (alt. ∠s, PQ // CD)

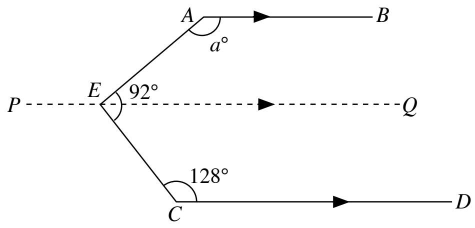
$b° = A\hat{E}P + C\hat{E}P$

$= 69° + 37°$

$= 106°$

$\therefore b = 106$

5. **(a)**



$C\hat{E}Q + 128° = 180°$ (int. ∠s, PQ // CD)

$C\hat{E}Q = 180° – 128°$

$= 52°$

$A\hat{E}Q = 92° – C\hat{E}Q$

$= 92° – 52°$

$= 40°$

$a° + A\hat{E}Q = 180°$ (int. ∠s, AB // PQ)

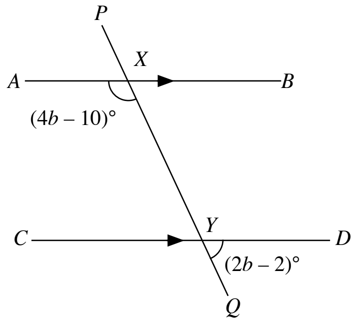
$a° + 40° = 180°$

$a° = 180° – 40°$

$= 140°$

$\therefore a = 140$

(b)



$C\hat{Y}P = (2b – 2)°$ (vert. opp. ∠s)

$C\hat{Y}P + (4b – 10)° = 180°$ (int. ∠s, AB // CD)

$(2b – 2)° + (4b – 10)° = 180°$

$2b° – 2° + 4b° – 10° = 180°$

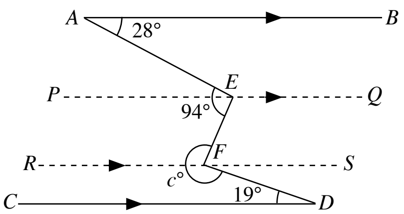
$2b° + 4b° = 180° + 2° + 10°$

$6b° = 192°$

$b° = 32$

$\therefore b = 32$

(c)



$A\hat{E}P = 28°$ (alt. ∠s, AB // PQ)

$F\hat{E}P = 94° – A\hat{E}P$

$= 94° – 28°$

$= 66°$

$E\hat{F}S = F\hat{E}P$

$= 66°$ (alt. ∠s, PQ // RS)

$D\hat{F}S = 19°$ (alt. ∠s, RS // CD)

$c° + E\hat{F}S + D\hat{F}S = 360°$ (∠s at a point)

$c° + 66° + 19° = 360°$

$c° = 360° – 66° – 19°$

$= 275°$

$\therefore c = 275$

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6. **(i)** $C\hat{D}F = 86°$ (alt. $\angle$s, CE // FG)

(ii) $H\hat{D}E = 86°$ (vert. opp. $\angle$s)

$E\hat{D}A = H\hat{D}E - 47°$
$= 86° - 47°$
$= 39°$

$B\hat{A}D + E\hat{D}A = 180°$ (int. $\angle$s, AB // CE)

$B\hat{A}D + 39° = 180°$
$B\hat{A}D = 180° - 39°$
$= 141°$

7. **(i)** $A\hat{E}B = 68°$ (alt. $\angle$s, BF // AD)

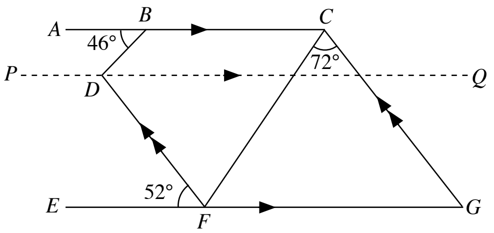
(ii) $E\hat{A}B = 58°$ (alt. $\angle$s, AB // CD)

$F\hat{B}A + E\hat{A}B = 180°$ (int. $\angle$s, BF // AD)

$F\hat{B}A + 58° = 180°$
$F\hat{B}A = 180° - 58°$
$= 122°$

$A\hat{B}E = F\hat{B}A - 68°$
$= 122° - 68°$
$= 54°$

8.



(i) $C\hat{G}F = 52°$ (corr. $\angle$s, FD // GC)

(ii) $B\hat{C}G + C\hat{G}F = 180°$ (int. $\angle$s, AC // EG)

$B\hat{C}G + 52° = 180°$
$B\hat{C}G = 180° - 52°$
$= 128°$

$B\hat{C}F = B\hat{C}G - 72°$
$= 128° - 72°$
$= 56°$

(iii) $B\hat{D}Q = 46°$ (alt. $\angle$s, AC // PQ)

$F\hat{D}Q = 52°$ (alt. $\angle$s, PQ // EG)

Reflex $B\hat{D}F + B\hat{D}Q + F\hat{D}Q = 360°$ ($\angle$s at a point)

Reflex $B\hat{D}F + 46° + 52° = 360°$

Reflex $B\hat{D}F = 360° - 46° - 52°$
$= 262°$

9. $F\hat{D}C + 58° = 180°$ (adj. $\angle$s on a str. line)

$F\hat{D}C = 180° - 58°$
$= 122°$

$D\hat{C}A = F\hat{D}C$ (alt. $\angle$s, DF // AC)
$= 122°$

$D\hat{C}A + 4x° = 360°$ ($\angle$s at a point)

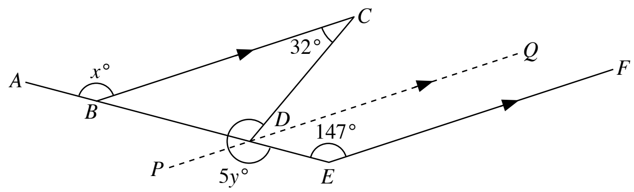
$122° + 4x° = 360°$
$4x° = 360° - 122°$
$= 238°$
$x° = 59.5°$
$\therefore x = 59.5$

$B\hat{A}C + D\hat{C}A = 180°$ (int. $\angle$s, AB // CE)

$B\hat{A}C + 122° = 180°$
$B\hat{A}C = 180° - 122°$
$= 58°$

$7y° + B\hat{A}C = 360°$
$7y° + 58° = 360°$
$7y° = 360° - 58°$
$= 302°$
$y° = 43.1°$ (to 1 d.p.)
$\therefore y = 43.1$

10.



$x° = 147°$ (corr. $\angle$s, BC // EF)

$\therefore x = 147$

$C\hat{D}Q = 32°$ (alt. $\angle$s, BC // PQ)

$Q\hat{D}E + 147° = 180°$ (int. $\angle$s, PQ // EF)

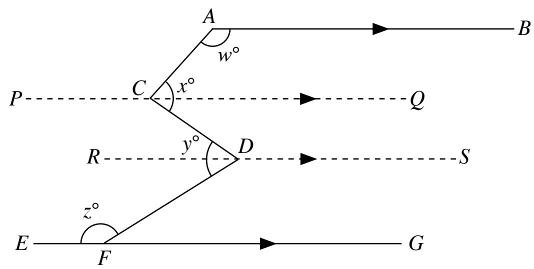
$Q\hat{D}E = 180° - 147°$
$= 33°$

$5y° + C\hat{D}Q + Q\hat{D}E = 360°$ ($\angle$s at a point)

$5y° + 32° + 33° = 360°$
$5y° = 360° - 32° - 33°$
$= 295°$
$y° = 59°$
$\therefore y = 59$

11. Since $A\hat{X}S + C\hat{Z}R = 104° + 76° = 180°$,
then AB // CD (converse of int. $\angle$s).
$\therefore B\hat{W}P = D\hat{Y}P = 46°$ (corr. $\angle$s, AB // CD)

12.



$Q\hat{C}A + w° = 180°$ (int. $\angle$s, AB // PQ)

$Q\hat{C}A = 180° - w°$

$Q\hat{C}D = x° - Q\hat{C}A$
$= x° - (180° - w°)$
$= x° - 180° + w°$

$C\hat{D}R = Q\hat{C}D$ (alt. $\angle$s, PQ // RS)
$= x° - 180° + w°$

$F\hat{D}R = y° - C\hat{D}R$
$= y° - (x° - 180° + w°)$
$= y° - x° + 180° - w°$

$F\hat{D}R + z° = 180°$ (int. $\angle$s, RS // EG)

$y° - x° + 180° - w° + z° = 180°$
$w° + x° = y° + z°$
$\therefore w + x = y + z$

Review Exercise 10

1. (a) $32° + 4a° + 84° = 180°$ (adj. $\angle$s on a str. line)

$4a° = 180° - 32° - 84°$

$= 64°$

$a° = 16°$

$\therefore a = 16$

$84° + 2b° = 180°$ (adj. $\angle$s on a str. line)

$2b° = 180° - 84°$

$= 96°$

$b° = 48°$

$\therefore b = 48$

(b) $c° + 68° = 180°$ (adj. $\angle$s on a str. line)

$c° = 180° - 68°$

$= 112°$

$\therefore c = 112$

$68° + 3d° - 5° + 30° = 180°$ (adj. $\angle$s on a str. line)

$3d° = 180° - 68° + 5° - 30°$

$= 87°$

$d° = 29°$

$\therefore d = 29$

2. (a) $4a° + 2a° + a° + a° + 2a° = 360°$ ($\angle$s at a point)

$10a° = 360°$

$a° = 36°$

$\therefore a = 36$

(b) $(3b - 14)° + (4b - 21)° + (2b + 1)° + (b + 34)°$

$= 360°$ ($\angle$s at a point)

$3b° - 14° + 4b° - 21° + 2b° + 1° + b° + 34° = 360°$

$3b° + 4b° + 2b° + b° = 360° + 14° + 21° - 1° - 34°$

$10b° = 360°$

$b° = 36°$

$\therefore b = 36$

3. (a) $C\hat{O}F = 4a° - 17°$ (vert. opp. $\angle$s)

$2a° + C\hat{O}F + 3a° - 10° = 180°$ (adj. $\angle$s on a str. line)

$2a° + 4a° - 17° + 3a° - 10° = 180°$

$2a° + 4a° + 3a° = 180° + 17° + 10°$

$9a° = 207°$

$a° = 23°$

$\therefore a = 23$

(b) $C\hat{O}F = 2b° + 15°$ (vert. opp. $\angle$s)

$2b° + 2b° + 15° + b° = 180°$ (adj. $\angle$s on a str. line)

$2b° + 2b° + b° = 180° - 15°$

$5b° = 165°$

$b° = 33°$

$\therefore b = 33$

$b° + 3c° + 2b° + 15° = 180°$ (adj. $\angle$s on a str. line)

$33° + 3c° + 2(33°) + 15° = 180°$

$33 + 3c° + 66° + 15° = 180°$

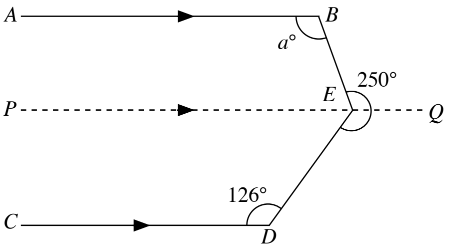
$3c° = 180° - 33° - 66° - 15°$

$= 66°$

$c° = 22°$

$\therefore c = 22$

4. (a)



$D\hat{E}P + 126° = 180°$ (int. $\angle$s, PQ // CD)

$D\hat{E}P = 180° - 126°$

$= 54°$

$B\hat{E}P + D\hat{E}P + 250° = 360°$ ($\angle$s at a point)

$B\hat{E}P + 54° + 250° = 360°$

$B\hat{E}P = 360° - 54° - 250°$

$= 56°$

$a° + B\hat{E}P = 180°$ (int. $\angle$s, AB // PQ)

$a° + 56° = 180°$

$a° = 180° - 56°$

$= 124°$

$\therefore a = 124$

(b) $(6b - 21)° + (5b - 52)° = 180°$ (int. $\angle$s, AB // CD)

$6b° - 21° + 5b° - 52° = 180°$

$6b° + 5b° = 180° + 21° + 52°$

$11b° = 253°$

$b° = 23°$

$\therefore b = 23$

$3c° = (6b - 21)°$

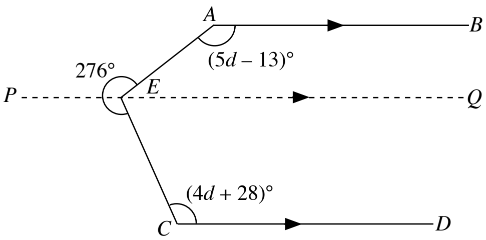
$= [6(23) - 21]°$

$= 117°$

$c° = 39°$

$\therefore c = 39$

(c)



$Q\hat{E}A + (5d - 13)° = 180°$ (int. $\angle$s, AB // PQ)

$Q\hat{E}A + 5d° - 13° = 180°$

$Q\hat{E}A = 180° - 5d° + 13°$

$= 193° - 5d°$

$Q\hat{E}C + (4d + 28)° = 180°$ (int. $\angle$s, PQ // CD)

$Q\hat{E}C + 4d° + 28° = 180°$

$Q\hat{E}C = 180° - 4d° - 28°$

$= 152° - 4d°$

$276° + Q\hat{E}A + Q\hat{E}C = 360°$ ($\angle$s at a point)

$276° + 193° - 5d° + 152° - 4d° = 360°$

$5d° + 4d° = 276° + 193° + 152° - 360°$

$9d° = 261°$

$d° = 29°$

$\therefore d = 29$

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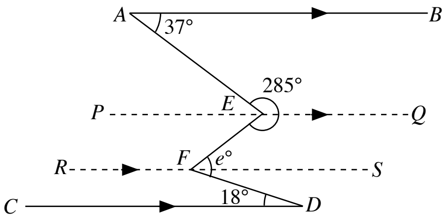
(d)



$A\hat{E}P = 37°$ (alt. $\angle$s, AB // PQ)

$A\hat{E}P + 285° + F\hat{E}P = 360°$ ($\angle$s at a point)

$37° + 285° + F\hat{E}P = 360°$

$F\hat{E}P = 360° - 37° - 285°$

$= 38°$

$E\hat{F}S = 38°$ (alt. $\angle$s, PQ // RS)

$D\hat{F}S = 18°$ (alt. $\angle$s, RS // CD)

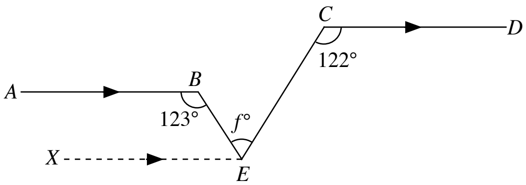
$e° = E\hat{F}S + D\hat{F}S$

$= 38° + 18°$

$= 56°$

$\therefore e = 56$

(e)



$X\hat{E}C = 122°$ (alt. $\angle$s, CD // XE)

$X\hat{E}B + 123° = 180°$ (int. $\angle$s, AB // XE)

$X\hat{E}B = 180° - 123°$

$= 57°$

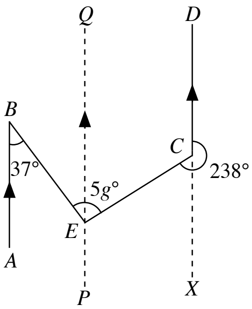
$f° = X\hat{E}C - X\hat{E}B$

$= 122° - 57°$

$= 65°$

$\therefore f = 65$

(f)



$B\hat{E}Q = 37°$ (alt. $\angle$s, AB // PQ)

$E\hat{C}X = 238° - 180°$

$= 58°$

$C\hat{E}Q = E\hat{C}X$ (alt. $\angle$s, PQ // XD)

$= 58°$

$5g° = B\hat{E}Q + C\hat{E}Q$

$= 37° + 58°$

$= 95°$

$g° = 19°$

$\therefore g = 19$

5. **(i)** $C\hat{D}F = 148°$ (alt. $\angle$s, GC // DF)

$C\hat{D}E + 84° + C\hat{D}F = 360°$ ($\angle$s at a point)

$C\hat{D}E + 84° + 148° = 360°$

$C\hat{D}E = 360° - 84° - 148°$

$= 128°$

(ii) $A\hat{B}C = C\hat{D}E$ (alt. $\angle$s, AB // DE)

$= 128°$

$A\hat{B}H = A\hat{B}C - 74°$

$= 128° - 74°$

$= 54°$

6. **(i)** $D\hat{E}H + 26° = 180°$ (adj. $\angle$s on a str. line)

$D\hat{E}H = 180° - 26°$

$= 154°$

(ii) $B\hat{E}H = 62°$ (alt. $\angle$s, EC // GH)

$D\hat{E}B + B\hat{E}H + D\hat{E}H = 360°$ ($\angle$s at a point)

$D\hat{E}B + 62° + 154° = 360°$

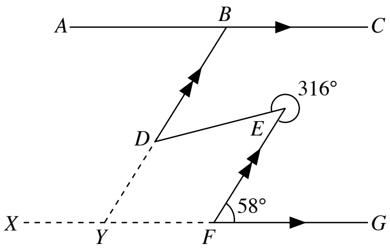
$D\hat{E}B = 360° - 62° - 154°$

$= 144°$

$A\hat{B}C = D\hat{E}B$ (corr. $\angle$s, AB // DF)

$= 144°$

7.



(i) $D\hat{E}F$ + reflex $D\hat{E}F = 360°$ ($\angle$s at a point)

$D\hat{E}F + 316° = 360°$

$D\hat{E}F = 360° - 316°$

$= 44°$

$B\hat{D}E = D\hat{E}F$ (alt. $\angle$s, DB // FE)

$= 44°$

(ii) $D\hat{Y}F = 58°$ (corr. $\angle$s, YB // FE)

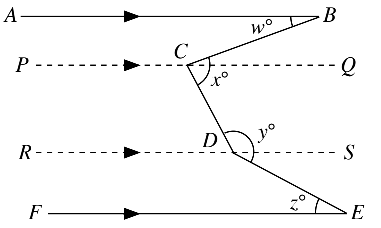
$A\hat{B}D = D\hat{Y}F$ (alt. $\angle$s, AC // XG)

$= 58°$

8. Since $B\hat{W}Q + D\hat{Y}P = 123° + 57° = 180°$,

then AB // CD (converse of int. $\angle$s).

$\therefore D\hat{Z}R = A\hat{X}S = 118°$ (alt. $\angle$s, AB // CD)

9. 

$B\hat{C}Q = w°$ (alt. $\angle$s, AB // PQ)
$D\hat{C}Q = x° - B\hat{C}Q$
$= x° - w°$
$C\hat{D}S + D\hat{C}Q = 180°$ (int. $\angle$s, PQ // RS)
$C\hat{D}S + x° - w° = 180°$
$C\hat{D}S = 180° - x° + w°$
$E\hat{D}S = y° - (180° - x° + w°)$
$= y° - 180° + x° - w°$
$E\hat{D}S = z°$ (alt. $\angle$s, RS // FE)
$y° - 180° + x° - w° = z°$
$w° + z° + 180° = x° + y°$
$\therefore w + z + 180 = x + y$

Challenge Yourself

1.

Number of Rays between *OA* and *OB*	**Number of Different Angles**
0	$1 = \frac{1}{2} \times 1 \times 2$
1	$3 = \frac{1}{2} \times 2 \times 3$
2	$6 = \frac{1}{2} \times 3 \times 4$
3	$10 = \frac{1}{2} \times 4 \times 5$
4	$15 = \frac{1}{2} \times 5 \times 6$
⋮	⋮
n	$\frac{1}{2}(n+1)(n+2)$

Number of different angles in the figure $= \frac{1}{2}(n+1)(n+2)$

2. Angle hour hand moves in 1 hour $= \frac{1}{12} \times 360°$
$= 30°$
Angle hour hand moves from 12 noon to 7 p.m. $= 7 \times 30°$
$= 210°$
Angle hour hand moves from 7 p.m. to 7.20 p.m. $= \frac{20}{60} \times 30°$
$= 10°$
Angle hour hand moves from 12 noon to 7.20 p.m. $= 210° + 10°$
$= 220°$
Angle minute hand moves from 7 p.m. to 7.20 p.m. $= \frac{20}{60} \times 360°$
$= 120°$
Smaller angle between minute hand and hour hand at 7.20 p.m.
$= 220° - 120°$
$= 100°$
Larger angle between minute hand and hour hand at 7.20 p.m.
$= 360° - 100°$ ($\angle$s at a point)
$= 260°$

3. From 9 a.m. to before 9 p.m. on any particular day, the bell will sound twice every hour, except for the hour from 1 p.m. to before 2 p.m. and the hour from 2 p.m. to before 3 p.m., when it only sounds once during each hour.
Likewise, from 9 p.m. on any particular day to before 9 a.m. the next day, the bell will sound twice every hour, except for the hour from 1 a.m. to before 2 a.m. and the hour from 2 a.m. to before 3 a.m, when it only sounds once during each hour.
$\therefore$ Number of times bell will sound from 9 a.m. on a particular day to before 9 p.m. the next day
$= 2 \times 30 + 1 \times 6$
$= 60 + 6$
$= 66$
Since the bell will sound at 9 p.m. the next day,
Number of times bell will sound from 9 a.m. on a particular day to 9 p.m. the next day
$= 66 + 1$
$= 67$

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Chapter 11 Triangles, Quadrilaterals and Polygons

TEACHING NOTES

Suggested Approach

Students have learnt about triangles, and quadrilaterals such as parallelograms, rhombuses and trapeziums in primary school. They would have learnt the properties and finding unknown angles involving these figures. In this chapter, students begin from 3-sided triangles, to 4-sided quadrilaterals and finally n-sided polygons. The incremental approach is to ensure that students have a good understanding before they move on to a higher level. Teachers may want to dedicate more time and attention to the section on polygons in the last section of this chapter.

Section 11.1: Triangles

Students have learnt about isosceles triangles, equilateral triangles and right-angled triangles in primary school. In this chapter, students should be aware that triangles can be classified by the number of equal sides or the types of angles. Teachers may want to check students' understanding on the classification of triangles (see Thinking Time on page 260). Teachers should highlight to the students that equilateral triangles are a special type of isosceles triangles while scalene triangles are triangles that are not isosceles, and are definitely not equilateral triangles.

Students should explore and discover that the longest side of a triangle is opposite the largest angle, and the sum of two sides is always larger than the third side (see Investigation: Basic Properties of a Triangle).

Teachers should ensure students are clear what exterior angles are before stating the relation between exterior angles and its interior opposite angles. Some may think that the exterior angle of a triangle is the same as the reflex angle at a vertex of a triangle.

Section 11.2: Quadrilaterals

Teachers may want to first recap students' knowledge of parallelograms, rhombuses and trapeziums based on what they have learnt in primary school. Teachers can use what students have learnt in Chapter 10, reintroduce and build up their understanding of the different types of quadrilaterals and their properties (see Investigation: Properties of Special Quadrilaterals and Investigation: Symmetric Properties of Special Quadrilaterals). For further understanding, teachers may wish to show the taxonomy of quadrilaterals to demonstrate their relations.

Before proceeding onto the next section, teachers may want to go through with the students the angle properties of triangles and quadrilaterals. This reinforces the students' knowledge as well as prepares them for the section on polygons.

Section 11.3: Polygons

Teachers should emphasise to the students that triangles and quadrilaterals are polygons so that they are aware that all the concepts which they have learnt so far remains applicable in this topic. Students should learn the different terms with regards to polygons. In this section, most polygons studied will be simple, convex polygons.

Students need to know the names of polygons with 10 sides or less and the general naming convention of polygons (see Class Discussion: Naming of Polygons). Through the class discussion, students should be able to develop a good understanding on polygons and be able to name them. They should also know and appreciate the properties of regular polygons (see Investigation: Properties of a Regular Polygon and Investigation: Symmetric Properties of Regular Polygons).

Teachers can ask students to recall the properties of triangles and quadrilaterals during the investigation of the sum of interior angles and sum of exterior angles of a polygon. Students should see a pattern in how the sum of interior angles differs as the number of sides increases and understand its formula, (see Investigation: Sum of Interior Angles of a Polygon) as well as discover that the sum of exterior angles is always equal to 360° regardless of the number of sides of the polygon (see Investigation: Sum of Exterior Angles of a Pentagon).

Challenge Yourself

Some of the questions (e.g. Questions 1 and 2) may be challenging for most students while the rest of the questions can be done with guidance from teachers.

Question 1: Two new points need to be added. The first point (say, E) is the midpoint of BC and the second point (say, F) lies on the line AE such that $\triangle BCF$ is equilateral. Draw the lines AE, CF and DF. Begin by finding $A\hat{B}C$ and continue from there.

Question 2: Draw DG such that BC // DG, and mark E at the point where DG cuts CD. Join E and F. Begin by finding $A\hat{C}B$ and continue from there.

WORKED SOLUTIONS

Thinking Time (Page 260)

A represents isosceles triangles.
B represents scalene triangles.
C represents acute-angled triangles.
D represents right-angled triangles.

Investigation (Basic Properties of a Triangle)

1. The side opposite $\angle B$ is b and the side opposite $\angle C$ is c.
2. The largest angle is $\angle C$ and the smallest angle is $\angle B$.
   The side opposite the largest angle, $\angle C$ is the longest side and the side opposite the smallest angle, $\angle B$ is the shortest side.
3. The bigger the angle, the longer the side opposite it. **The angle opposite the side shortest in length will be the smallest angle. This applies to the longest side as well i.e. the longest side is always opposite the largest angle**.
4. The sum of the lengths of the two shorter sides of a triangle is always longer than the length of the longest side.
5. Yes, since the sum of the angles facing the two shorter sides are greater than the largest angle facing the longest side, hence, the sum of the lengths of the two shorter sides of a triangle is always longer than the length of the longest side.
6. No, it is not possible to form a triangle.
7. $a + b = c$. It is still not possible to form a triangle.
8. The sum of the lengths of any two line segments has to be greater than the length of the third line segment

From the investigation, two basic properties of a triangle are:
- The largest angle of a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.
- The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Investigation (Properties of Special Quadrilaterals)

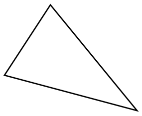
1. $AB = 2.8$ cm, $BC = 1.8$ cm, $DC = 2.8$ cm, $AD = 1.8$ cm
   $AB = DC$ and $BC = AD$ (Opposite sides are equal in length.)
2. $B\hat{A}D = 90°$, $A\hat{B}C = 90°$, $B\hat{C}D = 90°$, $A\hat{D}C = 90°$
   $B\hat{A}D = A\hat{B}C = B\hat{C}D = A\hat{D}C = 90°$ (All four interior angles are right angles.)
3. $AE = 1.7$ cm, $BE = 1.7$ cm, $CE = 1.7$ cm, $DE = 1.7$ cm
   $AE = BE = CE = DE = 1.7$ cm (Diagonals bisect each other.)
4. $AE + CE = 1.7 + 1.7 = 3.4$ cm,
   $BE + DE = 1.7 + 1.7 = 3.4$ cm
   Both of the sums are equal. (The two diagonals are equal in length.)
5. The following properties hold:
   - Opposite sides are equal in length.
   - All four interior angles are right angles.
   - Diagonals bisect each other.
   - The two diagonals are equal in length.
6. **(a)** Square : All sides are equal in length.
   Parallelogram : Opposite sides are equal in length.
   Rhombus : All sides are equal in length.
   Trapezium : All sides are not equal in length.
   Kite : There are two pairs of equal adjacent sides.
   (b) Square : All four interior angles are right angles.
   Parallelogram : Opposite interior angles are equal.
   Rhombus : Opposite interior angles are equal.
   Trapezium : All four interior angles are not equal.
   Kite : One pair of opposite interior angles is equal.
   (c) Square : The two diagonals are equal in length.
   Parallelogram : The two diagonals are not equal in length.
   Rhombus : The two diagonals are not equal in length.
   Trapezium : The two diagonals are not equal in length.
   Kite : The two diagonals are not equal in length.
   (d) Square : The diagonals bisect each other.
   Parallelogram : The diagonals bisect each other.
   Rhombus : The diagonals bisect each other.
   Trapezium : The diagonals do not bisect each other.
   Kite : The diagonals do not bisect each other.
   (e) Square : The diagonals are perpendicular to each other.
   Parallelogram : The diagonals are not perpendicular to each other.
   Rhombus : The diagonals are perpendicular to each other.
   Trapezium : The diagonals are not perpendicular to each other.
   Kite : The diagonals are perpendicular to each other.
   (f) Square : The diagonals bisect the interior angles.
   Parallelogram : The diagonals do not bisect the interior angles.
   Rhombus : The diagonals bisect the interior angles.
   Trapezium : The diagonals do not bisect the interior angles.
   Kite : One diagonal bisects the interior angles.

Thinking Time (Page 271)

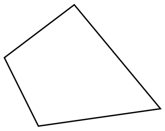
(a) Yes **(b)** Yes **(c)** Yes
(d) Yes **(e)** Yes

A represents kites.
B represents parallelograms.
C represents rhombus.
D represents squares.

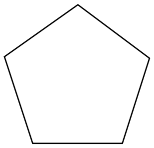
Class Discussion (Naming of Polygons)



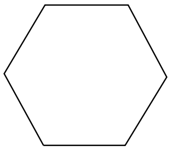
Triangle (3-sided)



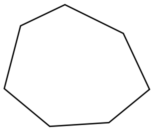
Quadrilateral (4-sided)



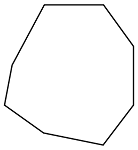
Pentagon (5-sided)



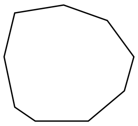
Hexagon (6-sided)



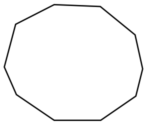
Heptagon (7-sided)



Octagon (8-sided)



Nonagon (9-sided)



Decagon (10-sided)

Thinking Time (Page 277)

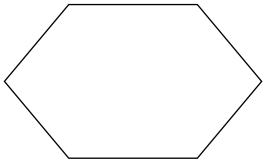
The name of a regular triangle is an equilateral triangle and the name of a regular quadrilateral is a square.

Investigation (Properties of a Regular Polygon)

1. Yes.

(a) Rhombus

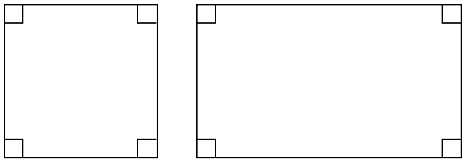
(b)



2. Yes.

(a) Square and Rectangle

(b)



Journal Writing (Page 278)

Since a regular polygon is a polygon with all sides equal and all angles equal, the statement made by Devi is correct as she stated one of the two *properties* of a regular polygon.

On the other hand, the statement made by Michael is wrong as he stated an incomplete *definition* of a regular polygon, i.e. the *conditions* of a regular polygon. A polygon with all sides equal may not be regular, e.g. a square is a regular polygon (see Fig. (a)) but a rhombus is not a regular polygon (see Fig. (b)). This is because even though a rhombus is a polygon with all sides equal, not all its angles are equal. The hexagon shown in Fig. (d) is a regular polygon but the hexagon shown in Fig. (e) is not a regular polygon because even though all its sides are equal, not all its angles are equal. Hence, it does not mean that a polygon with all sides equal is regular.

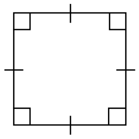


Fig. (a)

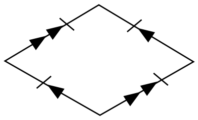


Fig. (b)

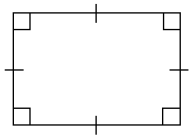


Fig. (c)

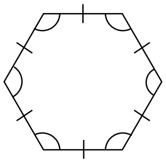


Fig. (d)

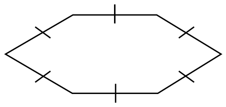


Fig. (e)

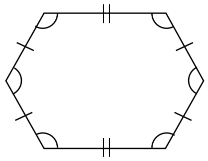


Fig. (f)

In addition, a polygon with all angles equal may not be regular. For example, a rectangle is a polygon (see Fig. (c)) but it is not regular because not all its sides are equal although all its angles are equal. Another example is the hexagon as shown in Fig. (f). It is not a regular polygon because even though all its angles are equal, not all its sides are equal. Hence, it does not mean that a polygon with all angles equal is regular.

In conclusion, a regular polygon is a polygon with all sides equal **and** all angles equal.

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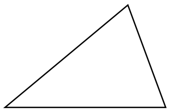
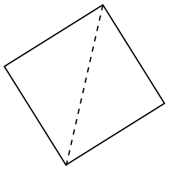
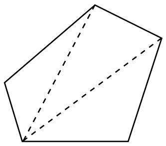
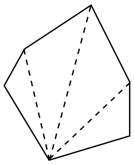
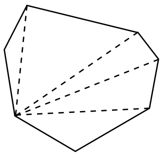
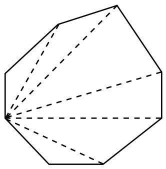
Investigation (Sum of Interior Angles of a Polygon)

1.

Polygon	Number of sides	Number of Triangle(s) formed	Sum of Interior Angles
 Triangle	3	1	$1 \times 180° = (3 - 2) \times 180°$
 Quadrilateral	4	2	$2 \times 180° = (4 - 2) \times 180°$
 Pentagon	5	3	$3 \times 180° = (5 - 2) \times 180°$
 Hexagon	6	4	$4 \times 180° = (6 - 2) \times 180°$
 Heptagon	7	5	$5 \times 180° = (7 - 2) \times 180°$
 Octagon	8	6	$6 \times 180° = (7 - 2) \times 180°$
n-gon	n	$(n - 2)$	$(n - 2) \times 180°$

2. If a polygon has n sides, then it will form $(n - 2)$ triangles.

Investigation (Tesellation)

1. The only regular polygons that tessellate on their own are equilateral triangles, squares and regular hexagons. Combinations of other regular polygons such as a square and a regular octagon can produce tessellations.

2. See Fig. 11.17 in the textbook for an example.

3. The sum of the corner angles will add up to 360°.

Investigation (Sum of Exterior Angles of a Pentagon)

1. –

2. The sum of exterior angles of a pentagon is 360° as all the exterior angles will meet at a vertex.
From the investigation, we observe that the sum of exterior angles of a pentagon is 360°.
A proof of the above result is given as follows:
Consider the pentagon in Fig. 11.24.
We have $\angle a + \angle p = 180°$, $\angle b + \angle q = 180°$,
$\angle c + \angle r = \underline{180°}$, $\angle d + \angle s = \underline{180°}$ and $\angle e + \angle t = \underline{180°}$.
$\therefore \angle a + \angle p + \angle b + \angle q + \angle c + \angle r + \angle d + \angle s + \angle e + \angle t$
$= \underline{5} \times 180°$
$(\angle a + \angle b + \angle c + \angle d + \angle e) + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$
Since the sum of interior angles of a pentagon
$= \angle a + \angle b + \angle c + \angle d + \angle e$
$= (5 - 2) \times 180° = 540°$,
$540° + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$.
$\therefore \angle p + \angle q + \angle r + \angle s + \angle t = 900° - \underline{540°} = \underline{360°}$
By using this method, we can show that the sum of exterior angles of a hexagon, of a heptagon and of an octagon is also 360°.

Thinking Time (Page 285)

1. (i) No. Since 70° is not an exact divisor of 360°, hence a regular polygon to have an exterior angle of 70° is not possible.

(ii) Since
$360° = 3 \times 120°$,
$360° = 4 \times 90°$,
$360° = 6 \times 60°$,
$360° = 8 \times 45°$,
$360° = 9 \times 40°$,
$360° = 10 \times 36°$,
$360° = 12 \times 30°$,
$360° = 15 \times 24°$,
$360° = 18 \times 20°$,
$360° = 20 \times 18°$,
$360° = 25 \times 15°$,
$360° = 30 \times 12°$,
$360° = 40 \times 9°$,
$360° = 45 \times 8°$,
$360° = 60 \times 6°$,
$360° = 90 \times 4°$,
$360° = 120 \times 3°$,
$360° = 180 \times 2°$,
All the possible values of the angle are 2°, 3°, 4°, 6°, 8°, 9°, 12°, 15°, 18°, 20°, 24°, 30°, 36°, 40°, 45°, 60°, 90° and 120°.

2. No, it is not possible as a concave polygon has one or more interior angles that are greater than 180° while as a regular polygons has all interior angles that are less than 180°.

Practise Now 1

1. $90° + 65° + a° = 180°$ ($\angle$ sum of $\triangle$)

$a° = 180° - 90° - 65°$

$= 25°$

$\therefore a = 25$

2. Since $AC = BC$, $\therefore C\hat{A}B = C\hat{B}A = b°$

$b° + 52° + b° = 180°$ ($\angle$ sum of $\triangle$)

$2b° = 180° - 52°$

$= 128°$

$b° = \dfrac{128°}{2}$

$= 64°$

$\therefore b = 64$

Practise Now 2

(a) $a° = 53° + 48°$ (ext. $\angle$ of $\triangle$)

$= 101°$

$\therefore a = 101$

(b) $F\hat{D}E = 93°$ (vert. opp. $\angle$s)

$b° + 33° + 93° = 180°$ ($\angle$ sum of $\triangle$)

$b° = 180° - 33° - 93°$

$= 54°$

$\therefore b = 54$

$c° = 41° + 93°$ (ext. $\angle$ of $\triangle ABD$)

$= 134°$

$\therefore c = 134$

Practise Now 3

1. **(i)** $D\hat{A}E = 90°$ (right angle)

$51° + 90° + A\hat{E}D = 180°$ ($\angle$ sum of $\triangle AED$)

$A\hat{E}D = 180° - 51° - 90°$

$= 39°$

(ii) $C\hat{D}E + 51° = 90°$ ($\angle ADC$ is a right angle)

$C\hat{D}E = 90° - 51°$

$= 39°$

$68° + 39° + C\hat{E}D = 180°$ ($\angle$ sum of $\triangle CDE$)

$C\hat{E}D = 180° - 68° - 39°$

$= 73°$

2. **(i)** Since $EB = EC$ (diagonals bisect each other), $\therefore E\hat{B}C = 63°$

$63° + B\hat{E}C + 63° = 180°$ ($\angle$ sum of $\triangle BEC$)

$B\hat{E}C = 180° - 63° - 63°$

$= 54°$

(ii) $D\hat{E}C + 54° = 180°$ (adj. $\angle$s on a str. line)

$D\hat{E}C = 180° - 54°$

$= 126°$

Since $ED = EC$ (diagonals bisect each other),

$\therefore C\hat{D}E = D\hat{C}E = x°$.

$x° + 126° + x° = 180°$ ($\angle$ sum of $\triangle CDE$)

$2x° = 180° - 126°$

$= 54°$

$x° = \dfrac{54°}{2}$

$= 27°$

$\therefore C\hat{D}E = 27°$

Practise Now 4

1. **(i)** $A\hat{B}C = 108°$ (opp. $\angle$s of // gram)

$9x° = 108°$

$x = \dfrac{108°}{9}$

$= 12°$

$\therefore x = 12$

(ii) $(D\hat{C}E + 38°) + 108° = 180°$ (int. $\angle$s, AD // BC)

$D\hat{C}E = 180° - 38° - 108°$

$= 34°$

2. $(5x + 6)° + (2x + 13)° = 180°$ (int. $\angle$s, AB // DC)

$7x° + 19° = 180°$

$7x° = 180° - 19°$

$= 161°$

$x° = \dfrac{161°}{7}$

$= 23°$

$\therefore x = 23$

$[5(23) + 6]° + (y + 17°) = 180°$ (int. $\angle$s, AB // DC)

$y° = 180° - 121° - 17°$

$= 42°$

$\therefore y = 42$

Practise Now 5

1. **(i)** $C\hat{A}B = 32°$ (alt. $\angle$s, AB // DC)

Since $BA = BC$, $\therefore A\hat{C}B = C\hat{A}B = 32°$

$32° + A\hat{B}C + 32° = 180°$ ($\angle$sum of $\triangle ABC$)

$A\hat{B}C = 180° - 32° - 32°$

$= 116°$

(ii) Since $AC = CE$, $\therefore C\hat{E}A = C\hat{A}E = 32°$

$32° + (32° + B\hat{C}E) + 32° = 180°$ ($\angle$ sum of $\triangle ABC$)

$B\hat{C}E = 180° - 32° - 32° - 32°$

$= 84°$

2. $B\hat{D}C = (3x + 13)°$ (diagonals bisect interior angles of a rhombus)

$D\hat{A}C = (x + 45)°$ (diagonals bisect interior angles of a rhombus)

$2(3x + 13)° + 2(x + 45)° = 180°$ (int. $\angle$ s, AB // DC)

$6x° + 26° + 2x° + 90° = 180°$

$8x° = 180° - 26° - 90°$

$8x° = 64°$

$x° = \dfrac{64°}{8}$

$= 8°$

$\therefore x = 8$

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Practise Now 6

1. Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$$a° + 121° + a° + a° + 107° = 540°$$
$$3a° = 540° - 121° - 107°$$
$$3a° = 312°$$
$$a° = \frac{312°}{3}$$
$$= 104°$$
$$\therefore a = 104$$

2. Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$$3b° + 4b° + 104° + 114° + 128° + 122° = 720°$$
$$7b° = 720° - 104° - 114° - 128° - 122°$$
$$7b° = 252°$$
$$b° = \frac{252°}{7}$$
$$= 36°$$
$$\therefore b = 36$$

Practise Now 7

(i) Sum of interior angles of a regular polygon with 24 sides

$= (n - 2) \times 180°$

$= (24 - 2) \times 180°$

$= 3960°$

(ii) Size of each interior angle of a regular polygon with 24 sides

$= \frac{3960°}{24}$

$= 165°$

Practise Now 8

1. (a) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{40°}$

$= 9$

(b) Size of each exterior angle of a regular polygon

$= 180° - 178°$

$= 2°$

The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{2°}$

$= 180$

2. The sum of exterior angles of the regular decagon is 360°.

$\therefore$ Size of each exterior angle of the regular decagon

$= \frac{360°}{10}$

$= 36°$

$\therefore$ Size of each interior angle of the regular decagon

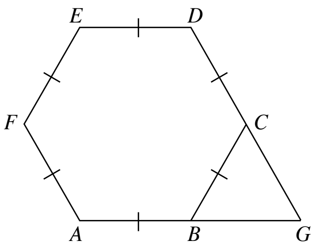
$= 180° - 36°$

$= 144°$

3. The sum of exterior angles of an n-sided polygon is 360°.

$$25° + 26° + 3(180° - 161°) + (n - 5)(180° - 159°) = 360°$$
$$25° + 26° + 3(19°) + (n - 5)(21°) = 360°$$
$$25° + 26° + 57° + n(21°) - 105° = 360°$$
$$n(21°) = 360° - 25° - 26° - 57° + 105°$$
$$= 357°$$
$$n = \frac{357°}{21°}$$
$$= 17$$

Practise Now 9



Size of each exterior angle of the hexagon

$= \frac{360°}{6}$

$= 60°$

$C\hat{B}G = B\hat{C}G = 60°$

$$B\hat{G}C + 60° + 60° = 180° \text{ (}\angle\text{ sum of } \triangle BCG)$$
$$B\hat{G}C = 180° - 60° - 60°$$
$$= 60°$$

Practise Now 10

(i) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

Since $P\hat{B}C$ is an interior angle of a pentagon,

$\therefore P\hat{B}C = \frac{540°}{5} = 108°.$

(ii) Since $C\hat{R}Q$ is an interior angle of a pentagon,

$\therefore C\hat{R}Q = 108°.$

Let $Q\hat{C}R = C\hat{Q}R = x°$ (base $\angle$s of isos. $\triangle CQR$)

$$x° + x° + 108° = 180° \text{ (}\angle\text{ sum of } \triangle CQR)$$
$$2x° = 180° - 108°$$
$$2x° = 72°$$
$$x° = \frac{72°}{2}$$
$$= 36°$$
$$\therefore Q\hat{C}R = 36°$$

(iii) $B\hat{C}D + 108° + 90° = 360°$ ($\angle$s at a point)

$B\hat{C}D = 360° - 108° - 90°$

$= 162°$

(iv) Let $B\hat{D}C = B\hat{C}D = y°$ (base $\angle$s of isos. $\triangle BCD$)

$y° + y° + 162° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2y° = 180° - 162°$

$2y° = 18°$

$y° = \dfrac{18°}{2}$

$= 9°$

$\therefore B\hat{D}C = 9°$

(v) Let the exterior angle of the n-sided polygon be $a°$.

$a° + 162° = 180°$ (adj $\angle$s on a str. line)

$a° = 180° - 162°$

$= 18°$

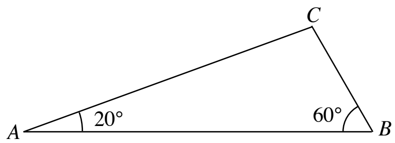
Since the sum of the exterior angles of the n-sided polygon is 360°,

$\therefore n = \dfrac{360°}{18°}$

$= 20$

Exercise 11A

1. **(a)**

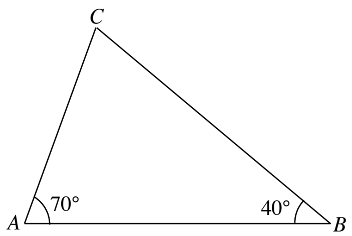


$\angle C = 180° - 20° - 60°$ ($\angle$ sum of $\triangle$)

$= 100°$

It is a scalene triangle and an obtuse-angled triangle.

(b)

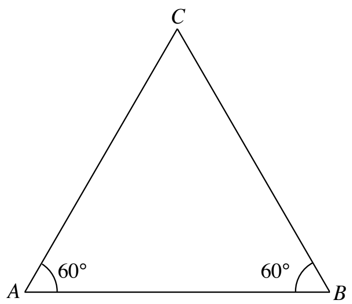


$\angle C = 180° - 70° - 40°$ ($\angle$ sum of $\triangle$)

$= 70°$

It is an isosceles triangle and acute-angled triangle.

(c)

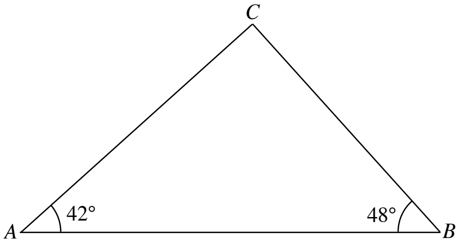


$\angle C = 180° - 60° - 60°$ ($\angle$ sum of $\triangle$)

$= 60°$

It is an equilateral triangle and acute-angled triangle.

(d)



$\angle C = 180° - 42° - 48°$ ($\angle$ sum of $\triangle$)

$= 90°$

It is a scalene triangle and right-angled triangle.

2. **(a)** Third angle of the triangle

$= 180° - 40° - 40°$ ($\angle$ sum of $\triangle$)

$= 100°$

(b) Third angle of the triangle

$= 180° - 87° - 87°$ ($\angle$ sum of $\triangle$)

$= 6°$

(c) Third angle of the triangle

$= 180° - 15° - 15°$ ($\angle$ sum of $\triangle$)

$= 150°$

(d) Third angle of the triangle

$= 180° - 79° - 79°$ ($\angle$ sum of $\triangle$)

$= 22°$

3. **(a)** $39° + 90° + a° = 180°$ ($\angle$ sum of $\triangle$)

$a° = 180° - 39° - 90°$

$= 51°$

$\therefore a = 51$

(b) $68° + 2b° + 64° = 180°$ ($\angle$ sum of $\triangle$)

$2b° = 180° - 68° - 64°$

$= 48°$

$b° = \dfrac{48°}{2}$

$= 24°$

$\therefore b = 24$

(c) $4c° + 3c° + 40° = 180°$ ($\angle$ sum of $\triangle$)

$4c° + 3c° = 180° - 40°$

$7c° = 140°$

$c = \dfrac{140°}{7}$

$= 20°$

$\therefore c = 20$

(d) $3d° + 4d° + d° = 180°$ ($\angle$ sum of $\triangle$)

$8d° = 180°$

$d° = \dfrac{180°}{8}$

$= 22.5°$

$\therefore d = 22.5$

(e) Since $BA = BC$, $\therefore B\hat{C}A = B\hat{A}C = 62°$

$62° + e° + 62° = 180°$ ($\angle$ sum of $\triangle$)

$e° = 180° - 62° - 62°$

$= 56°$

$\therefore e = 56$

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(f) Since $AC = BC = AB$, $\therefore$ $C\hat{A}B = C\hat{B}A = A\hat{C}B = f°$

$f° + f° + f° = 180°$ ($\angle$ sum of $\triangle$)

$3f° = 180°$

$f° = \dfrac{180°}{3}$

$= 60°$

$\therefore f = 60$

4. **(a)** $a° = 47° + 55°$ (ext. $\angle$ of $\triangle$)

$= 102°$

$\therefore a = 102$

(b) $90° + b° + 50° = 180°$ ($\angle$ sum of $\triangle$)

$b° = 180° - 90° - 50°$

$= 40°$

$\therefore b = 40$

$90° + c° + 35° = 180°$ ($\angle$ sum of $\triangle$)

$c° = 180° - 90° - 35°$

$= 55°$

$\therefore c = 55$

(c) $d° + 110° = 180°$ (adj. $\angle$s on a str. line)

$d° = 180° - 110°$

$= 70°$

$\therefore d = 70$

$2e° + 3e° = 110°$ (ext. $\angle$ of $\triangle$)

$5e° = 110°$

$e° = \dfrac{110°}{5}$

$= 22°$

$\therefore e = 22$

5. $3x° + 4x° + 5x° = 180°$ ($\angle$ sum of $\triangle$)

$12x° = 180°$

$x° = \dfrac{180°}{12}$

$= 15°$

$\therefore x = 15$

Smallest angle of the triangle

$= 3(15°)$

$= 45°$

6. **(i)** Let $A\hat{D}B = B\hat{D}C = x°$

$90° + 20° + 2x° = 180°$ ($\angle$ sum of $\triangle$)

$2x° = 180° - 90° - 20°$

$= 70°$

$x° = \dfrac{70°}{2}$

$= 35°$

$\therefore B\hat{D}C = 35°$

(ii) $C\hat{B}D + 20° + 35° = 180°$ ($\angle$ sum of $\triangle$)

$C\hat{B}D = 180° - 20° - 35°$

$= 125°$

7. **(a)** $a° + 90° = 115°$ (ext. $\angle$ of $\triangle BCE$)

$a° = 115° - 90°$

$= 25°$

$\therefore a = 25$

$b° = 90° + 32°$ (ext. $\angle$ of $\triangle EFG$)

$= 122°$

$\therefore b = 122$

(b) $A\hat{B}E = A\hat{B}D = 89° + 27°$ (ext. $\angle$ of $\triangle BCD$)

$= 116°$

$c° = 116° + 22°$ (ext. $\angle$ of $\triangle ABE$)

$= 138°$

$\therefore c = 138$

8. **(a)** $82° + 40° + a° = 180°$ ($\angle$ sum of $\triangle$)

$a° = 180° - 82° - 40°$

$= 58°$

$\therefore a = 58$

$A\hat{D}B = 82°$ (vert. opp. $\angle$s)

$b° = 45° + 82°$ (ext. $\angle$ of $\triangle ABD$)

$= 127°$

$\therefore b = 127$

(b) $E\hat{D}F + 44° + 57° = 180°$ ($\angle$ sum of $\triangle$)

$E\hat{D}F = 180° - 44° - 57°$

$= 79°$

$A\hat{D}B = 79°$ (vert. opp. $\angle$s)

$c° = 51° + 79°$ (ext. $\angle$ of $\triangle ABD$)

$= 130°$

$\therefore c = 130$

9. **(a)** $B\hat{A}C + A\hat{C}D = 180°$ (int. $\angle$s, AB // CD)

$108° + (a° + 37°) = 180°$

$a° = 180° - 108° - 37°$

$= 35°$

$\therefore a = 35$

$b° = 71° + 37°$ (ext. $\angle$ of $\triangle ABD$)

$= 108°$

$\therefore b = 108$

(b) $A\hat{H}F = 45°$ (vert. opp. $\angle$s)

$A\hat{H}I + C\hat{I}H = 180°$ (int. $\angle$s, AB // CD)

$(45° + 64°) + (32° + c°) = 180°$

$c° = 180° - 45° - 64° - 32°$

$= 39°$

$\therefore c = 39$

$d° + 39° + 64° = 180°$ ($\angle$ sum of $\triangle$)

$d° = 180° - 39° - 64°$

$= 77°$

$\therefore d = 77$

(c) Since $EB = EC$, $\therefore\ E\hat{C}B = E\hat{B}C = 2e°$

$$f° = 2e° + 2e° \text{ (ext. } \angle \text{ of } \triangle BCE)$$
$$= 4e°$$
$$e° + f° = 120° \text{ (ext. } \angle \text{ of } \triangle BEF)$$
$$e° + 4e° = 120°$$
$$5e° = 120°$$
$$e° = \frac{120°}{5}$$
$$= 24°$$
$$\therefore\ e = 24$$
$$f° = 4(24°)$$
$$= 96°$$
$$\therefore\ f = 96$$
$$A\hat{B}E = D\hat{E}B \text{ (alt } \angle\text{s, } AB \parallel CD)$$
$$g° + 2(24°) = 96°$$
$$g° = 96° - 48°$$
$$= 48°$$
$$\therefore\ g = 48$$

(d) $A\hat{F}E = C\hat{G}F = 68°$ (corr. $\angle$s, $AB \parallel CD$)

$$68° + h° = 180° \text{ (adj. } \angle\text{s on a str. line)}$$
$$h° = 180° - 68°$$
$$= 112°$$
$$\therefore\ h = 112$$
$$F\hat{J}I = K\hat{J}B = 65° \text{ (vert. opp. } \angle\text{s)}$$
$$i° = 65° \text{ (corr. } \angle\text{s, } AB \parallel CD)$$
$$\therefore\ i = 65$$
$$I\hat{G}H = C\hat{G}F = 68° \text{ (vert. opp. } \angle\text{s)}$$
$$68° + j° + 65° = 180° \text{ (} \angle \text{ sum of } \triangle GHI)$$
$$j° = 180° - 68° - 65°$$
$$= 47°$$
$$\therefore\ j = 47$$

10. $(x - 35)° + (x - 25)° + \left(\frac{1}{2}x - 10\right)° = 180°$ ($\angle$ sum of $\triangle$)

$$\frac{5}{2}x° - 70° = 180°$$
$$\frac{5}{2}x° = 180° + 70°$$
$$\frac{5}{2}x° = 250°$$
$$x° = \frac{250°}{\left(\frac{5}{2}\right)}$$
$$= 100°$$
$$\therefore\ x = 100$$

11. **(i)** $A\hat{B}C + 50° + 26° = 180°$ ($\angle$ sum of $\triangle$)

$$A\hat{B}C = 180° - 50° - 26°$$
$$= 104°$$

(ii) $C\hat{B}D = 50° + 26°$ (ext. $\angle$ of $\triangle$)

$$= 76°$$

12. **(i)** $D\hat{C}E + 61° + 41° = 180°$ ($\angle$ sum of $\triangle$)

$$D\hat{C}E = 180° - 61° - 41°$$
$$= 78°$$
$$A\hat{C}B = 78° \text{ (vert. opp. } \angle\text{s)}$$

(ii) $A\hat{B}C + 78° + 50° = 180°$ ($\angle$ sum of $\triangle$)

$$A\hat{B}C = 180° - 78° - 50°$$
$$= 52°$$

13. **(i)** $D\hat{E}C = B\hat{C}E = 47°$ (alt. $\angle$s, $AC \parallel ED$)

$$32° + 47° + D\hat{F}E = 180° \text{ (} \angle \text{ sum of } \triangle DEF)$$
$$D\hat{F}E = 180° - 32° - 47°$$
$$= 101°$$

(ii) $C\hat{B}D = B\hat{D}E = 32°$ (alt $\angle$s, $AC \parallel ED$)

$$106° + E\hat{B}D + 32° = 180° \text{ (adj. } \angle\text{s on a str. line)}$$
$$E\hat{B}D = 180° - 106° - 32°$$
$$= 42°$$
$$B\hat{D}C = E\hat{B}D = 42° \text{ (alt. } \angle\text{s, } BE \parallel CD)$$

14. Let $C\hat{B}O$ be $x°$.

Then $C\hat{A}O = \frac{1}{2}x°$ and $B\hat{A}O = 1\frac{1}{2}x°$.

Since $OA = OC$, $\therefore\ A\hat{C}O = C\hat{A}O = \frac{1}{2}x°$.

Since $OB = OC$, $\therefore\ C\hat{B}O = B\hat{C}O = x°$.

Since $OA = OB$, $\therefore\ B\hat{A}O = A\hat{B}O = 1\frac{1}{2}x°$.

Hence,

$C\hat{A}B + A\hat{B}C + B\hat{C}A = 180°$ ($\angle$ sum of $\triangle ABC$)

$$\left(\frac{1}{2}x° + 1\frac{1}{2}x°\right) + \left(1\frac{1}{2}x° + x°\right) + \left(\frac{1}{2}x° + x°\right) = 180°$$
$$6x° = 180$$
$$x° = \frac{180°}{6}$$
$$= 30°$$

$$\therefore\ C\hat{A}O = \frac{1}{2}(30°) = 15°.$$

15. Since $AB = AC$, then let $A\hat{B}C = A\hat{C}B = x°$.

$D\hat{B}E = 180° - x°$ (adj. $\angle$s on a str. line)

Since $BD = BE$, then

$$B\hat{D}E = B\hat{E}D = \frac{180° - (180° - x°)}{2} = \frac{x°}{2}.$$

Since $AF = DF$, $\therefore\ F\hat{A}D = F\hat{D}A$

$$F\hat{A}D = F\hat{D}A = B\hat{D}E = \frac{x°}{2}.$$

$$\frac{x°}{2} + x + x° = 180° \text{ (} \angle \text{ sum of } \triangle ABC)$$
$$2\frac{1}{2}x° = 180°$$
$$x = \frac{180°}{2\frac{1}{2}}$$
$$= 72°$$
$$\therefore\ A\hat{B}C = 72°$$

Exercise 11B

1. (a) $a° + 54° = 90°$ ($B\hat{C}D$ is a right angle)

$a° = 90° - 54°$

$= 36°$

$\therefore a = 36$

$b° = 36°$ (alt. $\angle$s, AB // DC)

$\therefore b = 36$

(b) $E\hat{B}C = 90°$ (right angle)

$90° + 39° + c° = 180°$ ($\angle$ sum of $\triangle BCE$)

$c° = 180° - 90° - 39°$

$= 51°$

$\therefore c = 51$

$D\hat{C}E + 39° = 90°$ ($B\hat{C}D$ is a right angle)

$D\hat{C}E = 90° - 39°$

$= 51°$

$51° + d° + 78° = 180°$ ($\angle$ sum of $\triangle CDE$)

$d° = 180° - 51° - 78°$

$= 51°$

$\therefore d = 51$

2. (a) $a° = 106°$ (opp. $\angle$s of // gram)

$\therefore a = 106$

$b° = 48°$ (alt. $\angle$s, AD // BC)

$\therefore b = 48$

(b) $4c° + 5c° = 180°$ (int. $\angle$s, AB // DC)

$9c° = 180°$

$c° = \dfrac{180°}{9}$

$= 20°$

$\therefore c = 20$

$2d° = 4(20°)$ (opp. $\angle$ s of // gram)

$d° = \dfrac{80°}{2}$

$= 40°$

$\therefore d = 40$

3. (a) Since $ABCD$ is a kite, $\therefore AD = CD$ and so $A\hat{C}D = C\hat{A}D = a°$

$a° + 100° + a° = 180°$ ($\angle$ sum of $\triangle ACD$)

$2a° = 180° - 100°$

$= 80°$

$a° = \dfrac{80°}{2}$

$= 40°$

$\therefore a = 40$

Since $ABCD$ is a kite, $\therefore AB = CB$ and so $C\hat{A}B = A\hat{C}B = 61°$.

$61° + b° + 61° = 180°$ ($\angle$sum of $\triangle ABC$)

$b° = 180° - 61° - 61°$

$= 58°$

$\therefore b = 58$

(b) Since $ABCD$ is a kite, $\therefore D\hat{A}C = B\hat{A}C = 40°$.

(One diagonal bisects the interior angles)

$40° + 26° + c° = 180°$ ($\angle$ sum of $\triangle ACD$)

$c° = 180° - 40° - 26°$

$= 114°$

$\therefore c = 114$

4. (a) Since $ABCD$ is a square, $\therefore D\hat{A}C = B\hat{A}C = 45°$ and hence $D\hat{A}E = 45°$.

(Diagonals bisect the interior angles)

$A\hat{E}D + 82° = 180°$ (adj. $\angle$s on a str. line)

$A\hat{E}D = 180° - 82°$

$= 98°$

$45° + 98° + a° = 180°$ ($\angle$ sum of $\triangle ADE$)

$a° = 180° - 45° - 98°$

$= 37°$

$\therefore a = 37$

Since $ABCD$ is a square, $\therefore B\hat{A}C = D\hat{A}C = 45°$ and hence $E\hat{A}F = 45°$.

(Diagonals bisect the interior angles)

$A\hat{E}F = 82°$ (vert. opp. $\angle$)

$b° = 45° + 82°$ (ext. $\angle$ of $\triangle AEF$)

$= 127°$

$\therefore b = 127$

(b) Since $ABCD$ is a square, $\therefore B\hat{C}A = D\hat{C}A = 45°$ and hence $E\hat{C}F = 45°$.

(Diagonals bisect the interior angles)

$c° + 45° + c° = 180°$ ($\angle$ sum of $\triangle CEF$)

$2c° = 180° - 45°$

$= 135°$

$c° = \dfrac{135°}{2}$

$= 67.5°$

$\therefore c = 67.5$

Since $ABCD$ is a square, $\therefore C\hat{E}D = 90°$.

(Diagonals bisect each other at right angles)

Hence,

$d° + 67.5° = 90°$

$d° = 90° - 67.5°$

$= 22.5°$

$\therefore d = 22.5$

5. (a) Since $ABCD$ is a rhombus, $\therefore A\hat{C}B = A\hat{D}C = 114°$ (Opposite angles are equal) and hence $a = 114$.

Since $ABCD$ is a rhombus, $\therefore AB = CB$ and hence $A\hat{C}B = C\hat{A}B = b°$.

$b° + 114° + b° = 180°$ ($\angle$ sum of $\triangle ABC$)

$2b° = 180° - 114°$

$= 66°$

$b° = \dfrac{66°}{2}$

$= 33°$

$\therefore b = 33$

(b) $C\hat{B}D = B\hat{D}A = 38°$ (alt. $\angle$s, AD // BC)

$c° = 38°$

$\therefore c = 38$

Since $ABCD$ is a rhombus, $\therefore AB = AD$ and hence $B\hat{D}A = D\hat{B}A = 38°$.

$38° + d° + 38° = 180°$ ($\angle$ sum of $\triangle ABD$)

$d° = 180° - 38° - 38°$

$= 104°$

$\therefore d = 104$

(c) $D\hat{C}A = C\hat{A}B = 42°$ (alt, $\angle$s, $AB \parallel DC$)

$e° = 42°$

$\therefore e = 42$

Since $ABCD$ is a rhombus, $\therefore A\hat{D}B = C\hat{D}B = f°$.

(Diagonals bisect the interior angles)

Also, $AD = CD$ and hence $C\hat{A}D = A\hat{C}D = 42°$

$42° + 2f° + 42° = 180°$ ($\angle$ sum of $\triangle ACD$)

$2f° = 180° - 42° - 42°$

$= 96°$

$f° = \dfrac{96°}{2}$

$= 48°$

$\therefore f = 48$

6. **(i)** $A\hat{E}D = 52°$ (vert. opp. $\angle$s)

Since $AE = DE$, $\therefore A\hat{D}E = D\hat{A}E = x°$.

$x° + 52° + x° = 180°$ ($\angle$ sum of $\triangle ADE$)

$2x° = 180° - 52°$

$= 128°$

$x° = \dfrac{128°}{2}$

$= 64°$

$\therefore A\hat{D}B = A\hat{D}E = 64°$

(ii) $A\hat{D}C = 90°$ (right angle of a rectangle)

$64° + 90° + A\hat{C}D = 180°$ ($\angle$ sum of $\triangle ACD$)

$A\hat{C}D = 180° - 64° - 90°$

$= 26°$

7. **(i)** $A\hat{D}E + 65° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$A\hat{D}E = 180° - 65°$

$= 115°$

(ii) $B\hat{C}D = 65°$ (opp. $\angle$s of // gram)

$C\hat{B}E + 65° = 125°$ (ext. $\angle$ of $\triangle BCE$)

$C\hat{B}E = 125° - 65°$

$= 60°$

8. **(i)** $A\hat{B}D = 46°$ (alt. $\angle$s, $AB \parallel DC$)

Since $ABCD$ is a rhombus, $\therefore AB = AD$ and hence

$B\hat{D}A = D\hat{B}A = 46°$.

$46° + B\hat{A}D + 46° = 180°$ ($\angle$ sum of $\triangle ABD$)

$B\hat{A}D = 180° - 46° - 46°$

$= 88°$

(ii) $D\hat{B}C = 46°$ (alt. $\angle$s, $AD \parallel BC$)

Since $BC = BE$, $\therefore B\hat{C}E = B\hat{E}C = x°$.

$x° + x° = 46°$ (ext. $\angle$ of $\triangle BCE$)

$2x° = 46°$

$x° = \dfrac{46°}{2}$

$= 23°$

$\therefore B\hat{C}E = 23°$

9. $A\hat{D}B = (3x + 7)°$ (diagonals bisect interior angles of a rhombus)

$D\hat{A}C = (2x + 53)°$ (diagonals bisect interior angles of a rhombus)

$2(3x + 7)° + 2(2x + 53)° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$6x° + 14° + 4x° + 106° = 180°$

$10x° = 180° - 14° - 106°$

$10x° = 60°$

$x° = \dfrac{60°}{10}$

$= 6°$

$\therefore x = 6$

10. $5x° + x° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$6x° = 180°$

$x° = \dfrac{180°}{6}$

$= 30°$

$\therefore x = 30$

$2.2(30°) + y° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$y° = 180° - 66°$

$= 114°$

$\therefore y = 114$

11. **(i)** Since $ABCD$ is a kite, $\therefore B\hat{A}C = D\hat{A}C = 25°$

(One diagonal bisects the interior angles) and

since $AB = AD$, $\therefore B\hat{D}A = D\hat{B}A = x°$

$x° + 2(25°) + x° = 180°$ ($\angle$ sum of $\triangle ABD$)

$2x° = 180° - 50°$

$= 130°$

$x° = \dfrac{130°}{2}$

$= 65°$

$\therefore A\hat{B}D = 65°$

(ii) Since $ABCD$ is a kite, $\therefore B\hat{C}A = D\hat{C}A = 44°$

One diagonal bisects the interior angles) and

since $CB = CD$, $\therefore B\hat{D}C = D\hat{B}C = y°$

$y° + 2(44°) + y° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2y° = 180° - 88°$

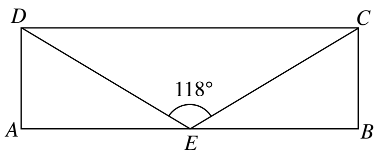
$= 92°$

$y° = \dfrac{92°}{2}$

$= 46°$

$\therefore C\hat{B}D = 46°$

12.



(i) Since E is the midpoint of AB, $\therefore CE = DE$ and hence

$C\hat{D}E = D\hat{C}E = x°$.

$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle CDE$)

$2x° = 180° - 118°$

$= 62°$

$x = \dfrac{62°}{2}$

$= 31°$

$A\hat{D}E + 31° = 90°$ ($A\hat{D}C$ is a right angle)

$= 59°$

(ii) From (i), $D\hat{C}E = x° = 31°$.

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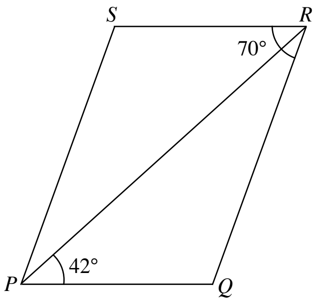
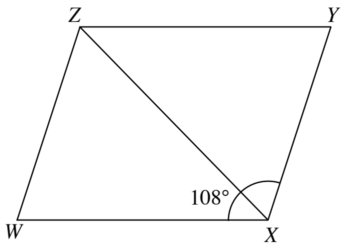
13.



(i) $P\hat{Q}R + 70° = 180°$ (int. $\angle$s, PQ // SR)
$P\hat{Q}R = 180° - 70°$
$= 110°$

(ii) $42° + 110° + P\hat{R}Q = 180°$ ($\angle$ sum of $\triangle PQR$)
$P\hat{R}Q = 180° - 42° - 110°$
$= 28°$

14.

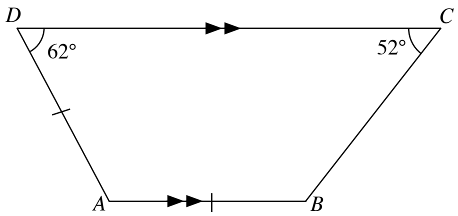


(i) Since $WXYZ$ is a rhombus, $W\hat{Z}Y = W\hat{X}Y = 108°$ (opp. $\angle$s of a // gram) and $X\hat{Z}Y = X\hat{Z}W = x°$ (Diagonal bisect the interior angles), hence $W\hat{Z}Y = 2x°$
$2x° = 108°$
$$x° = \frac{108°}{2}$$
$= 54°$
$\therefore X\hat{Z}Y = 54°$

(ii) $X\hat{Y}Z + 108° = 180°$ (int. $\angle$s, WX // ZY)
$X\hat{Y}Z = 180° - 108°$
$= 72°$

(iii) Since $WXYZ$ is a rhombus, $X\hat{W}Z = X\hat{Y}Z = 72°$ (opp. $\angle$s of a // gram) and $X\hat{W}Y = Z\hat{W}Y = y°$ (Diagonals bisect the interior angles), hence $X\hat{W}Z = 2y°$
$2y° = 72°$
$$y° = \frac{72°}{2}$$
$= 36°$
$\therefore X\hat{W}Y = 36°$

15.

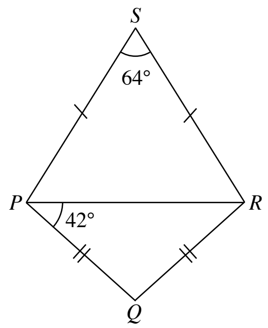


(i) $B\hat{A}D + 62° = 180°$ (int. $\angle$s, AB // DC)
$B\hat{A}D = 180° - 62°$
$= 118°$

Since $AB = AD$, $\therefore A\hat{B}D = A\hat{D}B = x°$
$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle ABD$)
$2x° = 180° - 118°$
$= 62°$
$$x° = \frac{62°}{2}$$
$= 31°$
$\therefore A\hat{B}D = 31°$

(ii) $A\hat{B}C + 52° = 180°$ (int. $\angle$s, AB // DC)
$A\hat{B}C = 180° - 52°$
$= 128°$
From (i), $A\hat{B}D = 31°$.
$36° + C\hat{B}D = 128°$
$C\hat{B}D = 128° - 31°$
$= 97°$

16.



(i) Since $PS = RS$, $\therefore R\hat{P}S = P\hat{R}S = x°$.
$x° + 64° + x° = 180°$ ($\angle$ sum of $\triangle PRS$)
$2x° = 180° - 64°$
$= 116°$
$$x° = \frac{116°}{2}$$
$= 58°$
$\therefore P\hat{R}S = 58°$

(ii) Since $PQ = QR$, $\therefore Q\hat{P}R = Q\hat{R}P = 42°$.
$42° + P\hat{Q}R + 42° = 180°$ ($\angle$ sum of $\triangle PQR$)
$P\hat{Q}R = 180° - 42° - 42°$
$= 96°$

Exercise 11C

1. **(a)** Sum of interior angles of a 11-gon
$= (n - 2) \times 180°$
$= (11 - 2) \times 180°$
$= 1620°$

(b) Sum of interior angles of a 12-gon
$= (n - 2) \times 180°$
$= (12 - 2) \times 180°$
$= 1800°$

(c) Sum of interior angles of a 15-gon
$= (n - 2) \times 180°$
$= (15 - 2) \times 180°$
$= 2340°$

(d) Sum of interior angles of a 20-gon
$= (n - 2) \times 180°$
$= (20 - 2) \times 180°$
$= 3240°$

2. **(a)** Sum of interior angles of a quadrilateral
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$
$78° + 62° + a° + 110° = 360°$
$a° = 360° - 78° - 62° - 110°$
$= 110°$
$\therefore a = 110$

(b) Sum of interior angles of a quadrilateral
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$
$b° + 78° + 2b° + 84° = 360°$
$3b° = 360° - 78° - 84°$
$= 198°$
$b° = \dfrac{198°}{3}$
$= 66°$
$\therefore b = 66$

(c) Sum of interior angles of a pentagon
$= (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$
$c° + 152° + 38° + 2c° + 101° = 540°$
$3c° = 540° - 152° - 38° - 101°$
$3c° = 249°$
$c° = \dfrac{249°}{3}$
$= 83°$
$\therefore c = 83$

(d) Sum of interior angles of a hexagon
$= (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720°$
$102° + 5d° + 4d° + 4d° + 108° + 4d° = 720°$
$17d° = 720° - 102° - 108°$
$= 510°$
$d° = \dfrac{510°}{17}$
$= 30°$
$\therefore d = 30$

3. **(a)** **(i)** Sum of interior angles of a hexagon
$= (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720°$

(ii) Hence, size of each interior angle of a hexagon
$= \dfrac{720°}{6}$
$= 120°$

(b) **(i)** Sum of interior angles of a regular polygon with 18 sides
$= (n - 2) \times 180°$
$= (18 - 2) \times 180°$
$= 2880°$

(ii) Hence, size of each interior angle of a regular polygon with 18 sides
$= \dfrac{2880°}{18}$
$= 160°$

4. **(a)** The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Size of each exterior angle of the regular polygon
$= \dfrac{360°}{24}$
$= 15°$
$\therefore$ Size of each interior angle of a regular polygon with 24 sides
$= 180° - 15°$
$= 165°$

(b) The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Size of each exterior angle of the regular polygon
$= \dfrac{360°}{36}$
$= 10°$
$\therefore$ Size of each interior angle of a regular polygon with 36 sides
$= 180° - 10°$
$= 170°$

5. **(a)** The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{90°}$
$= 4$

(b) The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{45°}$
$= 8$

(c) The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{12°}$
$= 30$

(d) The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{4°}$
$= 90$

6. **(a)** Size of each interior angle of a regular polygon
$= 180° - 140°$
$= 40°$
The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{40°}$
$= 9$

(b) Size of each interior angle of a regular polygon
$= 180° - 162°$
$= 18°$
The sum of exterior angles of the regular polygon is 360°
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{18°}$
$= 20$

(c) Size of each interior angle of a regular polygon
$= 180° - 172°$
$= 8°$
The sum of exterior angles of the regular polygon is 360°.
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{8°}$
$= 45$

(d) Size of each interior angle of a regular polygon
$= 180° - 175°$
$= 5°$
The sum of exterior angles of the regular polygon is 360°
$\therefore$ Number of sides of the polygon
$= \dfrac{360°}{5°}$
$= 72$

7. Sum of interior angles of a pentagon
$= (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$

$$2x° + 3x° + 4x° + 5x° + 6x° = 540°$$
$$20x° = 540°$$
$$x° = \frac{540°}{20}$$
$$= 27°$$

Hence, the largest interior angle of the pentagon
$= 6(27°)$
$= 162°$

8. **(i)** The sum of exterior angles of the triangle is 360°.

$$3y° + 4y° + 5y° = 360°$$
$$12y° = 360°$$
$$y° = \frac{360°}{12}$$
$$= 30°$$
$$\therefore y = 30$$

(ii) Smallest interior angle of the triangle
$= 180° - 5(30°)$
$= 180° - 150°$
$= 30°$

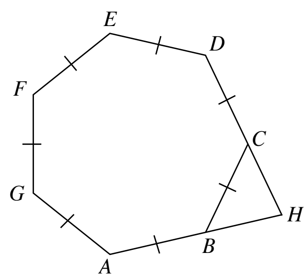
9. The sum of exterior angles of an n-sided polygon is 360°.

$$15° + 25° + 70° + (n - 3) \times 50° = 360°$$
$$15° + 25° + 70° + n(50°) - 150° = 360°$$
$$n(50°) = 360° - 15° - 25° - 70° + 150°$$
$$n(50°) = 400°$$
$$n = \frac{400°}{50°}$$
$$= 8$$

10. The sum of exterior angles of a n-sided polygon is 360°.

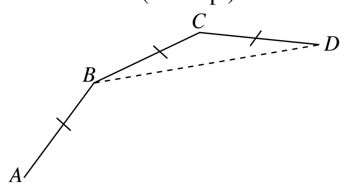
$$3(50°) + (180° - 127°) + (180° - 135°) + (n - 5)(180° - 173°)$$
$$= 360°$$
$$150° + 53° + 45° + (n - 5)(7°) = 360°$$
$$150° + 53° + 45° + n(7°) - 35° = 360°$$
$$n(7°) = 360° - 150° - 53° - 45° + 35°$$
$$= 147°$$
$$n = \frac{147°}{7°}$$
$$= 21$$

11.



Size of each exterior angle of the heptagon
$= \dfrac{360°}{7}$
$= 51.43°$
$B\hat{H}C + 51.43° + 51.43° = 180°$ ($\angle$ sum of $\triangle BCH$)
$B\hat{H}C = 180° - 51.43° - 51.43°$
$= 77.1°$ (to 1 d.p.)

12.



(i) Sum of interior angles of a regular polygon with 20 sides
$= (n - 2) \times 180°$
$= (20 - 2) \times 180°$
$= 3240°$
Hence, size of each interior angles of a regular polygon with 20 sides
$= \dfrac{3240°}{20}$
$= 162°$
$\therefore A\hat{B}C = 162°$

(ii) Since size of each interior angle of a regular polygon with 20 sides = 162°,

$\therefore B\hat{C}D = 162°$

Let $C\hat{B}D = C\hat{D}B = x°$ (base ∠s of isos. $\triangle BCD$)

$x° + x° + 162° = 180°$ (∠ sum of $\triangle BCD$)

$2x° = 180° - 162°$

$2x° = 18°$

$x° = \dfrac{18°}{2}$

$= 9°$

$\therefore x = 9$

Hence,

$A\hat{B}D = A\hat{B}C - C\hat{B}D$

$= 162° - 9°$

$= 153°$

13. (i) Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$\therefore$ Size of each interior angle of a hexagon

$= \dfrac{720°}{6}$

$= 120°$

Since $A\hat{B}P$ is an interior angle of a hexagon,

$\therefore A\hat{B}P = 120°$.

(ii) Since $P\hat{Q}R$ is an interior angle of a hexagon,

$\therefore P\hat{Q}R = 120°$.

$P\hat{Q}X = \dfrac{120°}{2}$ (QA is a line of symmetry)

$= 60°$

(iii) $A\hat{X}B = \dfrac{360°}{6}$ (∠s at a point)

$= 60°$

(iv) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$\therefore$ Size of each interior angle of a pentagon

$= \dfrac{540°}{5}$

$= 108°$

Since $A\hat{B}C$ is an interior angle of a pentagon,

$\therefore A\hat{B}C = 108°$.

(v) Since size of each interior angle of a pentagon = 108°,

$\therefore B\hat{C}D = 108°$

Let $B\hat{A}C = B\hat{C}A = x°$ (base ∠s of isos. $\triangle ABC$)

$x° + x° + 108° = 180°$ (∠ sum of $\triangle ABC$)

$2x° = 180° - 108°$

$2x° = 72°$

$x° = \dfrac{72°}{2}$

$= 36°$

$\therefore x = 36$

Hence,

$A\hat{C}D = B\hat{C}D - B\hat{C}A$

$= 108° - 36°$

$= 72°$

(vi) Since size of each interior angle of a hexagon = 120°,

$\therefore B\hat{A}S = 120°$

Since size of each interior angle of a pentagon = 108°,

$\therefore B\hat{A}E = 108°$

$120° + 108° + S\hat{A}E = 360°$ (∠s at a point)

$S\hat{A}E = 360° - 120° - 108°$

$= 132°$

Let $A\hat{S}E = A\hat{E}S = x°$ (base ∠ of isos. $\triangle AES$)

$x° + x° + 132° = 180°$ (∠ sum of $\triangle AES$)

$2x° = 180° - 132°$

$2x° = 48°$

$x° = \dfrac{48°}{2}$

$= 24°$

$\therefore A\hat{S}E = 24°$

14. (i) Let the interior angle be $5x°$ and the exterior angle be $x°$.

$5x° + x° = 180°$ (adj. ∠s on a str. line)

$6x° = 180°$

$x° = \dfrac{180°}{6}$

$= 30°$

Since sum of exterior angles of a n-sided polygon is 360°,

$\therefore n = \dfrac{360°}{30°} = 12$

(ii) $A\hat{B}C = 5(30°) = 150°$ (int. ∠ of a 12-sided polygon)

Let $B\hat{A}C = B\hat{C}A = x°$ (base ∠s of isos. $\triangle ABC$)

$x° + x° + 150° = 180°$ (∠ sum of $\triangle ABC$)

$2x° = 180° - 150°$

$2x° = 30°$

$x° = \dfrac{30°}{2}$

$= 15°$

Hence,

$A\hat{C}D = B\hat{C}D - B\hat{C}A$

$= 150° - 15°$

$= 135°$

(iii) $A\hat{B}C = B\hat{C}D = 150°$ (int. ∠ of a 12-sided polygon)

$B\hat{A}D = A\hat{D}C = y°$ (base ∠s of isos. quadrilateral, $BA = CD$)

$y° + y° + 150° + 150° = 360°$ (∠ sum of quadrilateral)

$2y° = 360° - 150° - 150°$

$2y° = 60°$

$y° = \dfrac{60°}{2}$

$= 30°$

$\therefore A\hat{D}C = 30°$

$C\hat{D}E = 150°$ (int. ∠ of a 12-sided polygon)

Hence,

$A\hat{D}E = C\hat{D}E - A\hat{D}C$

$= 150° - 30°$

$= 120°$

15. **(i)** Since sum of exterior angles of a n-sided polygon is 360°,

$\therefore n = \dfrac{360°}{36°} = 10$

(ii) Size of an interior angle of the n-sided polygon

$= 180° - 36°$ (adj. $\angle$s on a str. line)

$= 144°$

Let $C\hat{B}D = C\hat{D}B = x°$ (base $\angle$s of isos. $\triangle BCD$)

$x° + x° + 144° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2x° = 180° - 144°$

$2x° = 36°$

$x° = \dfrac{36°}{2}$

$= 18°$

$\therefore C\hat{D}B = 18°$

$C\hat{D}E = 144°$ (int. $\angle$ of a 10-sided polygon)

Hence,

$B\hat{D}E = C\hat{D}E - C\hat{D}B$

$= 144° - 18°$

$= 126°$

(iii) Let $X\hat{C}D = X\hat{D}C = 18°$ (base $\angle$s of isos. $\triangle CDX$, $CX = DX$)

$18° + 18° + C\hat{X}D = 180°$ ($\angle$ sum of $\triangle CDX$)

$C\hat{X}D = 180° - 18° - 18°$

$= 144°$

16. $A\hat{C}E = \angle a + \angle b$ (ext. $\angle$ of $\triangle ABC$)

$J\hat{C}E + \angle a + \angle b = 180°$ (adj. $\angle$s on a str. line)

$J\hat{C}E = 180° - \angle a - \angle b$

$D\hat{E}C = \angle c + \angle d$ (ext. $\angle$ of $\triangle DEF$)

$G\hat{E}C + \angle c + \angle d = 180°$ (adj. $\angle$s on a str. line)

$G\hat{E}C = 180° - \angle c - \angle d$

$H\hat{G}J = \angle e + \angle f$ (ext. $\angle$ of $\triangle GHI$)

$E\hat{G}J + \angle e + \angle f = 180°$ (adj. $\angle$s on a str. line)

$E\hat{G}J = 180° - \angle e - \angle f$

$C\hat{J}G = \angle g + \angle h$ (ext. $\angle$ of $\triangle JKL$)

$C\hat{G}J + \angle g + \angle h = 180°$ (adj. $\angle$s on a str. line)

$C\hat{J}G = 180° - \angle g - \angle h$

Sum of interior angles of quadrilateral $= (4 - 2) \times 180° = 360°$

$\therefore J\hat{C}E + G\hat{E}C + E\hat{G}J + C\hat{J}G = 360°$

$(180° - \angle a - \angle b) + (180° - \angle c - \angle d) + (180° - \angle e - \angle f) + (180° - \angle g - \angle h) = 360°$

$-\angle a - \angle b - \angle c - \angle d - \angle e - \angle f - \angle g - \angle h$

$= 360° - 180° - 180° - 180° - 180°$

$-\angle a - \angle b - \angle c - \angle d - \angle e - \angle f - \angle g - \angle h = -360°$

Hence,

$\angle a + \angle b + \angle c + \angle d + \angle e + \angle f + \angle g + \angle h = 360°$

17. Sum of interior angles of a pentagon = 540°

Let the exterior angle of the pentagon be $x°$.

$5(180° - x°) = 540°$

$900° - 5x° = 540°$

$-5x° = 540° - 900°$

$-5x° = -360°$

$x° = \dfrac{360°}{5}$

$= 72°$

$\angle a + 72° + 72° = 180°$

$\angle a = 180° - 72° - 72°$

$= 36°$

Hence, $\angle a + \angle b + \angle c + \angle d + \angle e = 5 \times 36° = 180°$

18. $a_1 + x_1 = 180°$ (adj. $\angle$s on a str. line)

$a_2 + x_2 = 180°$ (adj. $\angle$s on a str. line)

$a_3 + x_3 = 180°$ (adj. $\angle$s on a str. line)

$a_4 + x_4 = 180°$ (adj. $\angle$s on a str. line)

$a_n + x_n = 180°$ (adj. $\angle$s on a str. line)

Hence,

$a_1 + x_1 + a_2 + x_2 + a_3 + x_3 + a_4 + a_4 + \cdots + a_n + x_n = n \times 180°$

$\underbrace{a_1 + a_2 + a_3 + a_4 + \cdots + a_n} + \underbrace{x_1 + x_2 + x_3 + x_4 + \cdots + x_n} = n \times 180°$

$(n - 2) \times 180° + x_1 + x_2 + x_3 + x_4 + \cdots + x_n = n \times 180°$

$x_1 + x_2 + x_3 + x_4 + \cdots + x_n = n \times 180° - (n - 2) + 360°$

$x_1 + x_2 + x_3 + x_4 + \cdots + x_n = 180°n - 180°n + 360°$

$\therefore x_1 + x_2 + x_3 + x_4 + \cdots + x_n = 360°$

19. **(i)** Two regular polygons are equilateral triangles and squares.

(ii) The interior angles of the polygons meeting at a vertex must add to 360°.

(iii)

Shape	Interior Angle in degrees
Triangle	60
Square	90
Pentagon	108
Hexagon	120
More than six sides	More than 120 degrees

Since the interior angles of the polygon meeting at a vertex must add to 360°, hence the interior angle must be an exact divisor of 360°. This will work only for triangles, squares and hexagons as the interior angle are all divisor of 360°.

(iv) The reason is that the hexagon has the smallest perimeter for a given area as compared to the square and the triangle. This will allow the bees to make more honey using less wax and less work.

Review Exercise 11

1. **(a)** Since $AB = AC$, $\therefore\ A\hat{C}B = A\hat{B}C = 3a°$.

$3a° + 2a° + 3a° = 180°$ ($\angle$ sum of $\triangle ABC$)

$8a° = 180°$

$a° = \dfrac{180°}{8}$

$= 22.5°$

$\therefore\ a = 22.5$

(b) Since $DA = DB$, $\therefore\ D\hat{B}A = D\hat{A}B = 32°$.

$32° + A\hat{D}B + 32° = 180°$ ($\angle$ sum of $\triangle ABD$)

$A\hat{D}B = 180° - 32° - 32°$

$= 116°$

$116° + b° = 360°$ ($\angle$s at a point)

$b° = 360° - 116°$

$= 244°$

$\therefore\ b = 244$

Since $CA = CB$, $\therefore\ C\hat{A}B = C\hat{B}A = x°$.

$x° + 64° + x° = 180°$ ($\angle$ sum of $\triangle ABC$)

$2x° = 180° - 64°$

$= 116°$

$x° = \dfrac{116°}{2}$

$= 58°$

$c° + 32° = 58°$

$c° = 58° - 32°$

$= 26°$

$\therefore\ c = 26$

2. **(a)** Since $BA = BD$, $\therefore\ B\hat{D}A = B\hat{A}D = a°$.

$a° + 40° + a° = 180°$ ($\angle$ sum of $\triangle ABD$)

$2a° = 180° - 40°$

$= 140°$

$a° = \dfrac{140°}{2}$

$= 70°$

$\therefore\ a = 70$

$C\hat{B}D + 40° = 180°$ (adj. $\angle$ s on a str. line)

$C\hat{B}D = 180° - 40°$

$= 140°$

Since $BC = BD$, $\therefore\ B\hat{C}D = B\hat{D}C = b°$.

$b° + 140° + b° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2b° = 180° - 140°$

$= 40°$

$b° = \dfrac{40°}{2}$

$= 20°$

$\therefore\ b = 20$

(b) Since $BA = BD$, $\therefore\ B\hat{D}A = B\hat{A}D = c°$.

$c° + c° = 78°$ (ext. $\angle$ of $\triangle ABD$)

$2c° = 78°$

$c° = \dfrac{78°}{2}$

$= 39°$

$\therefore\ c = 39$

Since $DA = DC$, $\therefore\ D\hat{C}A = D\hat{A}C = 39°$.

$39° + A\hat{B}C + 39° = 180°$ ($\angle$ sum of $\triangle ACD$)

$A\hat{D}C = 180° - 39° - 39°$

$= 102°$

$39° + d° = 102°$

$d° = 102° - 39°$

$= 63°$

$\therefore\ d = 63$

(c) $e° + 62° + 52° = 180°$ ($\angle$ sum of $\triangle BCD$)

$e° = 180° - 62° - 52°$

$= 66°$

$\therefore\ e = 66$

$48° + f° + 66° = 180°$ (adj. $\angle$s on a str. line)

$f° = 180° - 48° - 66°$

$= 66°$

$\therefore\ f = 66$

(d) $110° + D\hat{B}C = 180°$ (adj. $\angle$s on a str. line)

$D\hat{B}C = 180° - 110°$

$= 70°$

Since $DB = DC$, $\therefore\ D\hat{C}B = D\hat{B}C = 70°$.

Hence, $g° = 70°$

$\therefore\ g = 70$

$70° + h° = 110°$ (ext. $\angle$ of $\triangle BCD$)

$h° = 110° - 70°$

$= 40°$

$\therefore\ h = 40$

(e) Since $DB = DC$, $\therefore\ D\hat{B}C = D\hat{C}B = 3i°$.

$(5i + 4)° + 3i° = 180°$ (adj. $\angle$s on a str. line)

$8i° = 180° - 4°$

$= 176°$

$i° = \dfrac{176°}{8}$

$= 22°$

$\therefore\ i = 22$

$3(22°) + 2j° = [5(22) + 4]°$ (ext. $\angle$ of $\triangle BCD$)

$2j° = 114° - 66°$

$= 48°$

$j° = \dfrac{48°}{2}$

$= 24°$

$\therefore\ j = 24$

(f) $k° + 78° = 3k°$ (ext. $\angle$ of $\triangle ABD$)

$3k° - k° = 78°$

$2k° = 78°$

$k° = \dfrac{78°}{2}$

$= 39°$

$\therefore\ k = 39$

$39° + l° + 78° = 180°$ ($\angle$ sum of $\triangle ABD$)

$l° = 180° - 39° - 78°$

$= 63°$

$\therefore\ l = 63$

3. **(a)** Since $AB = AC$, $A\hat{C}B = A\hat{B}C = a°$.

$a° + B\hat{A}C + a° = 180°$ ($\angle$ sum of $\triangle ABC$)

$B\hat{A}C = 180° - 2a°$

$D\hat{C}A = 180° - 2a°$ (alt. $\angle$s, $AB // DC$)

Since $AC = AD = CD$, $D\hat{C}A = C\hat{D}A = C\hat{A}D = 60°$

$180° - 2a° = 60°$

$-2a° = 60° - 180°$

$= -120°$

$a° = \dfrac{-120°}{-2}$

$= 60°$

$\therefore a = 60$

(b) $b° + b° + 76° = 180°$ (int. $\angle$s, $AB // DC$)

$2b° = 180° - 76°$

$= 104°$

$b° = \dfrac{104°}{2}$

$= 52°$

$\therefore b = 52$

$c° + c° + 118° = 180°$ (int. $\angle$s, $AB // DC$)

$2c° = 180° - 118°$

$= 62°$

$c° = \dfrac{62°}{2}$

$= 31°$

$\therefore c = 31$

$52° + 31° + d° = 180°$ ($\angle$ sum of $\triangle ABE$)

$d° = 180° - 52° - 31°$

$= 97°$

$\therefore d = 97$

(c) Since $EA = EB$, $E\hat{A}B = E\hat{B}A = 58°$.

$58° + e° = 180°$ (int. $\angle$s, $AB // DC$)

$e° = 180° - 58°$

$= 122°$

$\therefore e = 122$

$f° = 58°$ (corr. $\angle$s, $AB // DC$)

$\therefore f = 58$

Since $ED = EC$, $E\hat{D}C = E\hat{C}D = 58°$.

$58° + g° + 58° = 180°$ ($\angle$ sum of $\triangle CDE$).

$g° = 180° - 58° - 58°$

$= 64°$

$\therefore g = 64$

4. **(a)** $112° + A\hat{B}C = 180°$ (adj. $\angle$s on a str. line)

$A\hat{B}C = 180° - 112°$

$= 68°$

$62° + H\hat{E}D = 180°$ (adj. $\angle$s on a str. line)

$H\hat{E}D = 180° - 62°$

$= 118°$

$a° + B\hat{C}D = 180°$ (adj. $\angle$s on a str. line)

$B\hat{C}D = 180° - a°$

Sum of the interior angles of a pentagon $= (5 - 2) \times 180° = 540°$.

$\therefore 114° + 68° + 180° - a° + 95° + 118° = 540°$

$-a° = 540° - 114° - 68° - 180° - 95° - 118°$

$= -35°$

$\therefore a = 35$

(b) Sum of exterior angles of a hexagon = 360°

$\therefore 2b° + 4b° + 3b° + b° + b° + b° = 360°$

$12b° = 360°$

$b° = \dfrac{360°}{12}$

$= 30°$

$\therefore b = 30$

$c° + 3(30°) = 180°$ (adj. $\angle$s on a str. line)

$c° = 180° - 90°$

$= 90°$

$\therefore c = 90$

5. **(i)** $A\hat{C}D = 40°$ (alt. $\angle$s, $AB // DC$)

(ii) $C\hat{A}D + 108° + 40° = 180°$ (int. $\angle$s, $AD // BC$)

$C\hat{A}D = 180° - 108° - 40°$

$= 32°$

6. **(i)** Since $AB = AD$, $\therefore A\hat{D}B = A\hat{B}D = 62°$.

$62° + B\hat{A}D + 62° = 180°$ ($\angle$ sum of $\triangle ABD$)

$B\hat{A}D = 180° - 62° - 62°$

$= 56°$

(ii) Since $CB = CD$, $\therefore B\hat{D}C = D\hat{B}C = x°$.

$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2x° = 180° - 118°$

$= 62°$

$x° = \dfrac{62°}{2}$

$= 31°$

$\therefore B\hat{D}C = 31°$

7. Since $\triangle ABE$ is an equilateral triangle, $AB = AE = BE$ and $E\hat{A}B = E\hat{B}A = A\hat{E}B = 60°$.

$D\hat{A}E + 60° = 90°$ (right angle of a square)

$D\hat{A}E = 90° - 60°$
$= 30°$

Since $AD = AB$, $\therefore$ $AE = AD$ and $A\hat{E}D = A\hat{D}E = x°$.

$x° + 30° + x° = 180°$ ($\angle$ sum of $\triangle ADE$)

$2x° = 180° - 30°$
$= 150°$

$x° = \dfrac{150°}{2}$
$= 75°$

$C\hat{B}E + 60° = 90°$ (right angle of a square)

$C\hat{B}E = 90° - 60°$
$= 30°$

Since $BC = AB$, $\therefore$ $BE = BC$ and $B\hat{E}C = B\hat{C}E = y°$.

$y° + 30° + y° = 180°$ ($\angle$ sum of $\triangle BEC$)

$2y° = 180° - 30°$
$= 150°$

$y° = \dfrac{150°}{2}$
$= 75°$

$75° + 60° + 75° + C\hat{E}D = 360°$ ($\angle$s at a point)

$C\hat{E}D = 360° - 75° - 60° - 75°$
$= 150°$

8. Sum of interior angles of a $(2n - 3)$-sided polygon
$= [(2n - 3) - 2] \times 180°$

Hence,

$[(2n - 3) - 2] \times 180° = 62 \times 90°$
$(2n - 5) \times 180° = 5580°$
$360°n - 900° = 5580°$
$360°n = 5580° + 900°$
$360°n = 6480°$
$n = \dfrac{6480°}{360°}$
$= 18$

9. Sum of interior angles of a n-sided polygon
$= (n - 2) \times 180°$

$126° + (n - 1) \times 162° = (n - 2) \times 180°$
$126° + 162°n - 162° = 180°n - 360°$
$180°n - 162°n = 360° + 126° - 162°$
$18°n = 324°$
$n = \dfrac{324°}{18°}$
$= 18$

10. Sum of interior angles of a pentagon
$= (5 - 2) \times 180°$
$= 3 \times 180°$
$= 540°$

Let the 5 interior angles be $3x°$, $4x°$, $5x°$, $5x°$ and $7x°$.

$3x° + 4x° + 5x° + 5x° + 7x° = 540°$
$24x° = 540°$
$x° = \dfrac{540°}{24}$
$= 22.5°$

(i) Largest interior angle $= 7 \times 22.5°$
$= 157.5°$

(ii) Largest exterior angle $= 180° - 3 \times 22.5°$
$= 112.5°$

11. Sum of exterior angles of a n-sided polygon $= 360°$

$35° + 72° + (n - 2) \times 23° = 360°$
$23°n = 360° - 35° - 72° + 46°$
$= 299°$
$n = \dfrac{299°}{23°}$
$= 13$

12. Let the interior angle be $13x°$ and the exterior angle be $2x°$.

$13x° + 2x° = 180°$ (adj. $\angle$s on a str. line)
$15x° = 180°$
$x° = \dfrac{180°}{15}$
$= 12°$

Sum of exterior angles of a n-sided polygon $= 360°$

Hence,

$n = \dfrac{360°}{2(12°)}$
$= 15$

13. Sum of the interior angles of a n-sided polygon $= (n - 2) \times 180°$

Sum of the exterior angles of a n-sided polygon $= 360°$

$(n - 2) \times 180° = 4 \times 360°$
$180°n = 1440° + 360°$
$= 1800°$
$n = \dfrac{1800°}{180°}$
$= 10$

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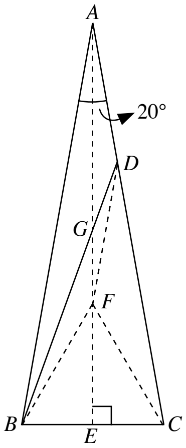
Challenge Yourself

1. $A\hat{B}C = \dfrac{180° - 20°}{2}$ (base $\angle$s of isos. $\triangle ABC$)

$= \dfrac{160°}{2}$

$= 80°$

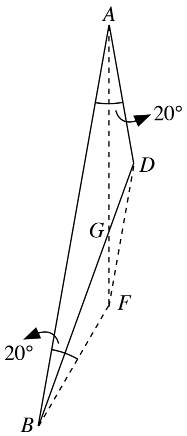


Draw E on BC such that $AE \perp BC$.

Draw F on AE such that $\triangle BCF$ is an equilateral triangle.

Then $A\hat{B}F = 80° - 60° = 20°$ and $BF = BC = AD$.

Consider the quadrilateral $ABFD$.



Since $A\hat{B}F = B\hat{A}D = B\hat{A}C = 20°$ and $BF = AD$,

then by symmetry, $AB \,//\, DF$ and $ABFD$ is an isosceles trapezium.

In the isosceles trapezium $ABFD$, by symmetry, $AG = BG$, so $\triangle ABG$ is an isosceles triangle.

Since $B\hat{A}G = B\hat{A}E = \dfrac{20°}{2} = 10°$ (AE bisects $B\hat{A}C$),

then $A\hat{B}G = B\hat{A}G = 10°$ (base $\angle$s of isos. $\triangle ABG$).

$\therefore A\hat{D}B + A\hat{B}D + B\hat{A}D = 180°$ ($\angle$ sum of $\triangle ABD$)

$A\hat{D}B + A\hat{B}G + 20° = 180°$

$A\hat{D}B + 10° + 20° = 180°$

$A\hat{D}B = 180° - 10° - 20°$

$= 150°$

Teachers may wish to note the usefulness of the symmetric properties of an isosceles trapezium. Otherwise, formal proofs using congruent triangles are beyond the scope of Secondary 1 syllabus.

2. $A\hat{C}B = \dfrac{180° - 20°}{2}$ (base $\angle$s of isos. $\triangle ABC$)

$= \dfrac{160°}{2}$

$= 80°$

$\therefore D\hat{C}F = D\hat{C}A$

$= A\hat{C}B - 60°$

$= 80° - 60°$

$= 20°$

$B\hat{F}C + F\hat{C}B + 50° = 180°$ ($\angle$ sum of $\triangle BCF$)

$B\hat{F}C + A\hat{C}B + 50° = 180°$

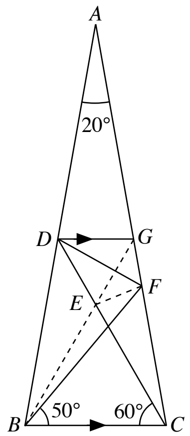
$B\hat{F}C + 80° + 50° = 180°$

$B\hat{F}C = 180° - 80° - 50°$

$= 50°$

Since $C\hat{B}F = B\hat{F}C = 50°$, i.e. $CB = CF$,

then $\triangle BCF$ is an isosceles triangle.



Draw G on AG such that $DG \,//\, BC$.

Draw BG to cut CD at E.

Draw EF.

By symmetry, $BE = CE$, so $\triangle BCE$ is an isosceles triangle.

Since the base angle of $\triangle BCE$ is 60°,

then $\triangle BCE$ is an equilateral triangle,

i.e. $B\hat{E}C = 60°$ and $E\hat{B}F = 60° - 50° = 10°$.

$\therefore CE = CB$ (sides of equilateral $\triangle BCE$)

$= CF$ (sides of isosceles $\triangle BCF$)

Since $CE = CF$,

then $\triangle CEF$ is an isosceles triangle.

$C\hat{F}E = \dfrac{180° - E\hat{C}F}{2}$ (base $\angle$s of isos. $\triangle CEF$)

$= \dfrac{180° - D\hat{C}F}{2}$

$= \dfrac{180° - 20°}{2}$

$= \dfrac{160°}{2}$

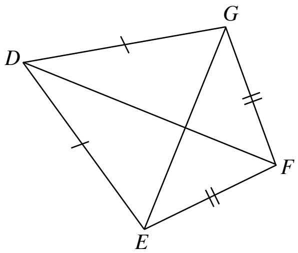
$= 80°$

$\therefore B\hat{F}E = C\hat{F}E - B\hat{F}C$

$= 80° - 50°$

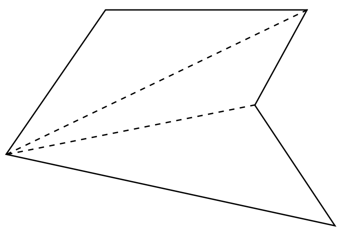
$= 30°$

$F\hat{E}G = E\hat{B}F + B\hat{F}E$ (ext. $\angle$ of $\triangle BEF$)
$= 10° + 30°$
$= 40°$
$D\hat{E}G = B\hat{E}C$ (vert. opp. $\angle$s)
$= 60°$
$D\hat{G}E = C\hat{B}E$ (alt. $\angle$s, $DG // BC$)
$= 60°$
Since the base angle of $\triangle DEG$ is 60°,
then $\triangle DEG$ is an equilateral triangle,
i.e. $E\hat{D}G = 60°$ and $DE = DG$.
$A\hat{G}D = A\hat{C}B$ (corr. $\angle$s, $DG // BC$)
$= 80°$
$\therefore\ F\hat{G}E + D\hat{G}E + A\hat{G}D = 180°$ (adj. $\angle$s on a str. line)
$F\hat{G}E + 60° + 80° = 180°$
$F\hat{G}E = 180° - 60° - 80°$
$= 40°$
Since $F\hat{E}G = F\hat{G}E = 40°$,
then $\triangle EFG$ is an isosceles triangle,
i.e. $FE = FG$.
Consider the quadrilateral $DEFG$.

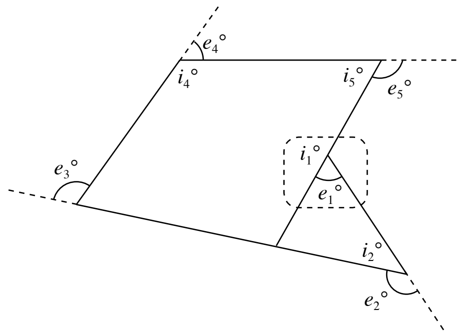


Since $DE = DG$ and $FE = FG$,
then $DEFG$ is a kite.
In the kite $DEFG$, the longer diagonal DF bisects $E\hat{D}G$.
$\therefore\ C\hat{D}F = E\hat{D}F$
$= \dfrac{60°}{2}$
$= 30°$

3. Yes. For any n-sided concave polygon, it can still form $(n - 2)$ triangles in the polygon.
Hence the sum of the interior angles is still the same.
E.g.

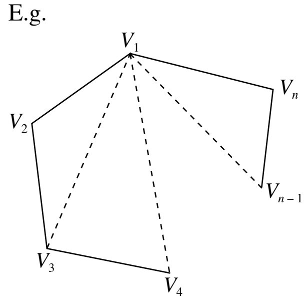


4. **(i)** An exterior angle of a concave polygon has a negative measure and is inside the polygon as shown in the diagram below.
E.g.



(ii) Yes. Exterior angle of the vertex which is "pushed in" will flip over into the inside of the polygon and becomes negative. Adding all the exterior angles as before, they will still add to 360°.
E.g. $i_1° + (-e_1°) + i_2° + e_2° + i_3° + e_3° + i_4° + e_4° + i_5° + e_5° +$
$= 5 \times 180°$
$[i_1° + i_2° + i_3° + i_4° + i_5°] + (-e_1°) + e_2° + e_3° + e_4° + e_5°$
$= 900°$
$(-e_1°) + e_2° + e_3° + e_4° + e_5°$
$= 900 - [i_1° + i_2° + i_3° + i_4° + i_5°]$
$(-e_1°) + e_2° + e_3° + e_4° + e_5° = 900° - (5 - 2) \times 180°$
$(-e_1°) + e_2° + e_3° + e_4° + e_5° = 900° - 540°$
$(-e_1°) + e_2° + e_3° + e_4° + e_5° = 360°$
The above proof holds for any n-sided polygon.

5. In a n-sided polygon, each diagonal connects one vertex to another vertex which is not its next-door neighbour. Since there are n vertices in an n-sided polygon, therefore there are n starting points for the diagonals. For each diagonal, it (e.g. V_1) can join to other $(n - 3)$ vertices since it cannot join itself (V_1) or either of the two neighbouring vertices (V_2 and V_n). So the total number of diagonals formed is $n \times (n - 3)$. However, in this way, each diagonal would be formed twice (to and from each vertex), so the product $n(n - 3)$ must be divided by 2. Hence the formula is $\dfrac{n(n-3)}{2}$.
E.g.



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Chapter 12 Geometrical Constructions

TEACHING NOTES

Suggested Approach

Students have learnt how to draw triangles and quadrilaterals using rulers, protractors and set squares in primary school. Teachers need to reintroduce these construction tools and demonstrate the use of these if students are still unfamiliar with them. When students are comfortable with the use of these construction tools and the compasses, teachers can proceed to the sections on construction of triangles and quadrilaterals.

Section 12.1: Introduction to Geometrical Constructions

Teachers may wish to recap with students how rulers, protractors and set squares are used. More emphasis should be placed on the use of protractors, such as the type of scale (inner or outer) to use, depending on the type of angle (acute or obtuse). Teachers need to impress upon students to avoid parallax errors when reading the length using a ruler, or an angle using a protractor.

Teachers should show and lead students on the use of compasses. Students are to know and be familiar with the useful tips in using the construction tools.

Section 12.2: Perpendicular Bisectors and Angle Bisectors

Teachers should state and define perpendicular bisectors and angle bisectors. Stating what perpendicular and bisect means individually will help students to remember their meanings.

For the worked examples in this section, teachers are encouraged to go through the construction steps one by one with the students. Students should follow and construct the same figures as shown in the worked examples.

Teachers should allow students to use suitable geometry software to explore and discover the properties of perpendicular bisectors and angle bisectors (see Investigation: Property of a Perpendicular Bisector and Investigation: Property of an Angle Bisector), that is, their equidistance from end-points and sides of angles respectively.

Section 12.3: Construction of Triangles

Students should be able to construct the following types of triangles at the end of this section:

- Given 2 sides and an included angle
- Given 3 sides
- Given 1 side and 2 angles

As a rule of thumb, students should draw the longest line as a horizontal line. Teachers are to remind their students to mark all angles, vertices, lengths and other markings (same angles, same sides, right angles etc.) clearly. Students should not erase any arcs that they draw in the midst of construction and check their figure at the end.

Section 12.4: Construction of Quadrilaterals

Students should be able to construct parallelograms, rhombuses, trapeziums and other quadrilaterals at the end of this section.

As a rule of thumb, students should draw the longest line as a horizontal line. Teachers are to remind their students to mark all angles, vertices, lengths and other markings (same angles, same sides, right angles etc.) clearly. Students should not erase any arcs they draw in the midst of construction and check their figure at the end.

WORKED SOLUTIONS

Investigation (Property of a Perpendicular Bisector)

4. The length of *AC* is equal to the length of *BC*.

5. Any point on the perpendicular bisector of *AB* is equidistant from *A* and *B*.

6. Any point which is not on the perpendicular bisector of *AB* is not equidistant from *A* and *B*.

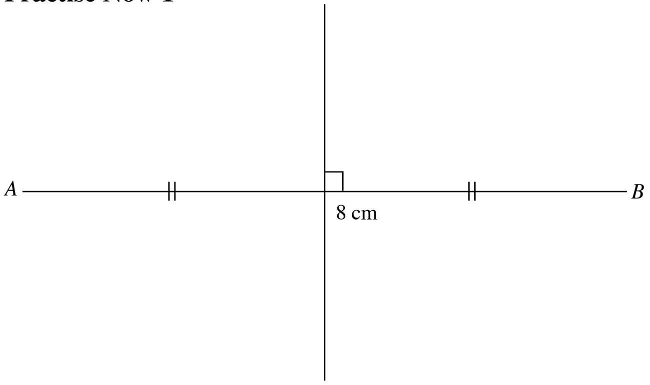
Investigation (Property of an Angle Bisector)

5. The length of *PR* is equal to the length of *QR*.

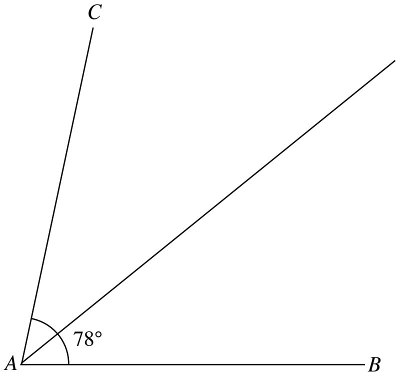
6. Any point on the angle bisector of $B\hat{A}C$ is equidistant from *AB* and *AC*.

7. Any point which is not on the angle bisector of $B\hat{A}C$ is not equidistant from *AB* and *AC*.

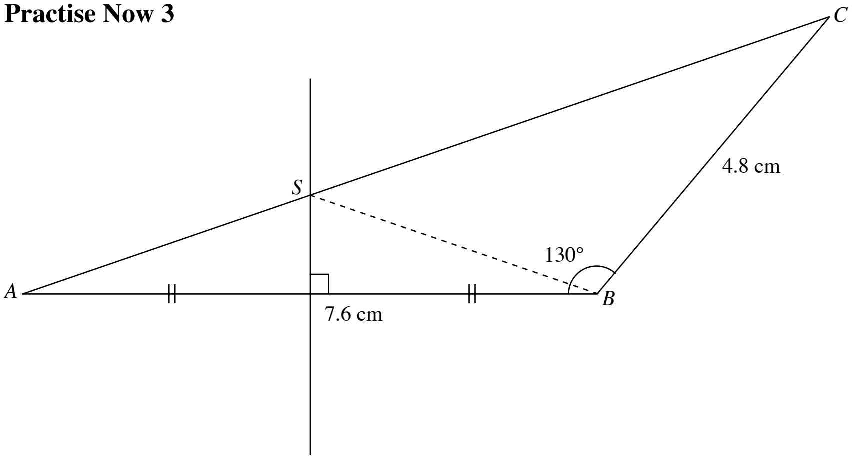
Practise Now 1



Practise Now 2



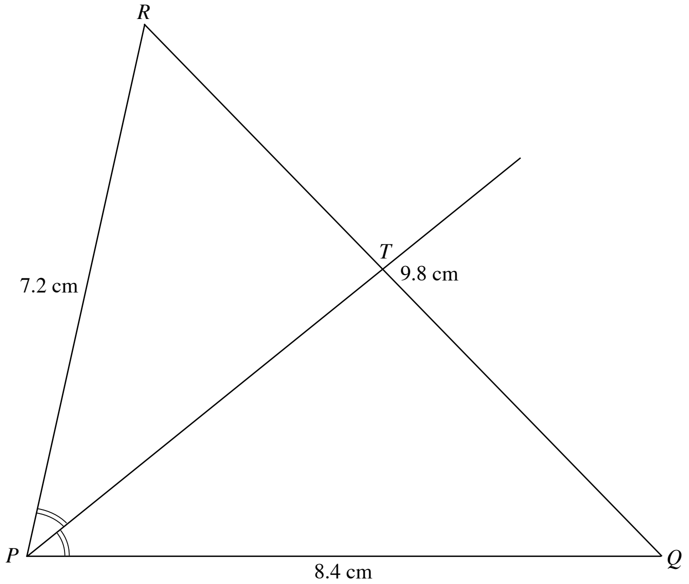
Practise Now 3



(i) Length of AC = 11.3 cm

(ii) Length of BS = 4.0 cm

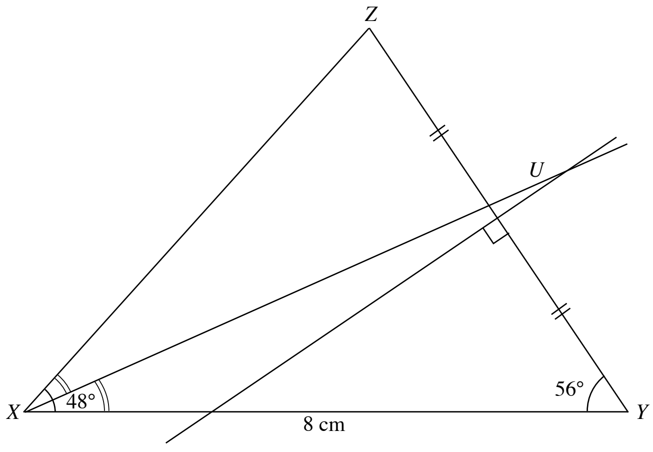
Practise Now 4



(i) Required angle, $Q\hat{P}R = 77°$

(ii) Length of QT = 5.3 cm

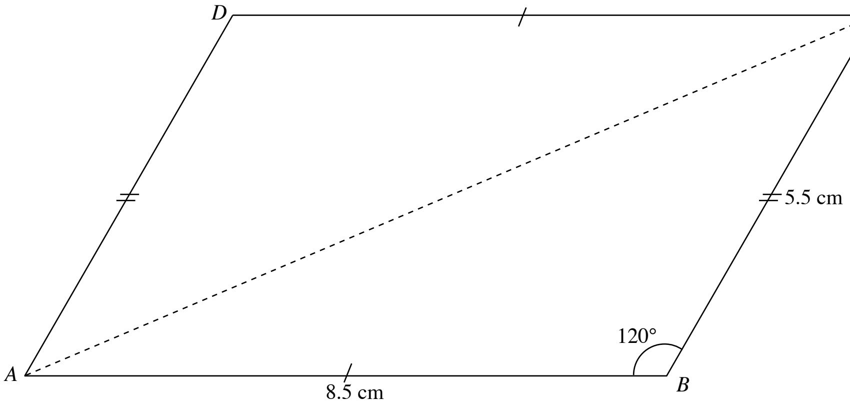
Practise Now 5



(iii) The point U is equidistant from the points Y and Z, and equidistant from the lines XY and XZ.

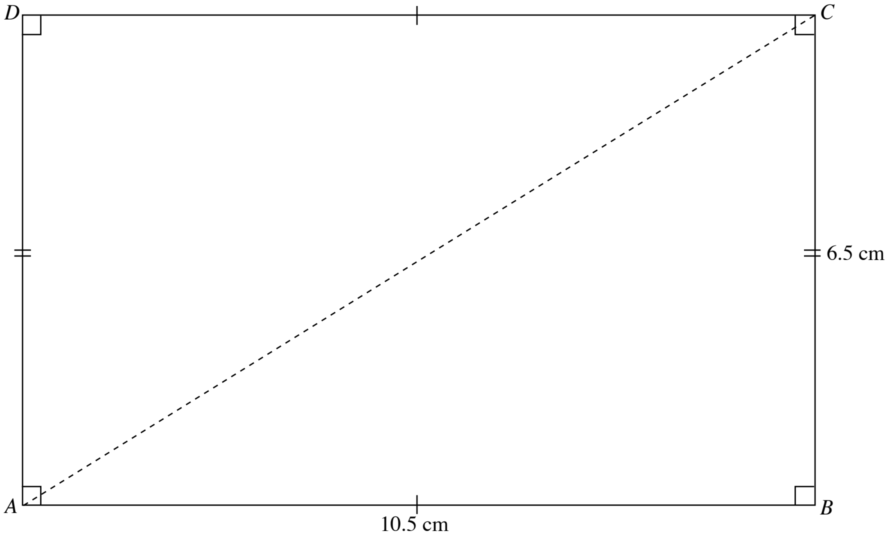
Practise Now 6

1.



Length of AC = 12.2 cm

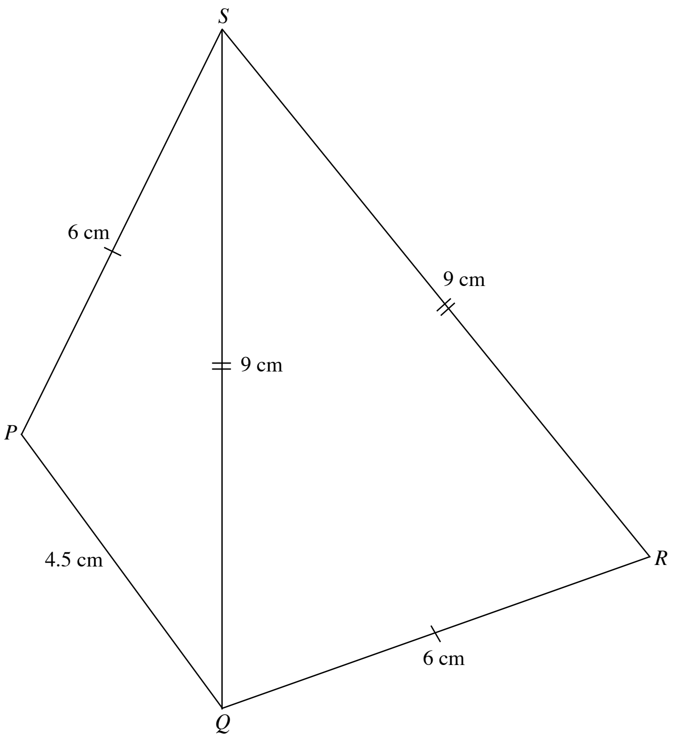
2.



Length of AC = 12.3 cm

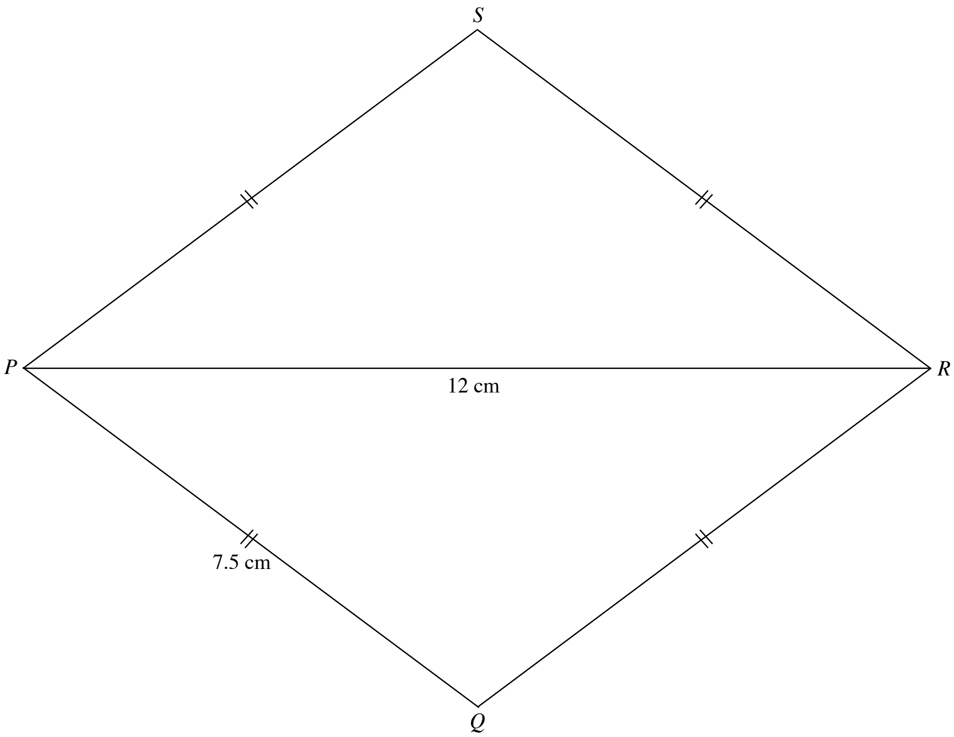
Practicse Now 7

1.



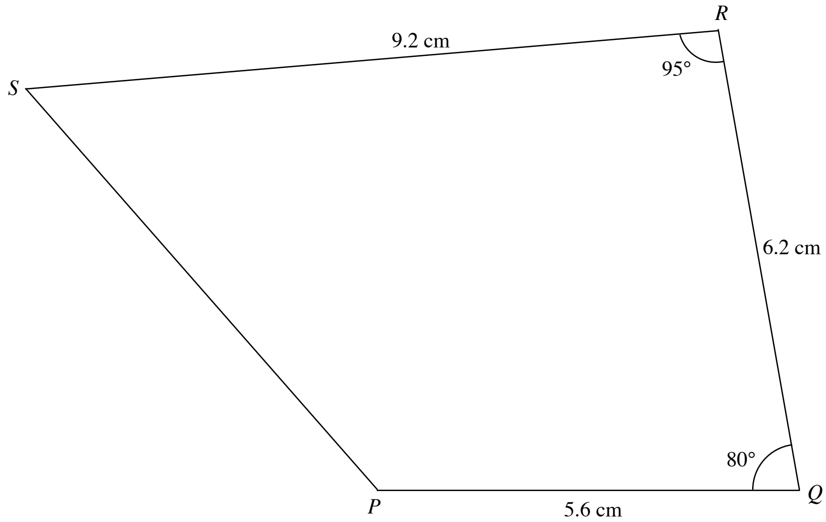
$Q\hat{R}S = 71°$

2.



$Q\hat{R}S = 74°$

Practise Now 8

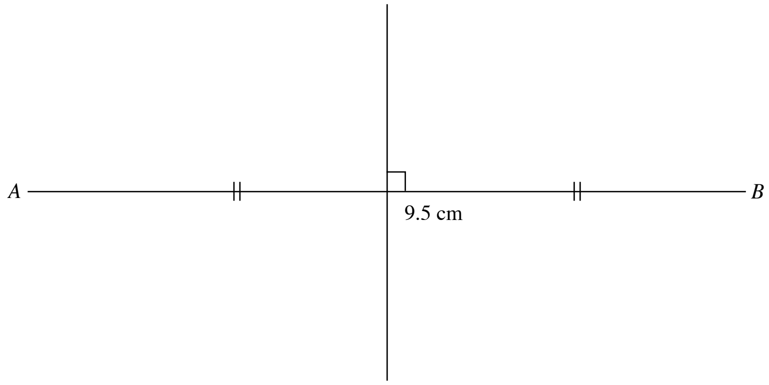


(i) Length of PS = 7.0 cm

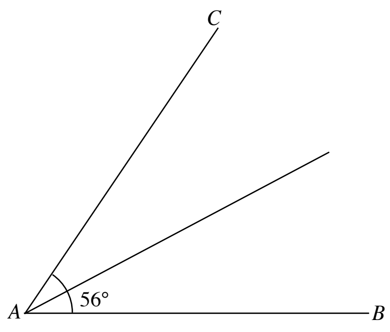
(ii) $P\hat{S}R = 54°$

Exercise 12A

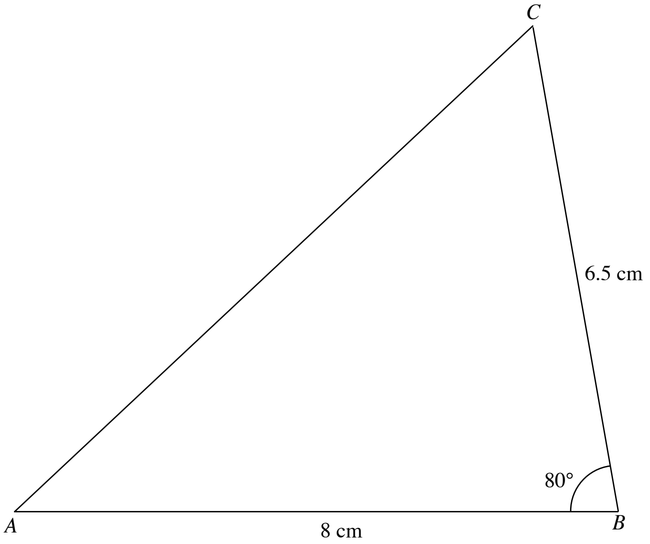
1.



2.

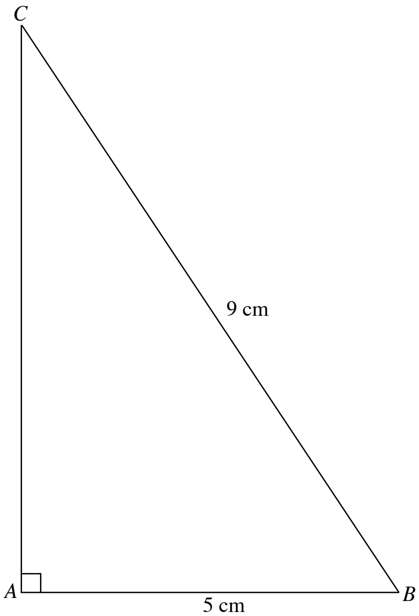


3.



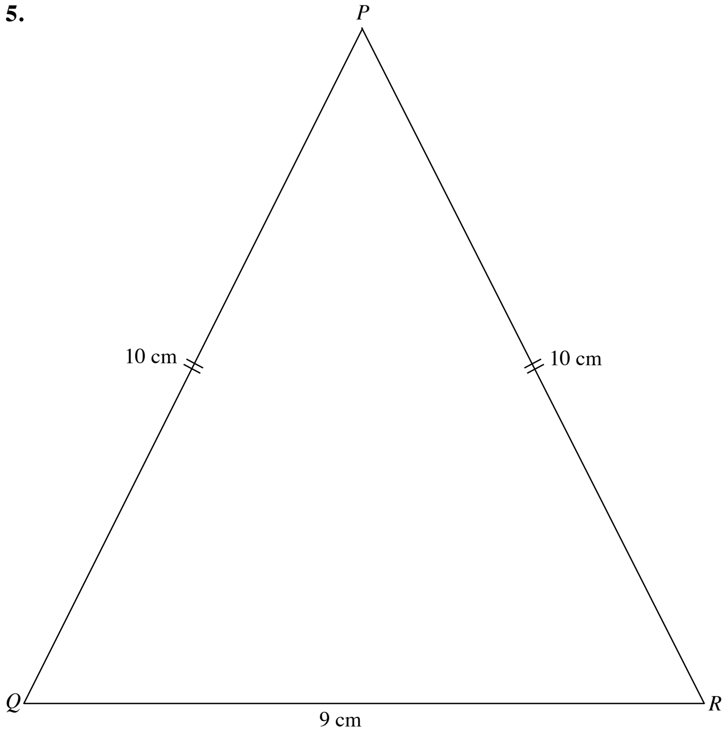
Length of AC = 9.4 cm

4.



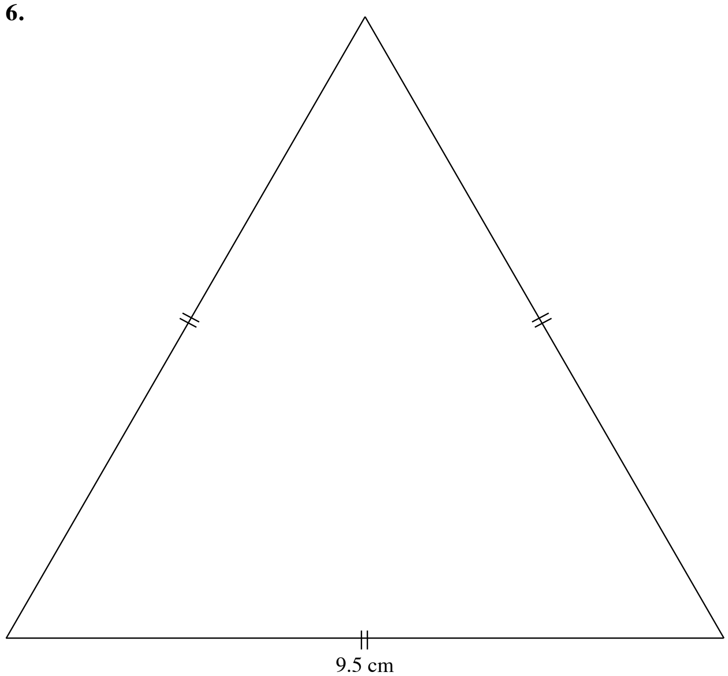
Length of AC = 7.5 cm

5.

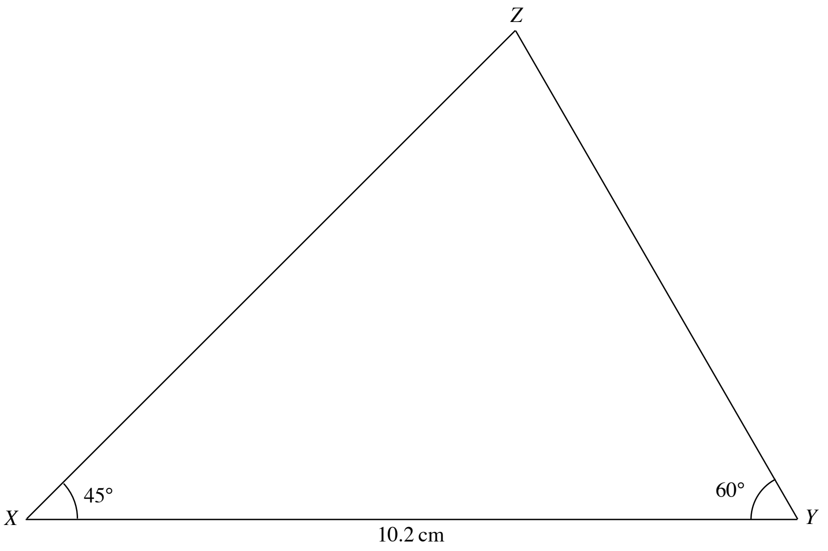


$Q\hat{P}R = 53°$

6.

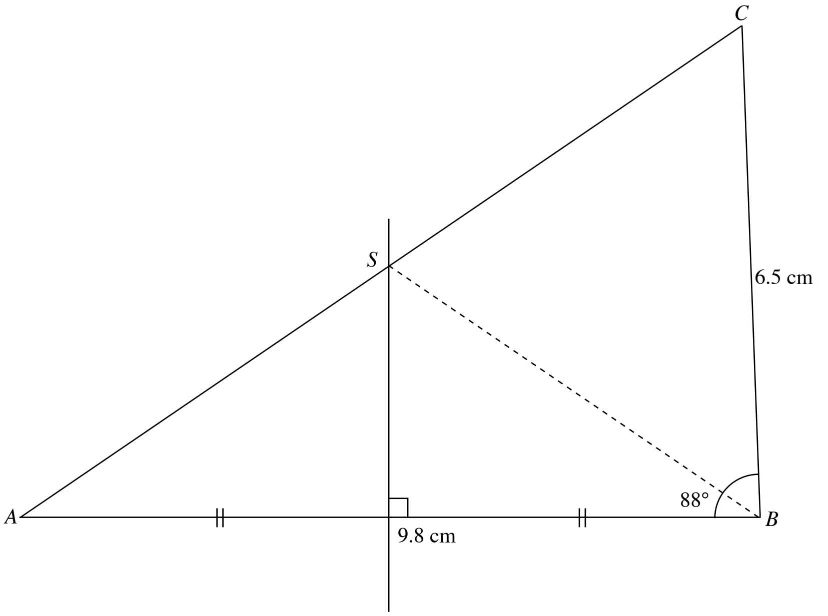


7.



Length of XZ = 9.1 cm

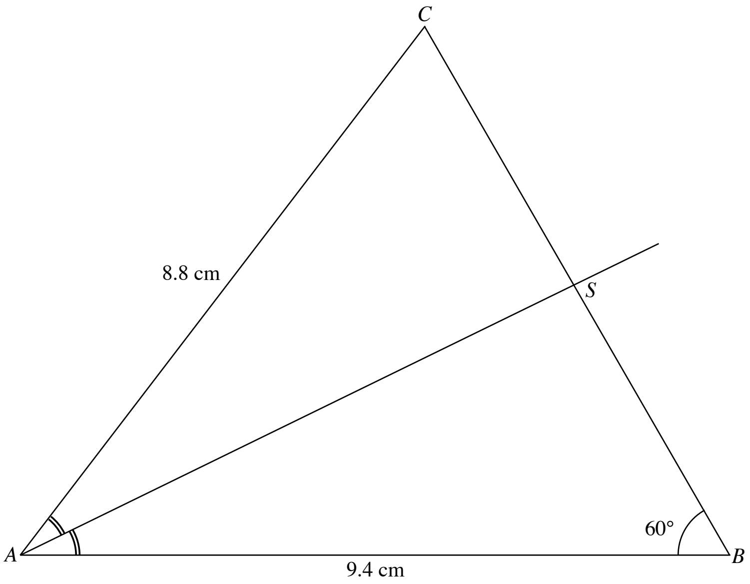
8.



(i) Length of AC = 11.6 cm

(ii) Length of BS = 5.9 cm

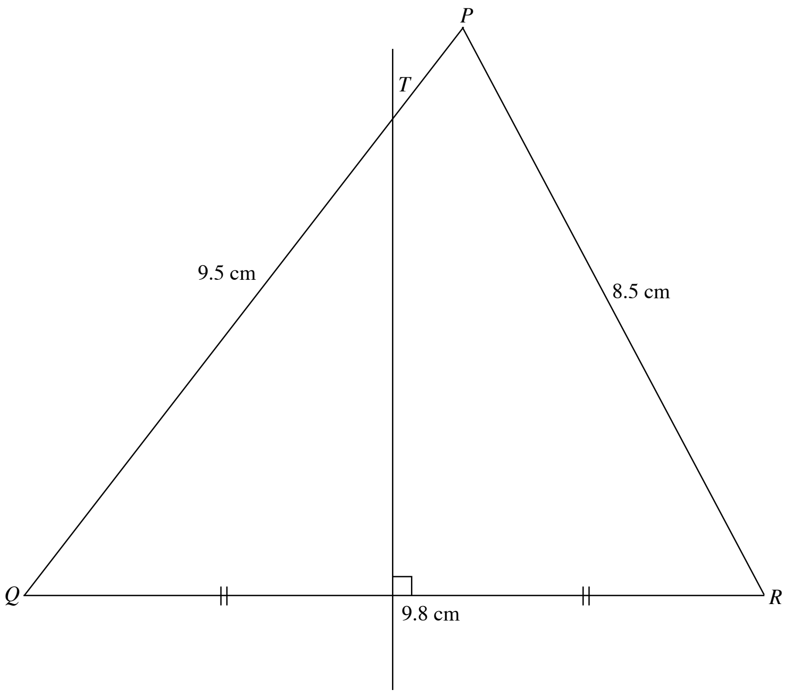
9.



(i) Required angle, $B\hat{A}C = 52°$

(ii) Length of $CS = 3.9$ cm

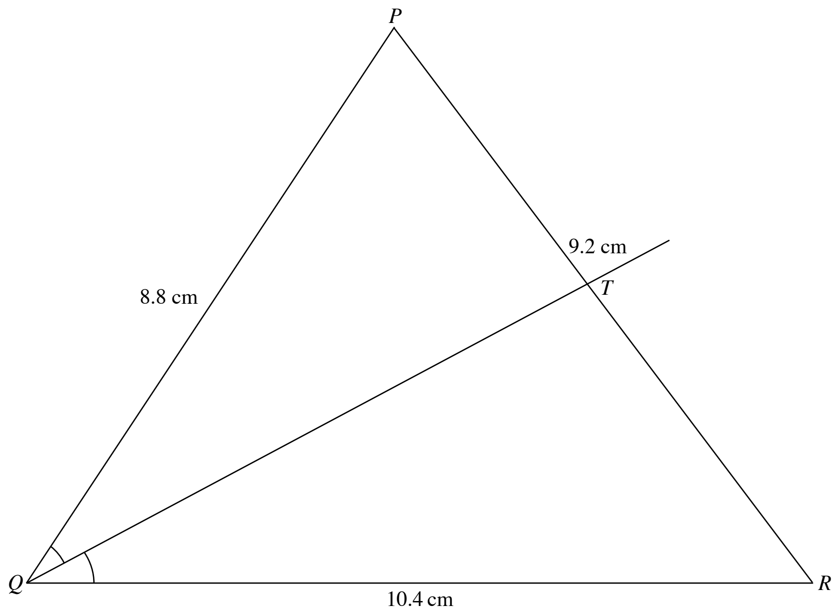
10.



(i) Required angle, $P\hat{Q}R = 52°$

(ii) Length of $QT = 8.0$ cm

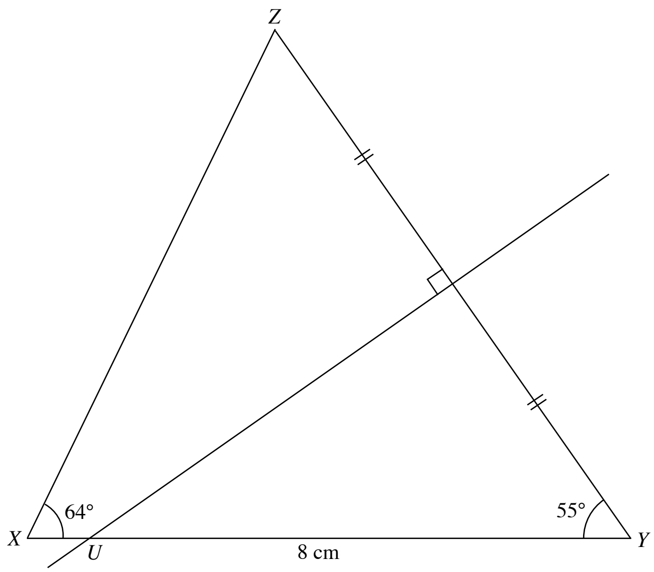
11.



(i) Required angle, $Q\hat{P}R = 71°$

(ii) $PT = 4.2$ cm

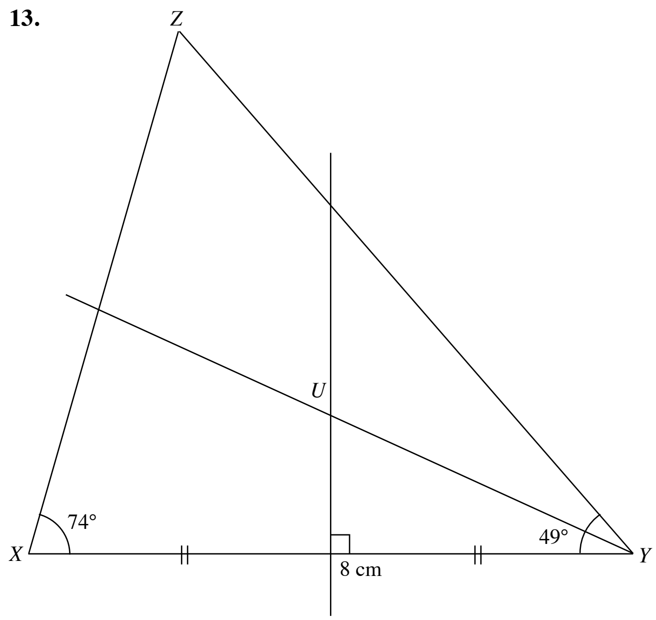
12.



(i) Length of $XZ = 7.5$ cm

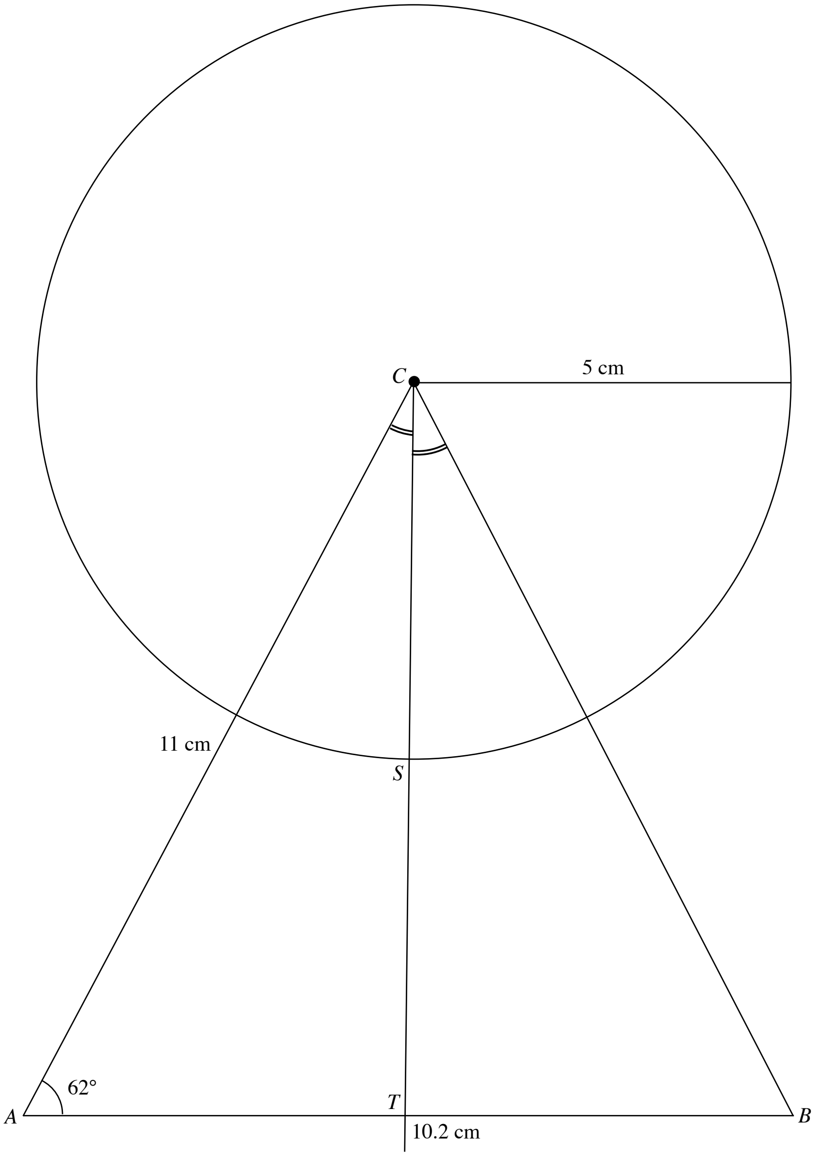
(ii) Length of $UY = 7.2$ cm

13.



(iii) The point U is equidistant from the points X and Y, and equidistant from the lines XY and YZ.

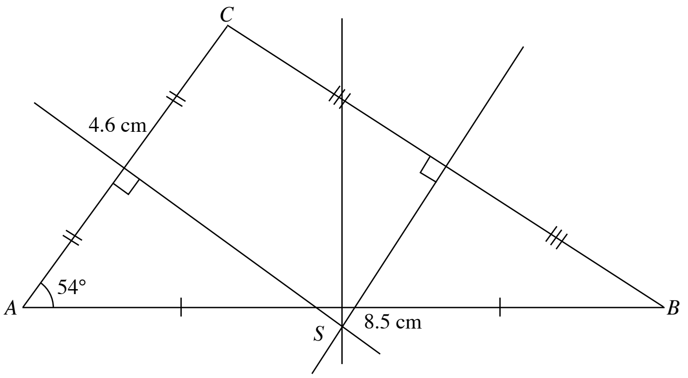
14.



(i) Length of BC = 10.9 cm

(iii) Length of ST = 4.7 cm

15.



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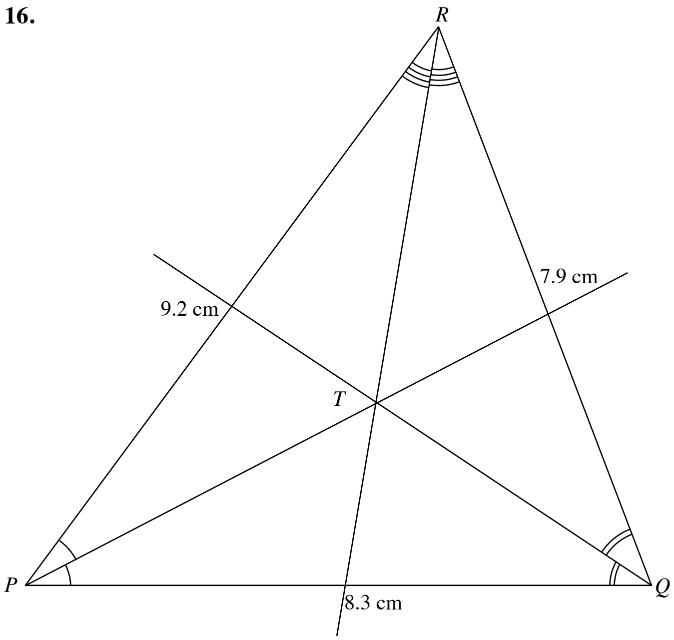
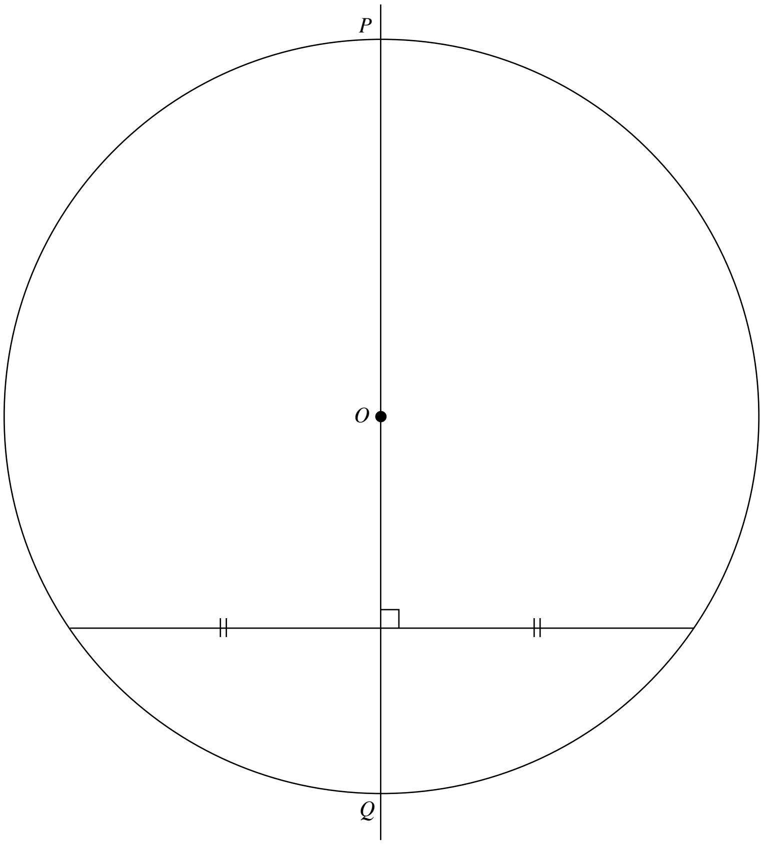


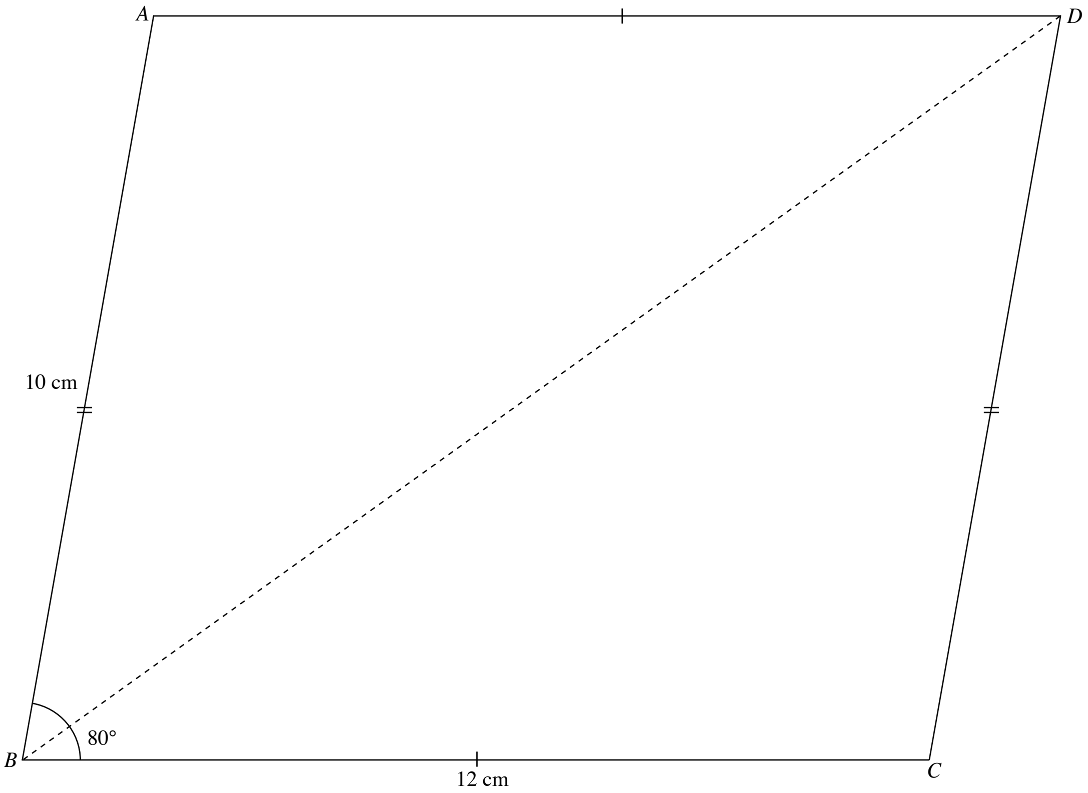
17.



(ii) Diameter

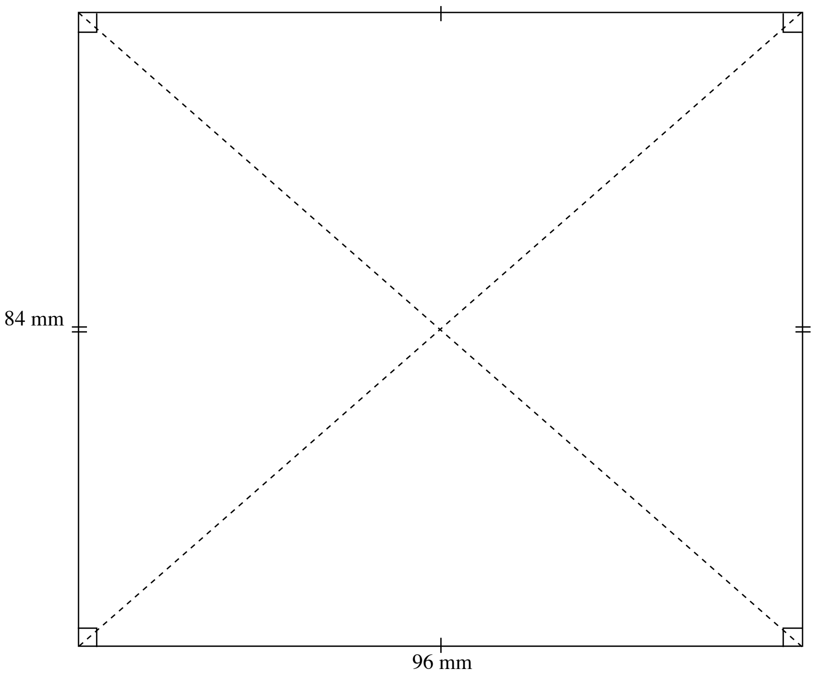
Exercise 12B

1.



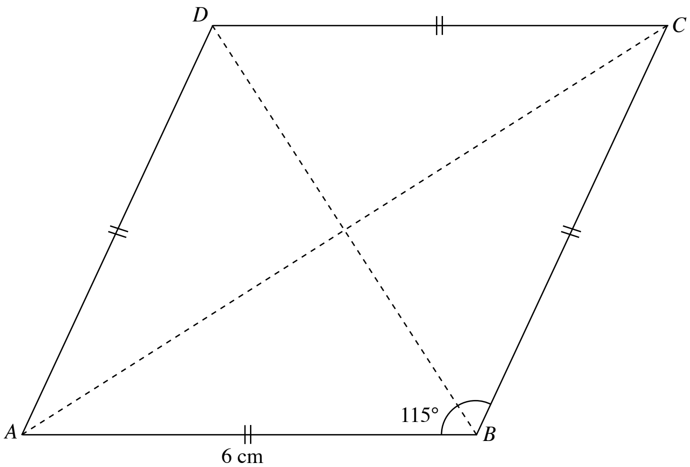
Length of diagonal BD = 16.9 cm

2. 96 mm = 9.6 cm
84 mm = 8.4 cm



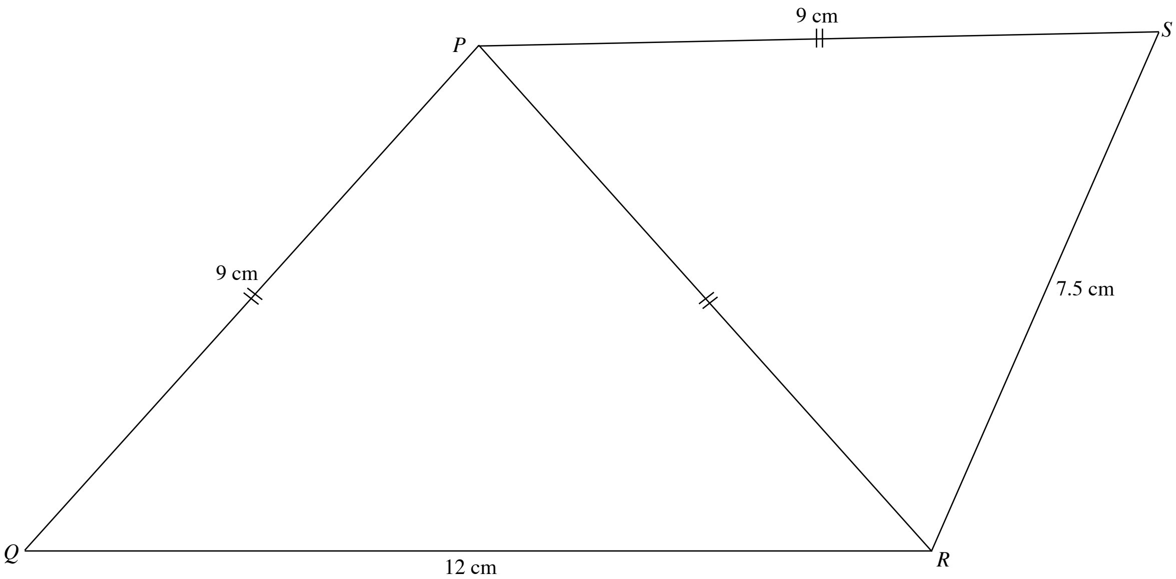
Length of each of the two diagonals = 12.8 cm

3.



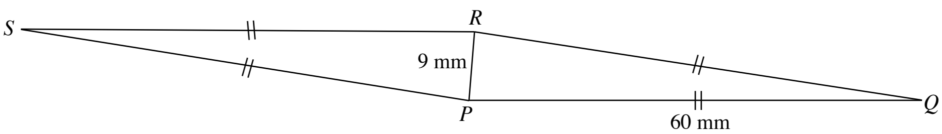
Length of each of the two diagonals = 10.1 cm, 6.5 cm

4.



$Q\hat{P}S = 133°$

5. 60 mm = 6 cm
9 mm = 0.9 cm



$Q\hat{P}S = 171°$

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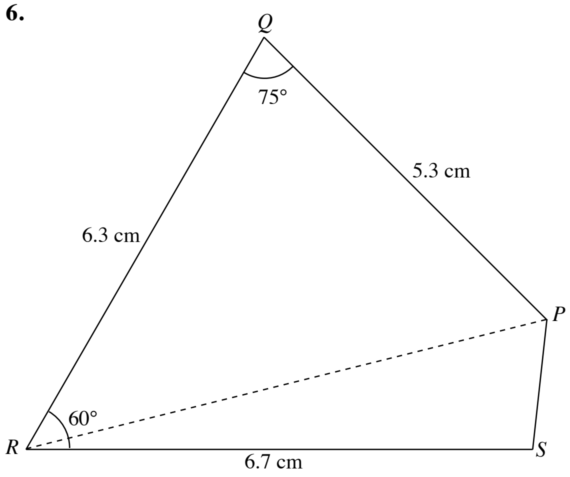
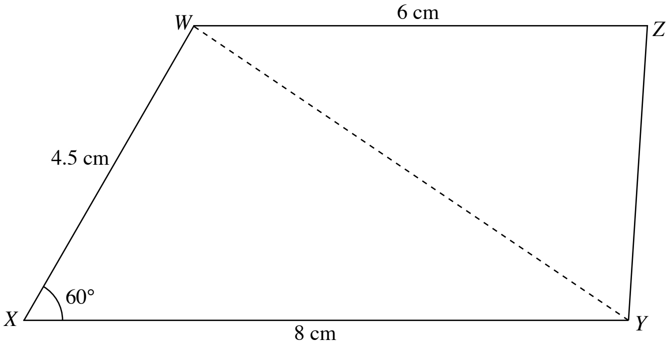
6.



(i) Length of $PR = 7.1$ cm

(ii) $R\hat{P}S = 70°$

7.

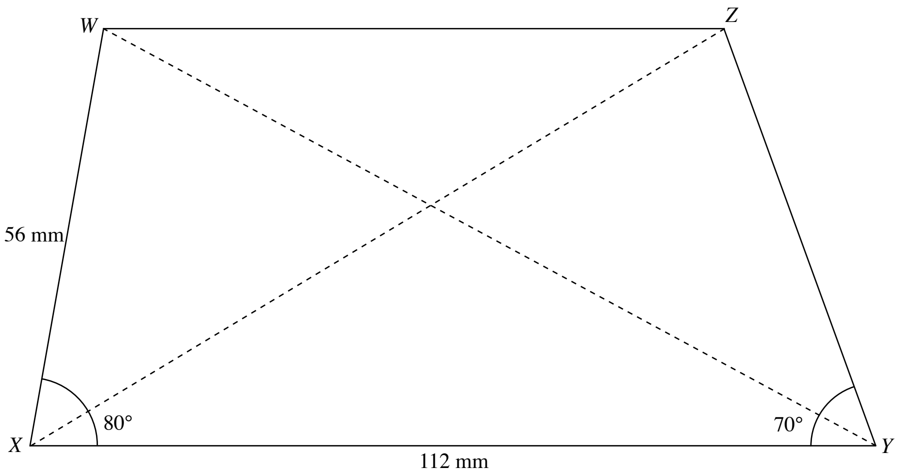


Length of $YZ = 3.9$ cm

Length of $WY = 6.9$ cm

8. 56 mm = 5.6 cm

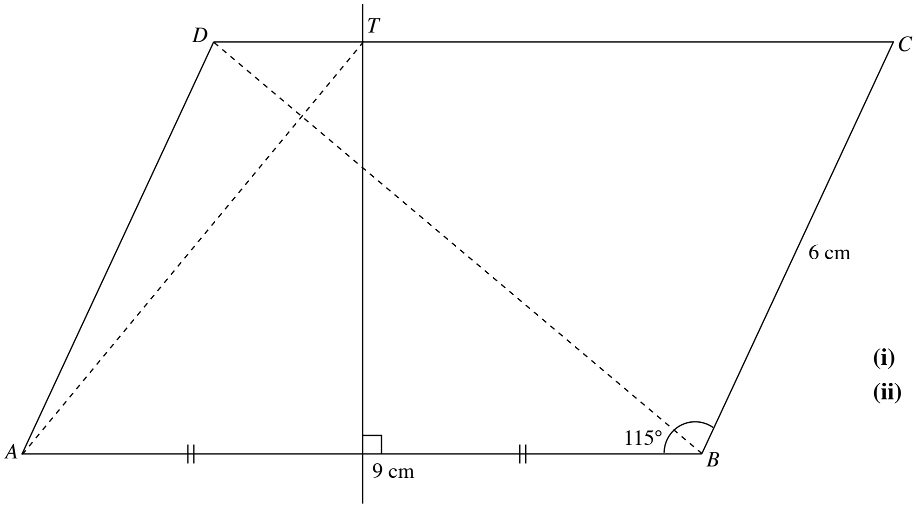
112 mm = 11.2 cm



Length of $WY = 11.6$ cm

Length of $XZ = 10.7$ cm

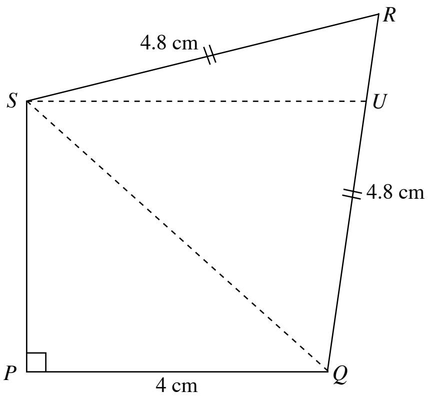
9.



(i) Length of diagonal BD = 8.4 cm

(ii) Length of AT = 7.1 cm

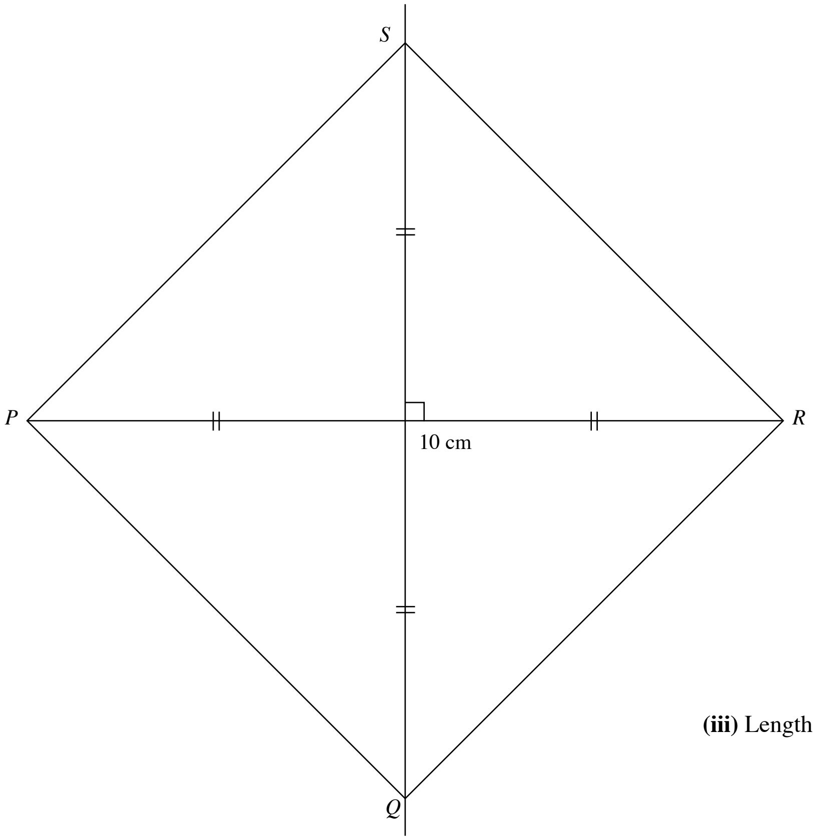
10.



(i) Length of QS = 5.4 cm

(ii) Length of SU = 4.5 cm

11.



(iii) Length of PQ = 7.0 cm

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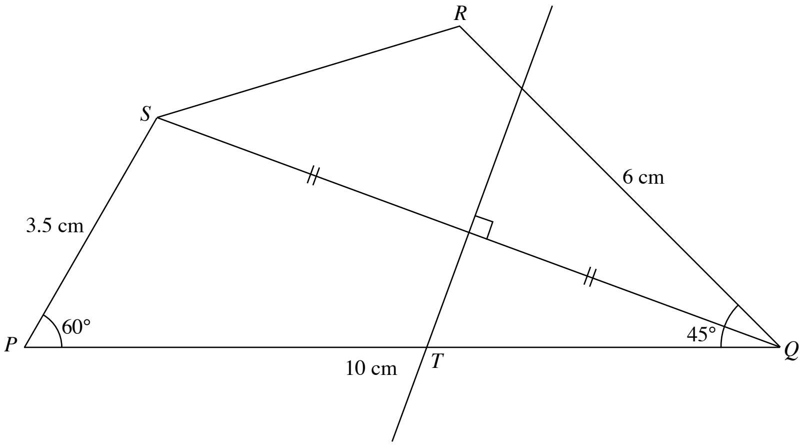
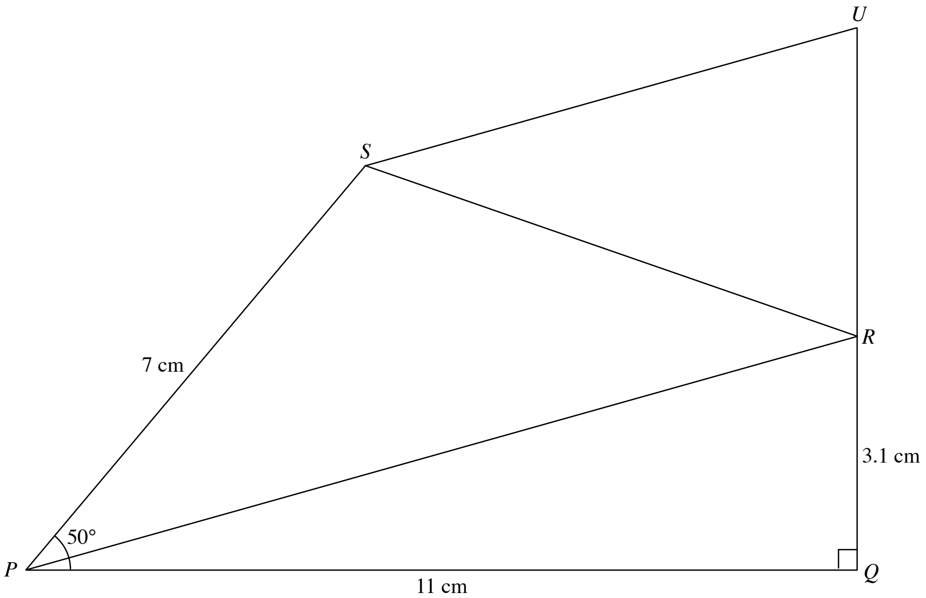
12.



(i) $Q\hat{R}S = 119°$

(ii) Length of PT = 5.4 cm

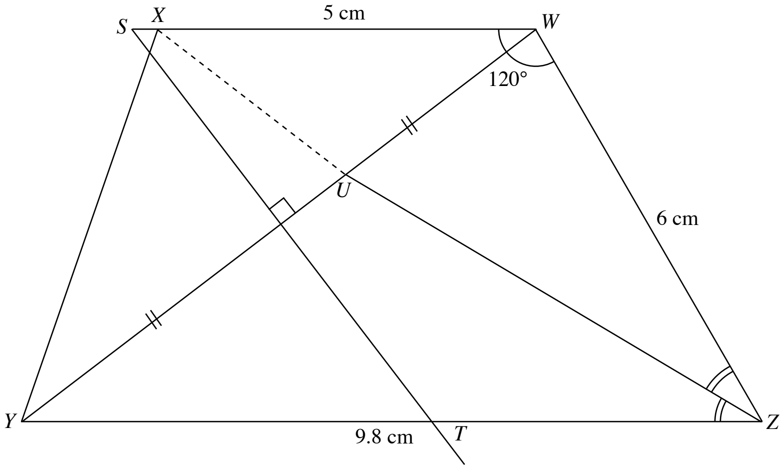
13.



(i) $Q\hat{R}S = 109°$

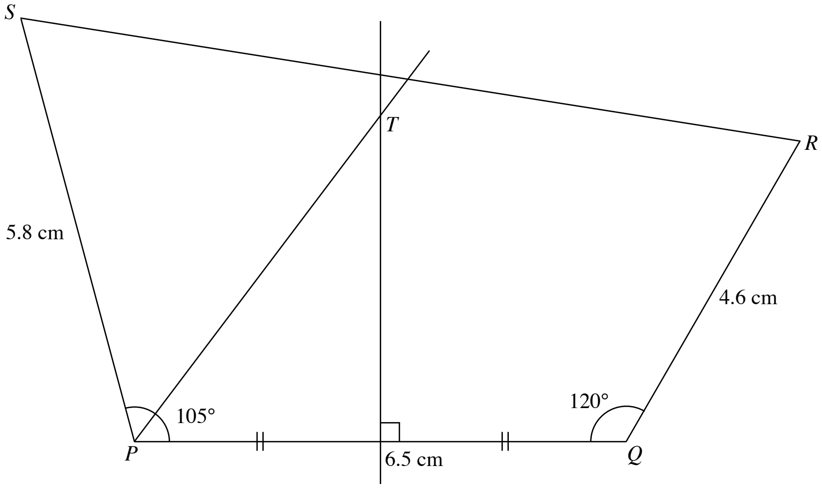
(ii) Length of RU = 4.1 cm

14.



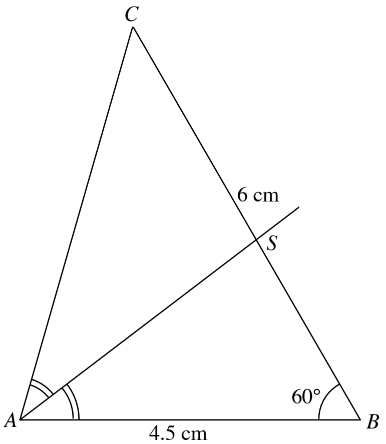
(i) Length of WY = 8.6 cm
(ii) Length of ST = 6.5 cm
(iii) $W\hat{U}X$ = 105°

15.



Review Exercise 12

1.



(i) Length of AC = 5.4 cm
(ii) Length of CS = 3.3 cm

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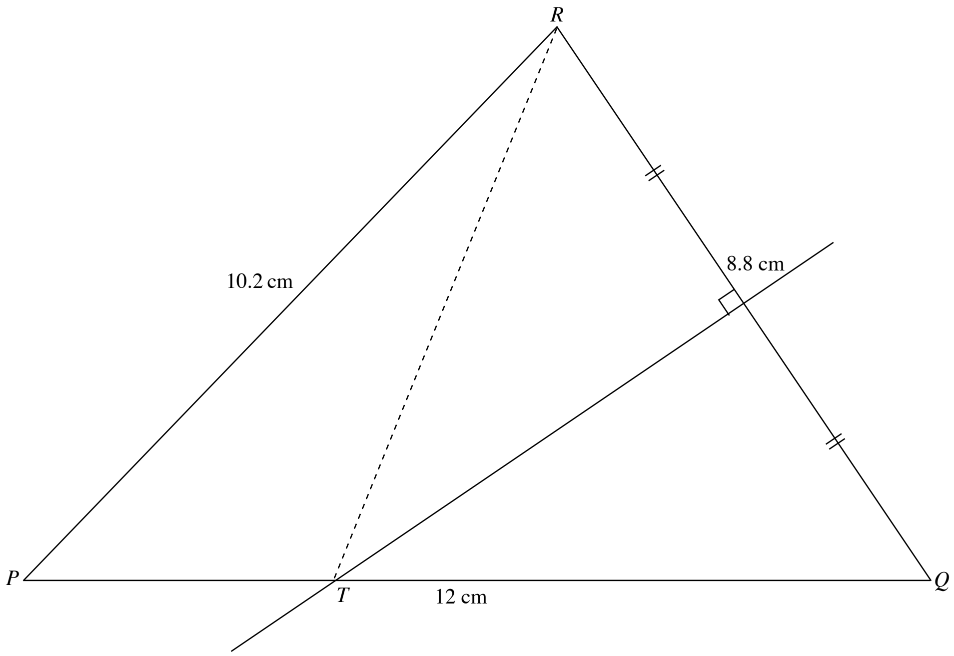
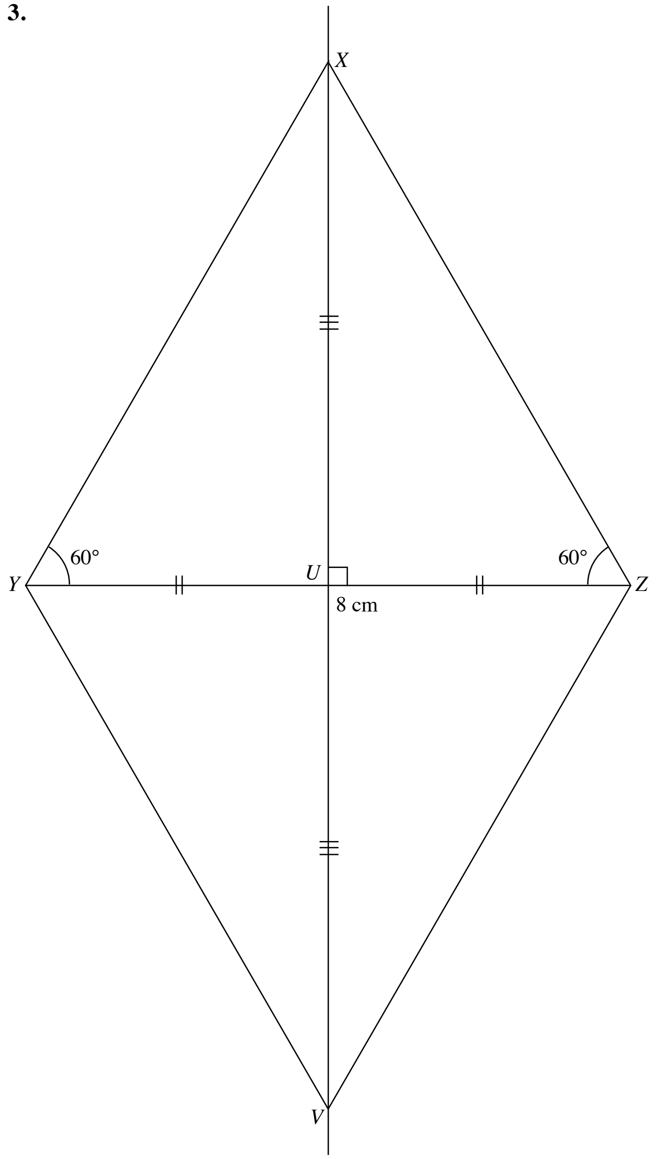
2.



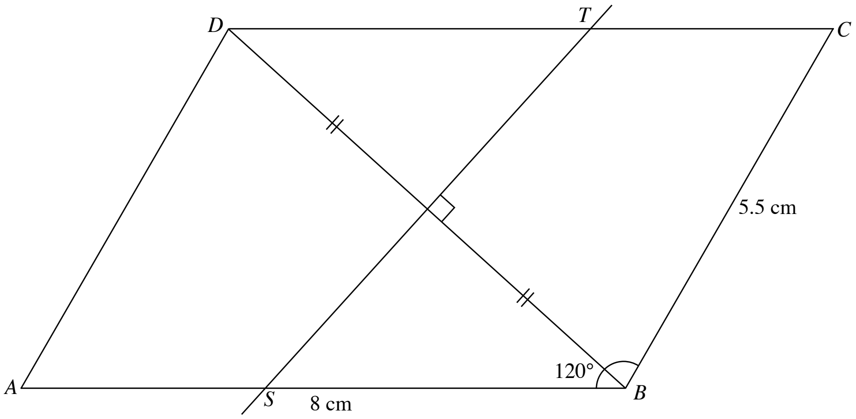
(i) Required angle, $Q\hat{P}R = 46°$
(ii) $RT = 7.9$ cm

3.



(iii) Rhombus

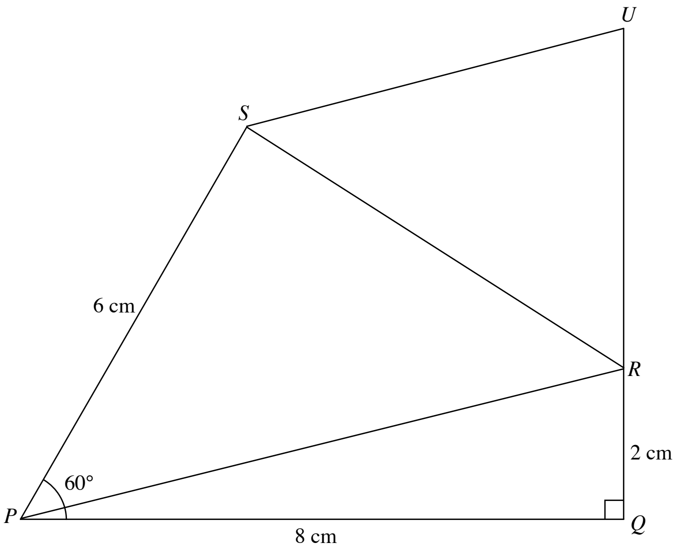
4.



(i) Length of BD = 7.1 cm

(ii) Length of ST = 6.5 cm

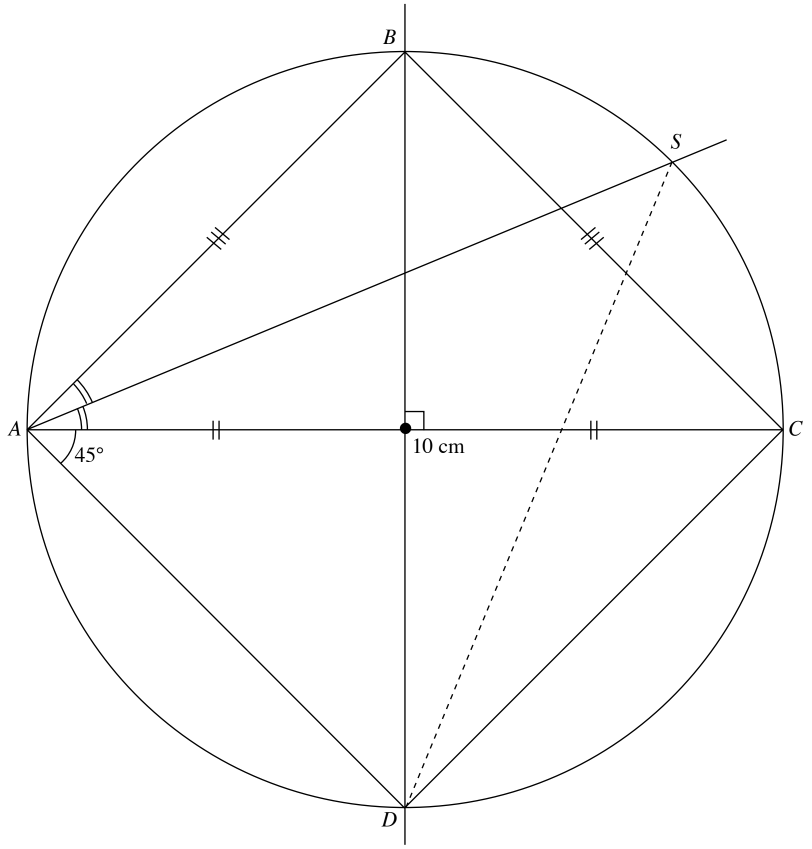
5.



(i) Required angle, $Q\hat{R}S = 123°$

(ii) Length of QU = 6.5 cm

6.



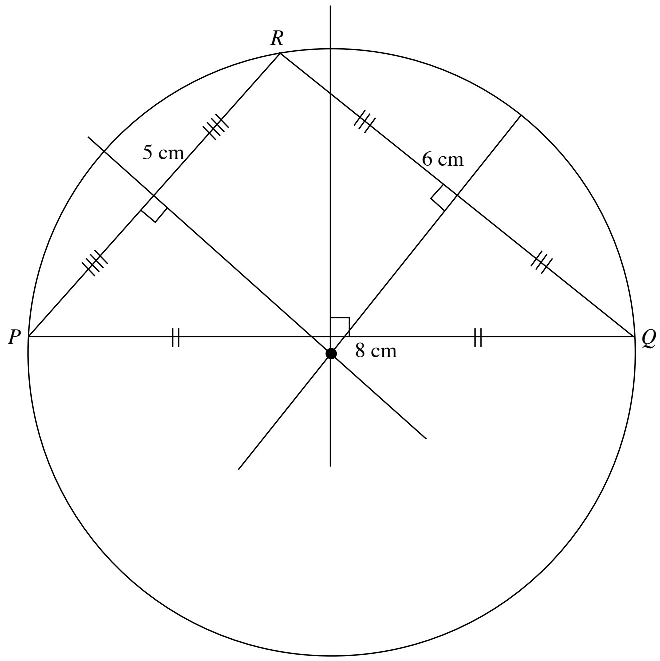
(iii) $AB = BC = AD = CD = 7.1$ cm

$ABCD$ is a square.

(iv) Length of $DS = 9.3$ cm

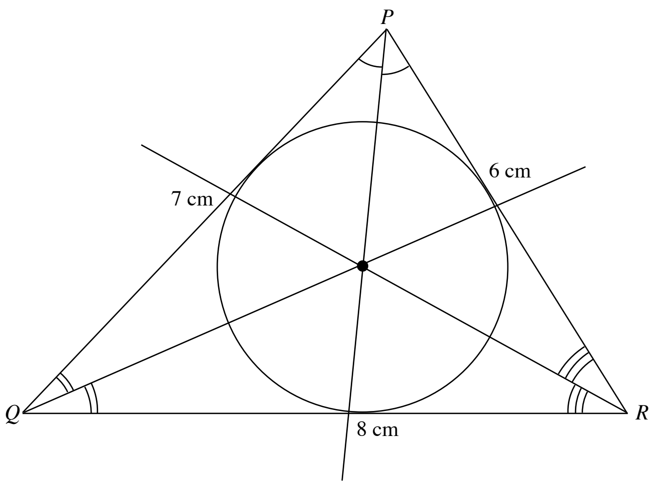
Challenge Yourself

1.



Circumcircle

2.



Incircle

Revision Exercise C1

1. 35% of students = 140

1% of students = $\frac{140}{35}$

100% of students = $\frac{140}{35} \times 100$

= 400

∴ The total number of students who take part in the competition is 400.

2. Value first obtained = (100 – 15)% of 5600

= 85% × 5600

= $\frac{85}{100} \times 5600$

= 4760

110% of 4760 = $\frac{1100}{100} \times 4760$

= 5236

∴ The final number is 5236.

3. $\frac{\text{Height of hall}}{28} = \frac{6}{7}$

Height of hall = $\frac{6}{7} \times 28$

= 24 m

Ratio of breadth of hall to height of hall = 21 : 24

= 7 : 8

4. 1035 hours $\xrightarrow{\text{53 minutes}}$ 1128 hours

Number of words in the report = $\frac{53}{25} \times 575$

= 1219

5. **(i)** 0845 hours $\xrightarrow{\text{6 hours 25 minutes}}$ 1510 hours

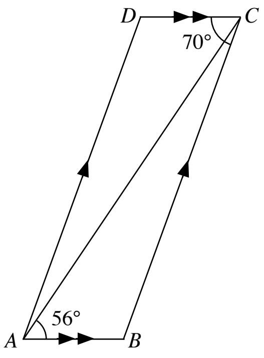
The train takes 6 hours 25 minutes to travel from Town *A* to Town *B*.

(ii) Distance between Town *A* and Town *B* = $108 \times 6\frac{25}{60}$

= $108 \times 6\frac{5}{12}$

= 693 km

6.



(i) $A\hat{B}C + 70° = 180°$ (int. ∠s, *AB* // *DC*)

$A\hat{B}C = 180° - 70°$

= 110°

(ii) $A\hat{C}D = 56°$ (alt. ∠s, *AB* // *DC*)

$A\hat{C}B = 70° - A\hat{C}D$

= 70° – 56°

= 14°

7. **(i)** $(2x + 17)° + (3x - 25)° + (2x + 49)° + (x + 40)° + (4x - 17)° + (3x - 4)° = (6 - 2) \times 180°$

$2x° + 17° + 3x° - 25° + 2x° + 49° + x° + 40° + 4x° - 17° + 3x° - 4°$
$= 4 \times 180°$

$2x° + 3x° + 2x° + x° + 4x° + 3x°$
$= 720° - 17° + 25° - 49° - 40° + 17° + 4°$

$15x° = 660°$

$x° = 44°$

∴ $x = 44$

(ii) Smallest interior angle = $(x + 40)°$

= (44 + 40)°

= 84°

(iii) Smallest exterior angle

= 180° – largest interior angle (adj. ∠s on a str. line)

= $180° - (4x - 17)°$

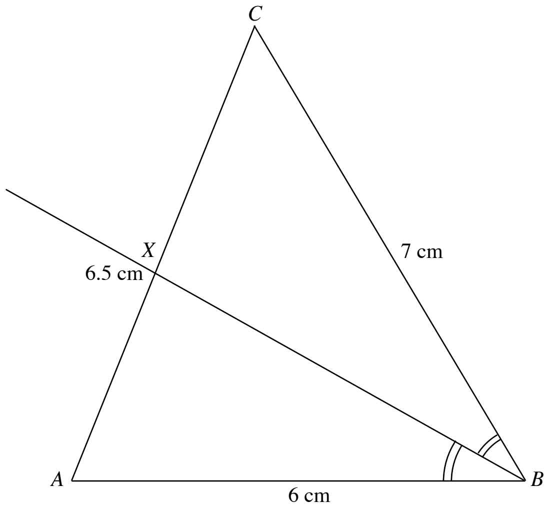
= 180° – [4(44) – 17]°

= 180° – (176 – 17)°

= 180° – 159°

= 21°

8. **(i)**



(ii) Length of *BX* = 5.6 cm

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Revision Exercise C2

1. Percentage increase in salary $= \frac{3780 - 3500}{3500} \times 100\%$

$= \frac{280}{3500} \times 100\%$

$= 8\%$

2. Percentage of students who did not have to stay back $= 100\% - 25\%$

$= 75\%$

75% of 40 $= \frac{75}{100} \times 40$

$= 30$

$\therefore$ 30 students did not have to stay back for detention.

3. $X : Y = 8 : 15$

$= 56 : 105$

$Y : Z = 21 : 32$

$= 105 : 160$

$\therefore X : Z = 56 : 160$

$= 7 : 20$

4. Number of times light can circle the world $= \frac{10}{4} \times 31$

$= 77\frac{1}{2}$

5. 0845 hours $\xrightarrow{\text{3 hours 45 minutes}}$ 1230 hours

Distance between Town A and Town $B = 52 \times 3\frac{45}{60}$

$= 52 \times 3\frac{3}{4}$

$= 195$ km

1455 hours $\xrightarrow{\text{3 hours 20 minutes}}$ 1815 hours

Average speed of lorry on the return journey $= \frac{195}{3\frac{20}{60}}$

$= \frac{195}{3\frac{1}{3}}$

$= 58\frac{1}{2}$ km/h

6. $26° + x° = 180°$ (adj. ∠s on a str. line)

$x° = 180° - 26°$

$= 154°$

$\therefore x = 154$

$B\hat{A}D = 62°$ (alt. ∠s, AB // CD)

$B\hat{A}Q = B\hat{A}D - 26°$

$= 62° - 26°$

$= 36°$

$R\hat{S}A = S\hat{A}Q$ (alt. ∠s, PQ // RS)

$= 36°$

$R\hat{S}A + y° + y° = 180°$ (adj. ∠s on a str. line)

$36° + y° + y° = 180°$

$y° + y° = 180° - 36°$

$2y° = 144°$

$y° = 72°$

$\therefore y = 72$

7. $95° + (n - 1) \times 169° = (n - 2) \times 180°$

$95 + 169n - 169 = 180n - 360$

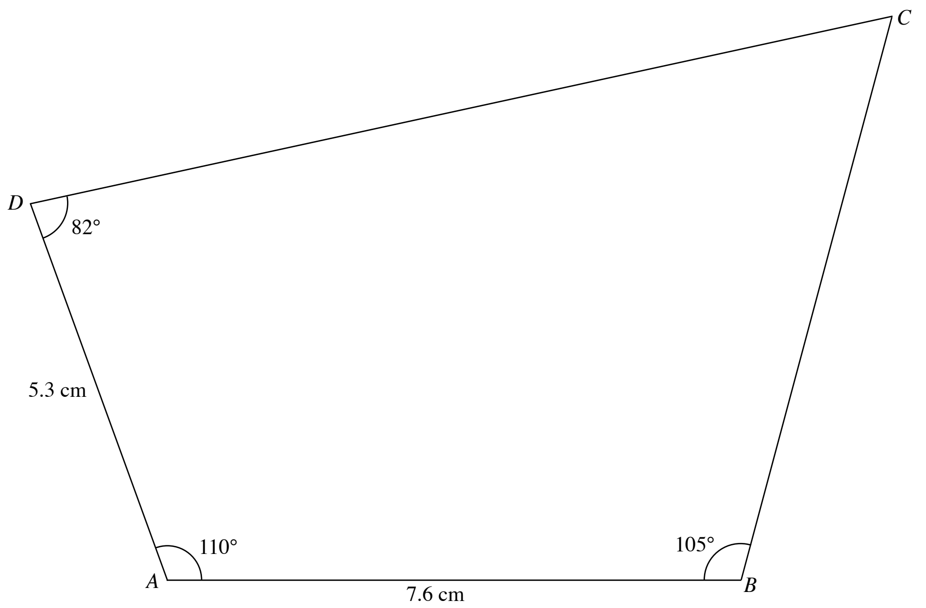
$169n - 180n = -360 - 95 + 169$

$-11n = -286$

$\therefore n = 26$

8. Length of $BC = 7.7$ cm

Length of $CD = 11.7$ cm



Chapter 13 Perimeter and Area of Plane Figures

TEACHING NOTES

Suggested Approach

In the previous chapter, students have learnt the construction of plane figures such as triangles and quadrilaterals. Here, they will learn how to convert units of area, as well as find the perimeter and area of triangles and quadrilaterals. Students will revise what they have learnt in primary school as well as learn the perimeter and area of parallelograms and trapeziums. Teachers should place more focus on the second half of the chapter and ensure students are able to solve problems involving the perimeter and area of parallelograms and trapeziums.

Section 13.1: Conversion of Units

Teachers may wish to recap with the students the conversion of unit lengths from one unit of measurement to another (i.e. mm, cm, m and km) before moving onto the conversion of units for areas.

Teachers may ask students to remember simple calculations such as $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$ to help them in their calculations when they solve problems involving the conversion of units.

Section 13.2: Perimeter and Area of Basic Plane Figures

This section is a recap of what students have learnt in primary school. Students are reminded to be clear of the difference in the units used for perimeter and area (e.g. cm and cm^2).

Teachers can impress upon the students that the value of π in calculators is used when its value is not stated in the question. Unless specified, all answers that are not exact should be rounded off to 3 significant figures.

Section 13.3: Perimeter and Area of Parallelograms

Teachers should illustrate the dimensions of a parallelogram to the students so that they are able to identify the base and height of parallelograms. It is important to emphasise to the students that the height of a parallelogram is with reference to the base and it must be perpendicular to the base chosen. Also, the height may lie within, or outside of the parallelogram. Teachers can highlight to the students that identifying the height of a parallelogram is similar to identifying the height of a triangle.

Teachers should guide students in finding the formula for the area of a parallelogram (see Investigation: Formula for Area of a Parallelogram). Both possible methods should be shown to students (The second method involves drawing the diagonal of the parallelogram and finding the area of the two triangles).

Section 13.4: Perimeter and Area of Trapeziums

Teachers should recap with students the properties of a trapezium. Unlike the parallelogram, the base of the trapezium is not required and the height must be with reference to the two parallel sides of the trapezium. Thus, the height lies either inside the trapezium, or it is one of its sides (this occurs in a right trapezium, where two adjacent angles are right angles).

Teachers should guide students in finding the formula for the area of a trapezium (see Investigation: Formula for Area of a Trapezium). Both possible methods should be shown to students (Again, the second method involves drawing the diagonal of the trapezium and finding the area of the two triangles).

Teachers can enhance the students' understanding and appreciation of the areas of parallelograms and trapeziums by showing them the link between the area of a trapezium, a parallelogram and a triangle (see Thinking Time on page 329).

WORKED SOLUTIONS

Class Discussion (International System of Units)

1. The seven basic physical quantities and their base units are shown in the following table:

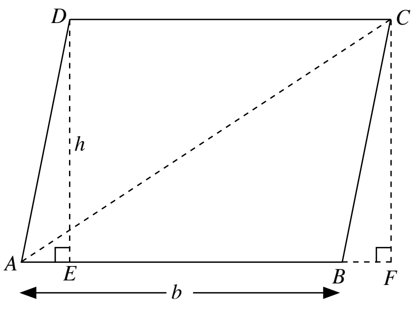
Basic Physical Quantity	Base Unit
Length	metre (m)
Mass	kilogram (kg)
Time	second (s)
Electric current	ampere (A)
Thermodynamic Temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Scientists developed the International System of Units (SI units) so that there is a common system of measures which can be used worldwide.

2. Measurements of Lengths:
1 foot (ft) = 0.3048 m
1 inch (in) = 0.0254 m
1 yard (yd) = 0.9144 m
1 mile = 1609.344 m
Measurement of Areas:
1 acre = 4046.8564 m^2

Investigation (Formula for Area of a Parallelogram)

1. The new quadrilateral *CDEF* is a rectangle.
2. Length of CF = length of $DE = h$
Length of EF = length of EB + length of BF
= length of EB + length of AE
$= b$
3. Area of parallelogram $ABCD$ = area of rectangle $CDEF$
$= EF \times CF$
$= bh$
4. Divide the parallelogram $ABCD$ into two triangles ABC and ADC by drawing the diagonal AC as shown below:



Length of CF = length of $DE = h$

Area of parallelogram $ABCD$ = area of $\triangle ABC$ + area of $\triangle ADC$

$$= \frac{1}{2} \times AB \times CF + \frac{1}{2} \times DC \times DE$$

$$= \frac{1}{2}bh + \frac{1}{2}bh$$

$$= bh$$

Thinking Time (Page 325)

From the geometry software template 'Area of Parallelogram', we can conclude that the formula for the area of parallelogram is also applicable to oblique parallelograms.

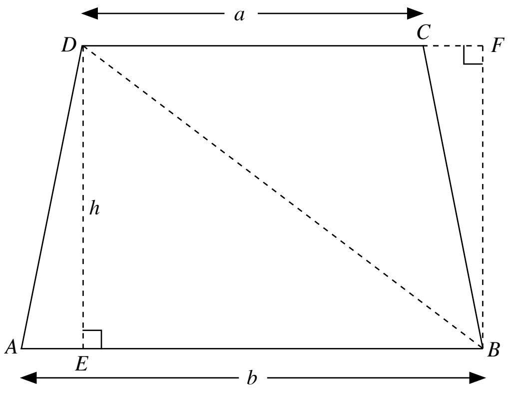
Investigation (Formula for Area of a Trapezium)

1. The new quadrilateral *AFGD* is a parallelogram.
2. Length of AF = length of AB + length of EF
$= b + a$
$= a + b$
3. Area of trapezium $ABCD = \frac{1}{2} \times$ area of parallelogram $AFGD$

$$= \frac{1}{2} \times AF \times h$$

$$= \frac{1}{2}(a + b)h$$

4. **Method 1:**
Divide the trapezium $ABCD$ into two triangles ABD and DCB by drawing the diagonal BD as shown below:



Length of FB = length of $DE = h$

Area of trapezium $ABCD$ = area of $\triangle ABD$ + area of $\triangle DCB$

$$= \frac{1}{2} \times AB \times DE + \frac{1}{2} \times DC \times FB$$

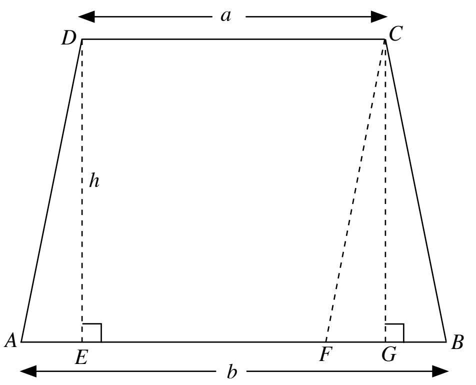
$$= \frac{1}{2} \times b \times h + \frac{1}{2} \times a \times h$$

$$= \frac{1}{2}(b + a)h$$

$$= \frac{1}{2}(a + b)h$$

Method 2:

Divide the trapezium *ABCD* into a parallelogram *AFCD* and a triangle *FBC* by drawing a line *FC* // *AD* as shown below:



Length of *CG* = length of *DE* = h

Length of *AF* = length of *DC* = a

$\therefore$ Length of *FB* = length of *AB* – length of *AF*

$= b - a$

Area of trapezium *ABCD*

= area of parallelogram *AFCD* + area of $\triangle FBC$

$= AF \times DE + \frac{1}{2} \times FB \times CG$

$= a \times h + \frac{1}{2} \times (b - a) \times h$

$= \frac{1}{2}(2a + b - a)h$

$= \frac{1}{2}(a + b)h$

Teachers may wish to get higher-ability students to come up with more methods to find a formula for the area of a trapezium.

Thinking Time (Page 329)

1. **(i)** The new figure is a parallelogram.

(ii) Area of trapezium = $\frac{1}{2}(a + b)h$

When $a = b$,

$\frac{1}{2}(a + b)h = \frac{1}{2}(b + b)h$

$= \frac{1}{2}(2b)h$

$= bh$

= area of parallelogram

2. **(i)** The new figure is a triangle.

(ii) Area of trapezium = $\frac{1}{2}(a + b)h$

When $a = 0$,

$\frac{1}{2}(a + b)h = \frac{1}{2}(0 + b)h$

$= \frac{1}{2}bh$

= area of triangle

Practise Now 1

(a) $16 \text{ m}^2 = 16 \times 10\,000 \text{ cm}^2$

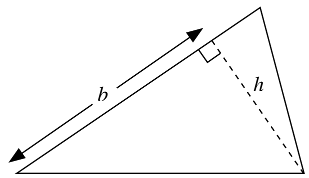
$= 160\,000 \text{ cm}^2$

(b) $357 \text{ cm}^2 = 357 \times 0.0001 \text{ m}^2$

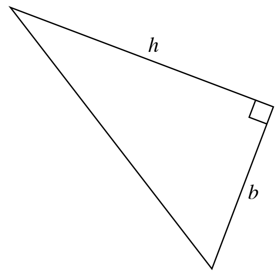
$= 0.0357 \text{ m}^2$

Practise Now (Page 318)

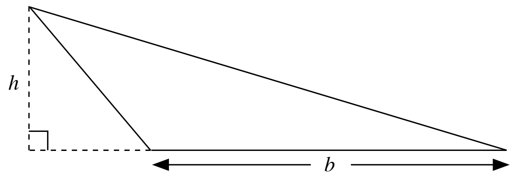
(a)



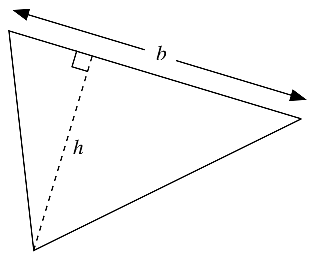
(b)



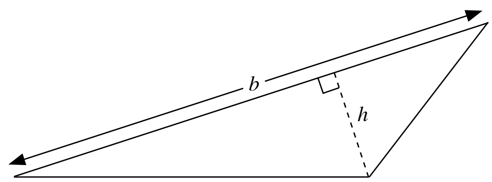
(c)



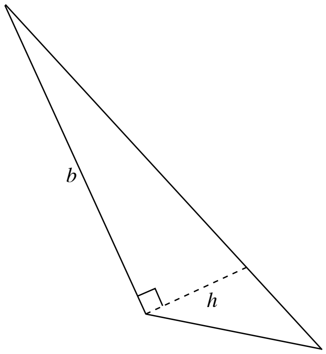
(d)



(e)



(f)



Practise Now 2

1. Length of each side of square field $= \dfrac{64}{4}$

$= 16$ m

Area of field $= 16^2$

$= 256\text{ m}^2$

Area of path $= (16 + 3.5 + 3.5)^2 - 256$

$= 23^2 - 256$

$= 529 - 256$

$= 273\text{ m}^2$

2. Area of shaded region

= area of rectangle $ABCD$ – area of $\triangle ARQ$ – area of $\triangle BRS$ – area of $\triangle CPS$ – area of $\triangle DPQ$

$= 25 \times 17 - \frac{1}{2} \times (25 - 14) \times 5 - \frac{1}{2} \times 14 \times 3$

$- \frac{1}{2} \times (25 - 8) \times (17 - 3) - \frac{1}{2} \times (17 - 5) \times 8$

$= 425 - \frac{1}{2} \times 11 \times 5 - 21 - \frac{1}{2} \times 17 \times 14 - \frac{1}{2} \times 12 \times 8$

$= 425 - 27\frac{1}{2} - 21 - 119 - 48$

$= 209\frac{1}{2}\text{ m}^2$

Practise Now 3

(i) Perimeter of unshaded region $= \frac{3}{4} \times 2\pi(14) + 2(14)$

$= 21\pi + 28$

$= 94.0$ cm (to 3 s.f.)

(ii) Area of unshaded region $= \frac{3}{4} \times \pi(14)^2$

$= 147\pi$

$= 462\text{ cm}^2$ (to 3 s.f.)

(iii) Area of shaded region = area of square – area of unshaded region

$= (2 \times 14)^2 - 147\pi$

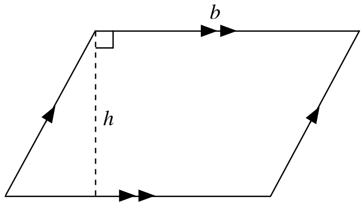
$= 28^2 - 147\pi$

$= 784 - 147\pi$

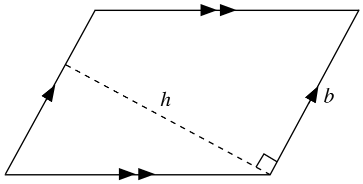
$= 322\text{ cm}^2$ (to 3 s.f.)

Practise Now (Page 324)

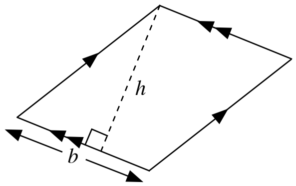
(a)



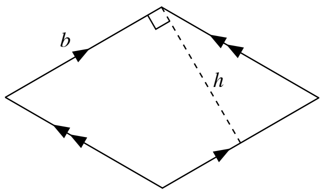
(b)



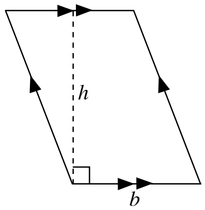
(c)



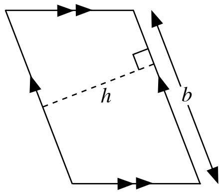
(d)



(e)



(f)



Practise Now 4

(i) Area of parallelogram $= 24 \times 7$

$= 168\text{ m}^2$

(ii) Perimeter of parallelogram $= 2(30 + 7)$

$= 2(37)$

$= 74$ m

Practise Now 5

$$\begin{aligned}\text{Area of parallelogram} = PQ \times ST &= 480 \text{ m}^2\\ 20 \times ST &= 480\\ ST &= 24\end{aligned}$$

Length of ST = 24 m

Practise Now 6

1. Total area of shaded regions

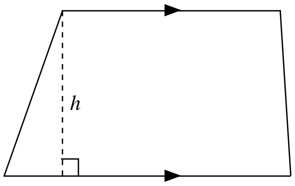
$= \text{area of parallelogram } ABJK + \text{area of parallelogram } CDIJ$
$\quad + \text{area of parallelogram } DEGH$
$= 4 \times 12 + (2 \times 4) \times 12 + 4 \times 12$
$= 48 + 8 \times 12 + 48$
$= 48 + 96 + 48$
$= 192 \text{ m}^2$

2. $$\begin{aligned}\text{Area of } \triangle CDF = \frac{1}{2} \times DC \times CF &= 60 \text{ cm}^2\\ \frac{1}{2} \times DC \times 3CG &= 60\\ \frac{3}{2} \times DC \times CG &= 60\\ DC \times CG &= 40\end{aligned}$$

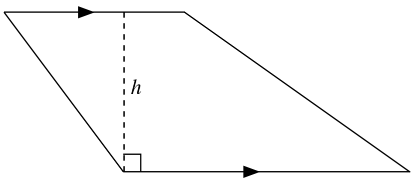
$$\begin{aligned}\text{Area of parallelogram } ABCD &= DC \times CG\\ &= 40 \text{ cm}^2\end{aligned}$$

Practise Now (Page 328)

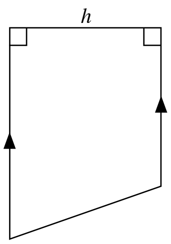
(a)



(b)



(c)



Practise Now 7

(i) $$\begin{aligned}\text{Area of trapezium} &= \frac{1}{2} \times (5 + 13.2) \times 4\\ &= \frac{1}{2} \times 18.2 \times 4\\ &= 36.4 \text{ m}^2\end{aligned}$$

(ii) $$\begin{aligned}\text{Perimeter of trapezium} &= 5 + 6 + 13.2 + 5.5\\ &= 29.7 \text{ m}\end{aligned}$$

Practise Now 8

(i) $$\begin{aligned}\text{Area of trapezium} = \frac{1}{2} \times (PQ + RS) \times PS &= 72 \text{ m}^2\\ \frac{1}{2} \times (14 + 10) \times PS &= 72\\ \frac{1}{2} \times 24 \times PS &= 72\\ 12 \times PS &= 72\\ PS &= 6\end{aligned}$$

Length of PS = 6 m

(ii) $$\begin{aligned}\text{Perimeter of trapezium} = PQ + QR + RS + PS &= 37.2 \text{ m}\\ 14 + QR + 10 + 6 &= 37.2\\ 30 + QR &= 37.2\\ QR &= 7.2\end{aligned}$$

Length of QR = 7.2 m

Practise Now 9

$$\begin{aligned}\text{Area of figure} &= \text{area of trapezium} + \text{area of semicircle}\\ &= \frac{1}{2} \times (48 + 16) \times 20 + \frac{1}{2}\pi\left(\frac{1}{2}\sqrt{1424}\right)^2\\ &= \frac{1}{2} \times 64 \times 20 + \frac{1}{2}\pi \times 356\\ &= 640 + 178\pi\\ &= 1200 \text{ m}^2 \text{ (to 3 s.f.)}\end{aligned}$$

Exercise 13A

1. **(a)** $40 \text{ m}^2 = 40 \times 10\,000 \text{ cm}^2$
$= 400\,000 \text{ cm}^2$

(b) $16 \text{ cm}^2 = 16 \times 0.0001 \text{ m}^2$
$= 0.0016 \text{ m}^2$

(c) $0.03 \text{ m}^2 = 0.03 \times 10\,000 \text{ cm}^2$
$= 300 \text{ cm}^2$

(d) $28\,000 \text{ cm}^2 = 28\,000 \times 0.0001 \text{ m}^2$
$= 2.8 \text{ m}^2$

2. **(i)** $$\begin{aligned}\text{Breadth of rectangle} &= \frac{259}{18.5}\\ &= 14 \text{ cm}\end{aligned}$$

(ii) $$\begin{aligned}\text{Perimeter of rectangle} &= 2(18.5 + 14)\\ &= 2(32.5)\\ &= 65 \text{ cm}\end{aligned}$$

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3. Area of figure = area of square – area of triangle

$$= 9^2 - \frac{1}{2} \times 3 \times 2.5$$
$$= 81 - 3.75$$
$$= 77.25 \text{ m}^2$$

4. **(a)** Diameter of circle = 2×10
$= 20$ cm

Circumference of circle = $2\pi(10)$
$= 20\pi$
$= 62.8$ cm (to 3 s.f.)

Area of circle = $\pi(10)^2$
$= 100\pi$
$= 314 \text{ cm}^2$ (to 3 s.f.)

(b) Radius of circle = $\frac{3.6}{2}$
$= 1.8$ m

Circumference of circle = $2\pi(1.8)$
$= 3.6\pi$
$= 11.3$ m (to 3 s.f.)

Area of circle = $\pi(1.8)^2$
$= 3.24\pi$
$= 10.2 \text{ m}^2$ (to 3 s.f.)

(c) Radius of circle = $\frac{176}{2\pi}$
$= \frac{88}{\pi}$
$= 28.0$ mm (to 3 s.f.)

Diameter of circe = $2 \times \frac{88}{\pi}$
$= \frac{176}{\pi}$
$= 56.0$ mm (to 3 s.f.)

Area of circle = $\pi\left(\frac{88}{\pi}\right)^2$
$= \pi\left(\frac{7744}{\pi^2}\right)$
$= \frac{7744}{\pi}$
$= 2460 \text{ mm}^2$ (to 3 s.f.)

(d) Radius of circle = $\sqrt{\frac{616}{\pi}}$
$= 14.0$ cm (to 3 s.f.)

Diameter of circle = $2 \times \sqrt{\frac{616}{\pi}}$
$= 28.0$ cm (to 3 s.f.)

Circumference of circle = $2\pi\left(\sqrt{\frac{616}{\pi}}\right)$
$= 88.0$ cm (to 3 s.f.)

5. Let the diameter of the semicircle be x cm.

$$\frac{1}{2} \times \pi \times x + x = 144$$
$$\frac{1}{2} \times \frac{22}{7} \times x + x = 144$$
$$\frac{11}{7}x + x = 144$$
$$\frac{18}{7}x = 144$$
$$x = 56$$

$\therefore$ Diameter of semicircle = 56 cm
= 0.56 m

6. **(a)** **(i)** Perimeter of figure = $2\pi\left(\frac{21}{2}\right) + 2(36 - 21)$
$= 2\pi(10.5) + 2(15)$
$= 21\pi + 30$
$= 96.0$ cm (to 3 s.f.)

(ii) Area of figure = area of two semicircles + area of rectangle
$= \pi(10.5)^2 + 15 \times 21$
$= 110.25\pi + 315$
$= 661 \text{ cm}^2$ (to 3 s.f.)

(b) **(i)** Perimeter of figure = $\frac{1}{2} \times 2\pi(5) + 2(5) + \sqrt{200}$
$= 5\pi + 10 + \sqrt{200}$
$= 39.9$ cm (to 3 s.f.)

(ii) Area of figure = area of semicircle + area of triangle
$= \frac{1}{2} \times \pi(5)^2 + \frac{1}{2} \times 10 \times 10$
$= \frac{25}{2}\pi + 50$
$= 89.3 \text{ cm}^2$ (to 3 s.f.)

(c) **(i)** Perimeter of figure = $\frac{1}{2} \times 2\pi\left(\frac{18}{2}\right) + 2\pi\left(\frac{18}{4}\right)$
$= \frac{1}{2} \times 2\pi(9) + 2\pi(4.5)$
$= 9\pi + 9\pi$
$= 18\pi$
$= 56.5$ cm (to 3 s.f.)

(ii) Area of figure
= area of big semicircle + area of two small semicircles
$= \frac{1}{2} \times \pi(9)^2 + \pi(4.5)^2$
$= \frac{81}{2}\pi + 20.25\pi$
$= 60.75\pi$
$= 191 \text{ cm}^2$ (to 3 s.f.)

7. **(i)** Perimeter of figure = $2\pi(2) + 2(9 - 2 \times 2) + 2(3)$
$= 4\pi + 2(5) + 6$
$= 4\pi + 10 + 6$
$= 4\pi + 16$
$= 28.6$ m (to 3 s.f.)

(ii) Area of figure = area of rectangle – area of four quadrants

$= 9 \times [2(2) + 3] - \pi(2)^2$

$= 9 \times 7 - 4\pi$

$= 63 - 4\pi$

$= 50.4 \text{ m}^2$ (to 3 s.f.)

8. Let the breadth of the rectangular field be x m.

Then the length of the field is $(x + 15)$ m.

$2[(x + 15) + x] = 70$

$2(2x + 15) = 70$

$2x + 15 = 35$

$2x = 20$

$x = 10$

$\therefore$ Breadth of field = 10 m

Length of field = 10 + 15

= 25 m

Area of field = 25×10

= 250 m^2

Area of path $= (25 + 2.5 + 2.5) \times (10 + 5 + 5) - 250$

$= 30 \times 20 - 250$

$= 600 - 250$

$= 350 \text{ m}^2$

9. Area of shaded region = area of quadrilateral $PQRS$

$= \frac{1}{2} \times AR \times RP + \frac{1}{2} \times RB \times RP$

$= \frac{1}{2} \times RP \times (AR + RB)$

$= \frac{1}{2} \times AD \times AB$

$= \frac{1}{2} \times 23 \times (7 + 13.5)$

$= \frac{1}{2} \times 20.5 \times 23$

$= 235.75 \text{ m}^2$

10. Area of shaded region = area of $\triangle ABC$ – area of $\triangle ADE$

$= \frac{1}{2} \times 20 \times 21 - \frac{1}{2} \times 10 \times 10.5$

$= 210 - 52.5$

$= 157.5 \text{ m}^2$

11. Area of $\triangle ACD = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times CD \times AE$

$\frac{1}{2} \times 20 \times BD = \frac{1}{2} \times 22 \times 16$

$10 \times BD = 176$

$BD = 17.6$

Length of BD = 17.6 cm

12. (i) Area of surface of circular pond $= \pi\left(\frac{12}{2}\right)^2$

$= \pi(6)^2$

$= 36\pi$

$= 113 \text{ m}^2$ (to 3 s.f.)

(ii) Area of path $= \pi(6 + 2)^2 - 36\pi$

$= \pi(8)^2 - 36\pi$

$= 64\pi - 36\pi$

$= 28\pi \text{ m}^2$

Cost incurred $= 28\pi \times \$55$

= \$4838.05 (to the nearest cent)

13. (i) Perimeter of figure $= \frac{1}{2} \times 2\pi\left(\frac{7}{2}\right) + 2(5.7)$

$= \frac{1}{2} \times 2\pi(3.5) + 11.4$

$= 3.5\pi + 11.4$

$= 22.4$ cm (to 3 s.f.)

(ii) Area of figure = area of semicircle BCD + area of $\triangle ABD$

$= \frac{1}{2} \times \pi(3.5)^2 + \frac{1}{2} \times 7 \times (8 - 3.5)$

$= 6.125\pi + \frac{1}{2} \times 7 \times 4.5$

$= 6.125\pi + 15.75$

$= 35.0 \text{ cm}^2$ (to 3 s.f.)

14. (i) Perimeter of shaded region

$= \frac{3}{4} \times 2\pi(10) + \frac{1}{2} \times 2\pi\left(\frac{10}{2}\right) + \frac{1}{4} \times 2\pi(10 - 3) + 3 + (10 - 3)$

$= 15\pi + \frac{1}{2} \times 2\pi(5) + \frac{1}{4} \times 2\pi(7) + 3 + 7$

$= 15\pi + 5\pi + \frac{7}{2}\pi + 10$

$= \frac{47}{2}\pi + 10$

$= 83.8$ cm (to 3 s.f.)

(ii) Area of shaded region = area of big semicircle
+ area of small semicircle
+ area of region $ABCE$

$= \frac{1}{2} \times \pi(10)^2 + \frac{1}{2} \times \pi(5)^2 + \frac{1}{4} \times \pi(10^2 - 7^2)$

$= 50\pi + \frac{25}{2}\pi + \frac{1}{4} \times \pi(100 - 49)$

$= 50\pi + \frac{25}{2}\pi + \frac{1}{4} \times \pi(51)$

$= 50\pi + \frac{25}{2}\pi + \frac{51}{4}\pi$

$= \frac{301}{4}\pi$

$= 236 \text{ cm}^2$ (to 3 s.f.)

15. (i) Perimeter of shaded region $= \frac{1}{2} \times 2\pi\left(\frac{\sqrt{200}}{2}\right) + 2(10)$

$= \frac{\sqrt{200}}{2}\pi + 20$

$= 42.2$ m (to 3 s.f.)

(ii) Area of shaded region

= area of semicircle BCD – area of $\triangle BCD$

$= \frac{1}{2} \times \pi\left(\frac{\sqrt{200}}{2}\right)^2 - \frac{1}{2} \times 10 \times 10$

$= 25\pi - 50$

$= 28.5 \text{ m}^2$ (to 3 s.f.)

16. Radius of each circle = $\sqrt{\dfrac{0.785}{\pi}}$ cm

Area of shaded region $= \dfrac{1}{2} \times 2\sqrt{\dfrac{0.785}{\pi}} \times \sqrt{\dfrac{0.785}{\pi}}$

$= \dfrac{0.785}{\pi}$

$= 0.250 \text{ cm}^2$ (to 3 s.f.)

17. Area of grass within the goat's reach $= \pi(1.5)^2$

$= 2.25\pi \text{ m}^2$

Time the goat needs $= 2.25\pi \times 14$

$= 99.0$ minutes (to 3 s.f.)

Exercise 13B

1. **(a)** Area of parallelogram $= 12 \times 7$

$= 84 \text{ cm}^2$

(b) Base of parallelogram $= \dfrac{42}{6}$

$= 7$ m

(c) Height of parallelogram $= \dfrac{42.9}{7.8}$

$= 5.5$ mm

2. **(a)** Area of trapezium $= \dfrac{1}{2} \times (7 + 11) \times 6$

$= \dfrac{1}{2} \times 18 \times 6$

$= 54 \text{ cm}^2$

(b) Height of trapezium $= \dfrac{126}{\frac{1}{2} \times (8 + 10)}$

$= \dfrac{126}{\frac{1}{2} \times 18}$

$= \dfrac{126}{9}$

$= 14$ m

(c) Length of parallel side 2 of trapezium $= \dfrac{72}{\frac{1}{2} \times 8} - 5$

$= \dfrac{72}{4} - 5$

$= 18 - 5$

$= 13$ mm

3. **(i)** Area of parallelogram $= 6 \times 9$

$= 54 \text{ cm}^2$

(ii) Perimeter of parallelogram $= 2(10 + 6)$

$= 2(16)$

$= 32$ cm

4. Area of parallelogram $= PQ \times ST = QR \times SU$

$PQ \times 8 = 10 \times 11.2$

$PQ \times 8 = 112$

$PQ = 14$

Length of $PQ = 14$ m

5. **(i)** Area of trapezium $= \dfrac{1}{2} \times (35.5 + 20) \times 15$

$= \dfrac{1}{2} \times 55.5 \times 15$

$= 416.25 \text{ cm}^2$

(ii) Perimeter of trapezium $= 35.5 + 18 + 20 + 16$

$= 89.5$ cm

6. **(i)** Area of trapezium $= \dfrac{1}{2} \times (PQ + RS) \times PT = 150 \text{ m}^2$

$\dfrac{1}{2} \times (12 + RS) \times 10 = 150$

$5 \times (12 + RS) = 150$

$12 + RS = 30$

$RS = 18$

Length of $RS = 18$ m

(ii) Perimeter of trapezium $= PQ + QR + RS + PS = 54.7$ m

$12 + QR + 18 + 13 = 54.7$

$43 + QR = 54.7$

$QR = 11.7$

Length of $QR = 11.7$ m

7. Area of shaded regions = area of trapezium $ABCD$ – area of $\triangle BCE$

$= \dfrac{1}{2} \times (10 + 14) \times 12 - \dfrac{1}{2} \times 14 \times 12$

$= \dfrac{1}{2} \times 24 \times 12 - 84$

$= 144 - 84$

$= 60 \text{ cm}^2$

8. Area of parallelogram $ABFG = \dfrac{702}{2}$

$= 351 \text{ m}^2$

Height of parallelogram $ABFG$ with reference to base $FG = \dfrac{351}{27}$

$= 13$ m

Area of shaded region $= \dfrac{1}{2} \times (2 \times 27) \times 13$

$= \dfrac{1}{2} \times 54 \times 13$

$= 351 \text{ m}^2$

9. **(a)** Total area of shaded regions

= area of rectangle – area of parallelogram

– area of circle – area of triangle

$= (12 + 14) \times (15 + 10)$

$- (12 + 14 - 5 - 2) \times 10 - \pi(4)^2 - \dfrac{1}{2} \times 12 \times 15$

$= 26 \times 25 - 19 \times 10 - 16\pi - 90$

$= 650 - 190 - 16\pi - 90$

$= 370 - 16\pi$

$= 320 \text{ cm}^2$ (to 3 s.f.)

(b) Area of shaded region = area of trapezium – area of circle

$= \dfrac{1}{2} \times (35 + 18) \times 18 - \pi(6)^2$

$= \dfrac{1}{2} \times 53 \times 18 - 36\pi$

$= 477 - 36\pi$

$= 364 \text{ cm}^2$ (to 3 s.f.)

10. Area of figure = area of trapezium $ABCE$

– area of parallelogram $GHDE$ – area of semicircle

$$= \frac{1}{2} \times (12 + 13 + 15) \times 24 - 13 \times 16 - \frac{1}{2} \times \pi\left(\frac{15}{2}\right)^2$$
$$= \frac{1}{2} \times 40 \times 24 - 208 - \frac{1}{2} \times \pi(7.5)^2$$
$$= 480 - 208 - 28.125\pi$$
$$= 272 - 28.125\pi$$
$$= 184 \text{ cm}^2 \text{ (to 3 s.f.)}$$

11. Area of $\triangle AED = \frac{1}{2} \times AE \times ED = 25 \text{ cm}^2$

$$AE \times ED = 50$$

Area of trapezium $BCDE = \frac{1}{2} \times (EB + DC) \times ED$
$$= \frac{1}{2} \times (3AE + 4AE) \times ED$$
$$= \frac{1}{2} \times 7AE \times ED$$
$$= \frac{7}{2} \times AE \times ED$$
$$= \frac{7}{2} \times 50$$
$$= 175 \text{ cm}^2$$

12. **(i)** Let the height of the parallelogram $ABCD$ with reference to the base BC be h cm.

Area of parallelogram $ABCD = BC \times h = 80 \text{ cm}^2$

Area of $\triangle ABE = \frac{1}{2} \times BE \times h$
$$= \frac{1}{2} \times 2BC \times h$$
$$= BC \times h$$
$$= 80 \text{ cm}^2$$

(ii) Let the height of the parallelogram $ABCD$ with reference to the base DC be h' cm.

Area of parallelogram $ABCD = DC \times h' = 80 \text{ cm}^2$

Area of $\triangle ADF = \frac{1}{2} \times DF \times h'$
$$= \frac{1}{2} \times \frac{1}{2} DC \times h'$$
$$= \frac{1}{4} \times DC \times h'$$
$$= \frac{1}{4} \times 80$$
$$= 20 \text{ cm}^2$$

Review Exercise 13

1. **(a)** Area of shaded region
$$= 11 \times 13 + 7 \times (14 + 13) + 8 \times (35 - 20) + 9 \times 35 - 12 \times 9$$
$$= 143 + 7 \times 27 + 8 \times 15 + 315 - 108$$
$$= 143 + 189 + 120 + 315 - 108$$
$$= 659 \text{ cm}^2$$

(b) Total area of shaded regions

= area of circle – area of triangle – area of rectangle
$$= \pi(13.6)^2 - \frac{1}{2} \times (2 \times 13.6) \times 13.6 - 16 \times 11$$
$$= 184.96\pi - \frac{1}{2} \times 27.2 \times 13.6 - 176$$
$$= 184.96\pi - 184.96 - 176$$
$$= 184.96\pi - 360.96$$
$$= 220 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(c) Total area of shaded regions
$$= \frac{1}{2} \times (48 + 16) \times 20 + \frac{1}{2} \times (30 + 20) \times 16$$
$$= \frac{1}{2} \times 64 \times 20 + \frac{1}{2} \times 50 \times 16$$
$$= 640 + 400$$
$$= 1040 \text{ cm}^2$$

(d) Area of shaded region = area of trapezium – area of triangle
$$= \frac{1}{2} \times (17 + 9) \times (2 \times 6) - \frac{1}{2} \times 17 \times 6$$
$$= \frac{1}{2} \times 26 \times 12 - 51$$
$$= 156 - 51$$
$$= 105 \text{ cm}^2$$

2. **(i)** Perimeter of shaded region $= \frac{1}{2} \times 2\pi\left(\frac{28}{2}\right) + 2\pi\left(\frac{28}{4}\right)$
$$= \frac{1}{2} \times 2\pi(14) + 2\pi(7)$$
$$= 14\pi + 14\pi$$
$$= 28\pi$$
$$= 88.0 \text{ cm (to 3 s.f.)}$$

(ii) Area of shaded region

= area of big semicircle – area of two small semicircles
$$= \frac{1}{2} \times \pi(14)^2 - \pi(7)^2$$
$$= 98\pi - 49\pi$$
$$= 49\pi$$
$$= 154 \text{ cm}^2 \text{ (to 3 s.f.)}$$

3. Area of shaded region = area of one square of sides (2×12) cm
$$= (2 \times 12)^2$$
$$= 24^2$$
$$= 576 \text{ cm}^2$$

4. **(i)** Area of parallelogram $= 9 \times 25$
$$= 225 \text{ m}^2$$

(ii) Perimeter of parallelogram $= 2(9 + 30.8)$
$$= 2(39.8)$$
$$= 79.6 \text{ m}$$

5. Let $AB = BC = CD = DE = EF = AF = x$ cm.
$$(x + x) \times x = 24$$
$$2x \times x = 24$$
$$2x^2 = 24$$
$$x^2 = 12$$

Since $x > 0$, $x = \sqrt{12}$

Area of parallelogram $BCEF = \sqrt{12} \times \sqrt{12}$
$$= 12 \text{ cm}^2$$

6. Area of trapezium $ABPQ = \frac{1}{2} \times (8 + 8 \div 2) \times (6 \div 2)$

$= \frac{1}{2} \times (8 + 4) \times 3$

$= \frac{1}{2} \times 12 \times 3$

$= 18 \text{ cm}^2$

7. Area of figure

= area of rectangle $ABCF$ + area of trapezium $FCDE$

$= 20 \times 15 + \frac{1}{2} \times (20 + 3.5) \times 7$

$= 300 + \frac{1}{2} \times 23.5 \times 7$

$= 300 + 82.25$

$= 382.25 \text{ m}^2$

$= 382.25 \times 0.0001$ ha

$= 0.038\,225$ ha

8. **(i)** $x + y = \dfrac{36}{\frac{1}{2} \times 6}$

$= \frac{36}{3}$

$= 12$

(ii) Since $x = 2y$,

$2y + y = 12$

$3y = 12$

$y = 4$

$\therefore x = 2 \times 4$

$= 8$

9. Length of each side of square $= \sqrt{1}$

$= 1$ m

Perimeter of square $= 4 \times 1$

$= 4$ m

Radius of circle $= \sqrt{\frac{1}{\pi}}$ m

Circumference of circle $= 2\pi\left(\sqrt{\frac{1}{\pi}}\right)$ m

Required difference $= 4 - 2\pi\left(\sqrt{\frac{1}{\pi}}\right)$

$= 0.455$ m (to 3 s.f.)

10. Circumference of drum $= 2\pi\left(\frac{21}{2}\right)$

$= 21\pi$ cm

Number of complete turns of handle required

$= \frac{9.89 \times 100}{21\pi}$

$= \frac{989}{21\pi}$

= 15 (rounded up to the nearest whole number)

Challenge Yourself

1. Let the length of AB be x cm.

Then the length of BC = the length of $AC = 2x$ cm.

Area of $\triangle ABC$

= area of $\triangle ABD$ + area of $\triangle BCD$ + area of $\triangle ACD$

$= \frac{1}{2} \times x \times 9 + \frac{1}{2} \times 2x \times 7 + \frac{1}{2} \times 2x \times 7$

$= 4.5x + 7x + 7x$

$= 18.5x \text{ cm}^2$

Case 1: The base of $\triangle ABC$ is taken to be AB.

$\frac{1}{2} \times x \times h_1 = 18.5x$

$\therefore h_1 = 37$ cm

Case 2: The base of $\triangle ABC$ is taken to be BC or AC.

$\frac{1}{2} \times 2x \times h_2 = 18.5x$

$\therefore h_2 = 18.5$ cm

2. **(i)** Perimeter of figure $= \pi r_1 + \pi r_2 + \pi r_3 + \pi r_4 + \pi r_5 + AB$

$= \pi(r_1 + r_2 + r_3 + r_4 + r_5) + AB$

$= \pi \times \frac{AB}{2} + AB$

$= \pi \times \frac{70}{2} + 70$

$= 35\pi + 70$

$= 180$ cm (to 3 s.f.)

(ii) Perimeter of figure $= \pi r + AB$

$= \pi \times \frac{AB}{2} + AB$

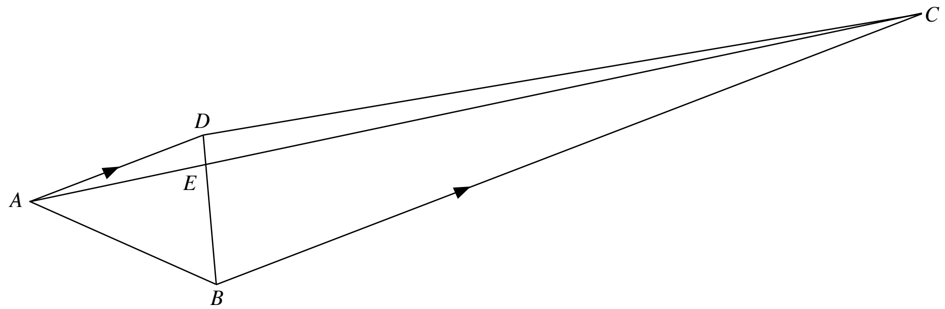
$= \pi \times \frac{70}{2} + 70$

$= 35\pi + 70$

$= 180$ cm (to 3 s.f.)

(iii) Given a line segment AB of fixed length, regardless of the number of semicircles drawn on the line segment, the perimeter of the figure will be the same.

3.



$$\frac{BD}{BE} = \frac{5}{4} \Leftrightarrow \frac{ED}{BE} = \frac{1}{4}$$

$$\frac{\text{Area of } \triangle AED}{\text{Area of } \triangle ABE} = \frac{ED}{BE} = \frac{1}{4}$$

$$\frac{\text{Area of } \triangle AED}{20} = \frac{1}{4}$$

$\therefore$ Area of $\triangle AED = 5\text{ cm}^2$

Since $\triangle ACD$ shares the same base AD and the same height as $\triangle ABD$,

area of $\triangle ACD$ = area of $\triangle ABD$.

Since $\triangle AED$ is a common part of $\triangle ACD$ and $\triangle ABD$,

area of $\triangle DCE$ = area of $\triangle ABE = 20\text{ cm}^2$.

$$\frac{\text{Area of } \triangle BCE}{\text{Area of } \triangle DCE} = \frac{BE}{ED} = \frac{4}{1}$$

$$\frac{\text{Area of } \triangle BCE}{20} = \frac{4}{1}$$

$\therefore$ Area of $\triangle BCE = 80\text{ cm}^2$

Area of trapezium

= area of $\triangle ABE$ + area $\triangle AED$ + area of $\triangle DCE$ + area of $\triangle BCE$

$= 20 + 5 + 20 + 80$

$= 125\text{ cm}^2$

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Chapter 14 Volume and Surface Area of Prisms and Cylinders

TEACHING NOTES

Suggested Approach

Students have learnt the conversion of unit area and perimeter and area of plane figures in the last chapter. This chapter will be dealing with the conversion of unit volumes and the volume and surface area of solids, which is a natural transition from the last chapter, from two-dimensional to three-dimensional. To assist in the students' understanding, teachers should continually remind students to be aware of the linkages between both topics, as well as introducing real-life applications that can reinforce learning.

Section 14.1: Conversion of Units

Teachers should recap the unit conversion of lengths and areas, proceed to introduce of volume by stating actual applications (see Class Discussion: Measurement in Daily Lives), and then stating the different units associated with volume (e.g. m, cm^3 and m^3).

Students should recognise how the number of dimensions and the unit representation for lengths, areas and volumes are related (e.g. cm, cm^2 and cm^3). Students should recall calculations such as $1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$ and solve problems involving conversion of unit volumes.

Section 14.2: Nets

Teachers should first define and explain that nets are basically flattened figures that can be folded to its three-dimensional solids.

Teachers should show the nets of the various solids. Students are encouraged to make their own nets and form the different three-dimensional solids. They should also be able to visualise the solids from different viewpoints.

Section 14.3: Volume and Surface Area of Cubes and Cuboids

Teachers can state that the volume of an object refers to the space it occupies, so the greater the volume, the more space the object occupies.

Students should be informed and know that the volume of cubes and cuboids is the product of its three sides (base × height = (length × breadth) × height).

The formulas for the total surface area of cubes and cuboids can be explored and discovered by students (see Class Discussion: Surface Area of Cubes and Cuboids). It is important for the students to observe that the total surface area is the total area of all its faces.

Section 14.4: Volume and Surface Area of Prisms

Teachers can introduce prisms to the students by stacking a few cubes to form a prism and show them how a prism looks like. Students should know terms like lateral faces and cross-sections, and learn that prisms are solids with uniform polygonal cross-sections. Teachers can ask the students to name some real-life examples of prisms and use this opportunity to get them to explain why certain objects are not prisms so that they can get a better understanding about prisms.

Observant students should realise that cuboids are prisms. Teachers can highlight to the students that prisms do not necessarily have square bases and challenge students to think of bases of other possible shapes (see Fig. 14.2 on page 348).

Teachers should illustrate and derive the formulas for the volume and total surface area. Students need to understand the definitions of volume and total surface area rather than memorise the formulas.

Section 14.5: Volume and Surface Area of Cylinders

Similar to the last section, teachers can introduce cylinders by stacking coins or showing students real-life examples of cylindrical objects. Only right circular cylinders are covered in this syllabus.

Some students may think that cylinders are also prisms since both have uniform cross-sections. Teachers need to impress upon students that this is not the case even though cylinders and prisms share similarities (see Investigation: Comparison between a Cylinder and a Prism).

Teachers should also cover the formulas for the volume and total surface area of cylinders. Again, students need to understand the definitions of volume and total surface area rather than memorise formulas.

Section 14.6: Volume and Surface Area of Composite Solids

Teachers should go through Worked Example 10 closely with students. Other than assessing their understanding, teachers can inform students to be aware of any sides that should be omitted in finding total surface areas.

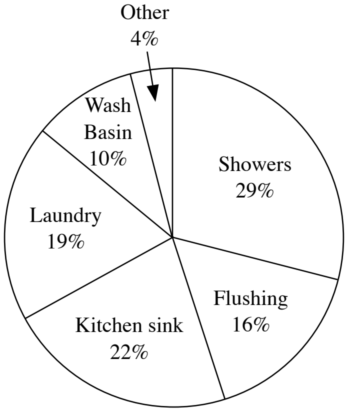
Challenge Yourself

Teachers should challenge students to think how the cross-section of the cuboid looks like in finding the volume and surface area.

WORKED SOLUTIONS

Class Discussion (Measurements in Daily Lives)

1. (i)



Source:
http://www.pub.gov.sg/conserve/Households/Pages/Watersavinghabits.aspx

The activity which requires the greatest amount of water is shower.

(ii) –

Some measures:

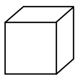
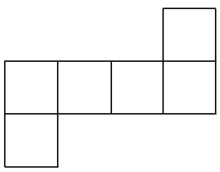
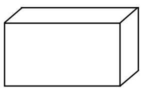
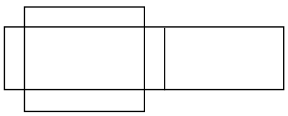
- Take shorter showers.
- Turn off the shower tap while soaping.
- Use a tumbler when brushing your teeth.
- Do not thaw food under running water. Let it defrost overnight inside the refrigerator instead.
- Wash vegetables and dishes in a sink or container filled with water.
- Install thimbles or water saving devices at taps with high flow rate.
- Turn off taps tightly to ensure they do not drip.
- Do not leave the tap running when not in use.

2. (i) The volume of one teaspoon of liquid is 5 ml.

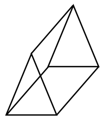
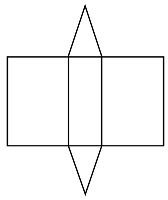
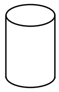
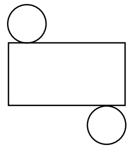
(ii) This corresponds to 2 litres of water.

Investigation (Cubes, Cuboids Prisms and Cylinders)

Part II:

Name	Figure	Net
Cube		
Cuboid		

Part III:

Name	Figure	Net
Triangular Prism		
Cylinder		

Class Discussion (Surface Area of Cubes and Cuboids)

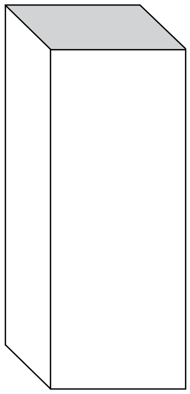
1. A cube has <u>6</u> surfaces. Each surface is in the shape of a <u>square</u>.
   The area of each face is <u>equal</u>.
   $\therefore$ The total surface area of a cube is $6l^2$.
   A cuboid has <u>6</u> surfaces. Each surface is in the shape of a <u>rectangle</u>.
   $\therefore$ The total surface area of a cube is $2(b \times l + b \times h + l \times h)$.
2. The total surface of the object is equal to the total area of all the faces of the net.

Thinking Time (Page 349)

1. (i) The shape of all the lateral faces of a right prism is a rectangle.

(ii) The shape of all the lateral faces of an oblique prism is a parallelogram.

2.



3. Examples of building structures and items are European-style houses and chocolates. They are shaped as prisms as they have a uniform cross-section.

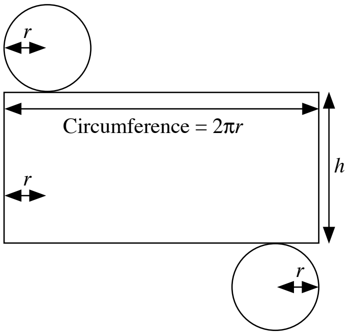
Thinking Time (Page 354)

Examples are can drinks, toilet rolls and iron rods. They are shaped as cylinders as they have a uniform circular cross-section.

Investigation (Comparison between a Cylinder and a Prism)

1. The polygon will become a circle.
2. The prism will become like a cylinder.

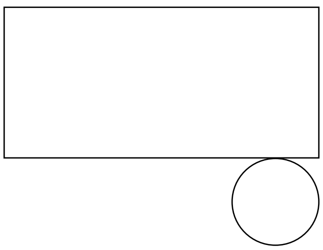
Thinking Time (Page 358)



Total outer surface of a closed cylinder
$= \pi r^2 + 2\pi rh + \pi r^2$
$= 2\pi r^2 + 2\pi rh$

Class Discussion (Total Surface Area of Other Types of Cylinders)

(a) an open cylinder



Total outer surface of an open cylinder $= 2\pi rh + \pi r^2$

(b) a pipe of negligible thinkness

Total outer surface of a pipe of negligible thickness $= 2\pi rh$

Practise Now 1

(a) (i) $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$
$10 \text{ m}^3 = 10 \times 1\,000\,000 \text{ cm}^3$
$= 10\,000\,000 \text{ cm}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$
$10\,000\,000 \text{ cm}^3 = 10\,000\,000 \text{ m}l$
$10 \text{ m}^3 = 10\,000\,000 \text{ m}l$

(b) (i) $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$
$165\,000 \text{ cm}^3 = \dfrac{165\,000}{1\,000\,000} \text{ m}^3$
$= 0.165 \text{ m}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$
$165\,000 \text{ cm}^3 = 165\,000 \text{ m}l$
$= \dfrac{165\,000}{1000}\ l$
$= 165\ l$

Practise Now 2

1. (i) Volume of the cuboid $= l \times 18 \times 38 = 35\,568$
$l = \dfrac{35\,568}{18 \times 38}$
$= 52$

(ii) Volume of each small cube $= 2 \times 2 \times 2 = 8 \text{ cm}^3$
Number of cubes to be obtained
$= \dfrac{35\,568}{8}$
$= 4446$

2. Volume of the open rectangular tank
$= 55 \times 35 \times 36$
$= 69\,300 \text{ cm}^3$
Volume of water in the open rectangular tank initally
$= \dfrac{1}{2} \times 69\,300$
$= 34\,650 \text{ cm}^3$
Total volume of water in the open rectangular tank after 7700 cm^3 of water are added to it
$= 34\,650 + 7700$
$= 42\,350 \text{ cm}^3$
Let the depth of water in the tank be d cm.
$55 \times 35 \times d = 42\,350$
$1925d = 42\,350$
$d = 22$
Depth of water = 22 cm

Practise Now 3

External volume $= (180 + 30 + 30) \times (80 + 30 + 30) \times (120 + 30)$
$= 240 \times 140 \times 150$
$= 5\,040\,000 \text{ cm}^3$
Internal volume $= 180 \times 80 \times 120$
$= 1\,728\,000 \text{ cm}^3$
Volume of concrete used $= 5\,040\,000 - 1\,728\,000$
$= 3\,312\,00 \text{ cm}^3$

Practise Now 4

1. (i) Volume of cuboid $= 8 \times 5 \times 10$
$= 400 \text{ cm}^3$

(ii) Surface area of the cuboid $= 2(8 \times 5 + 8 \times 10 + 5 \times 10)$
$= 340 \text{ cm}^2$

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2. **(i)** Volume of water in the tank
$= 16 \times 9 \times 8$
$= 1152 \text{ cm}^3$
$= 1152 \text{ m}l$
$= \dfrac{1152}{1000}\ l$
$= 1.152\ l$
(ii) Surface area of the tank that is in contact with the water
$= (16 \times 9) + 2(16 \times 8 + 9 \times 8)$
$= 544 \text{ cm}^2$

3. Let the length of the cube be l cm.
$l \times l \times l = 27 \text{ cm}^3$
$l^3 = 27$
$l = \sqrt[3]{27}$
$l = 3$
Total area of the faces that will be coated with paint
$= 6(3 \times 3)$
$= 54 \text{ cm}^2$

Practise Now 5

1. Base area = area of square
$= 4 \times 4$
$= 16 \text{ m}^2$
Volume of the prism = base area × height
$= 16 \times 10$
$= 160 \text{ m}^3$

2. Base area = area of triangle
$= \dfrac{1}{2} \times 5.6 \times x$
$= 2.8x \text{ cm}^2$
Volume of the prism = base area × height = $2.8x \times 12 = 151.2$
$33.6x = 151.2$
$x = 4.5$

Practise Now 6

(i) Volume of the prism = base area × height
$$= \left[\left(\frac{1}{2} \times 3 \times 4\right) + (6 \times 5)\right] \times 4.5$$
$= 36 \times 4.5$
$= 162 \text{ cm}^3$

(ii) Total surface area of the prism
= perimeter of the base × height + 2 × base area
$= (3 + 4 + 6 + 5 + 6) \times 4.5 + 2 \times 36$
$= 180 \text{ cm}^2$

Practise Now 7

1. Base radius = $18 \div 2 = 9$ cm
Height of the cylinder = $2.5 \times 9 = 22.5$ cm
Volume of the cylinder = $\pi r^2 h$
$= \pi(9)^2(22.5)$
$= 5730 \text{ cm}^3$ (to 3 s.f.)

2. Base radius = $12 \div 2 = 6$ cm
Volume of the cylinder = $\pi(6)^2 h = 1000$
$h = \dfrac{1000}{\pi(6)^2}$
$h = 8.84$ cm (to 3 s.f.)

Practise Now 8

1. Since petrol is discharged through the pipe at a rate of 2.45 m/s, i.e. 245 cm/s, in 1 second, the volume of petrol discharged is the volume of petrol that fills the pipe to a length of 245 cm.
In 1 second, volume of petrol discharged
= volume of pipe of length 245 cm
$= \pi r^2 h$
$= \pi(0.6)^2(245)$
$= 88.2\pi \text{ cm}^3$
In 3 minutes, volume of petrol discharged
$= 88.2\pi \times 3 \times 60$
$= 49\,900 \text{ cm}^3$
$= 49.9\ l$ (to 3 s.f.)

2. Base radius = $0.036 \div 2 = 0.018$ m
Since water is discharged through the pipe at a rate of 1.6 m/s, i.e. in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 1.6 m.
In 1 second, volume of water discharged
= volume of pipe of length 1.6 m
$= \pi r^2 h$
$= \pi(0.018)^2(1.6)$
$= 0.000\,518\,4\pi \text{ cm}^3$
Volume of the cylindrical tank
$= \pi r^2 h$
$= \pi(3.4)^2(1.4)$
$= 16.184\pi \text{ cm}^3$
Time required to fill the tank
$= \dfrac{16.184\pi}{0.000\,518\,4\pi}$
$= 3\,1219\dfrac{11}{81}$ s
= 520 min (to the nearest minute)

Practise Now 9

1. **(i)** Total surface area of the can
$= 2\pi r^2 + 2\pi rh$
$= 2\pi(3.5)^2 + 2\pi(3.5)(10)$
$= 24.5\pi + 70\pi$
$= 94.5\pi$
$= 297 \text{ cm}^2$ (to 3 s.f.)

(ii) Area of the can that is painted

$= \pi r^2 + 2\pi rh$ (An open cylinder has only one base and a curved surface)

$= \pi(3.5)^2 + 2\pi(3.5)(10)$

$= 12.25\pi + 70\pi$

$= 82.25\pi$

Ratio of the area of the can that is painted, to the total surface area found in **(i)**.

$= 94.5\pi : 82.25\pi$

$= 94.5 : 82.25$

$= 54 : 47$

2. **(i)** Area of the cross section of the pipe

$= \pi(2.5)^2 - \pi(2.1)^2$

$= 6.25\pi - 4.41\pi$

$= 1.84\pi \text{ cm}^2$

(ii) Internal curved surface area of the pipe

$= 2\pi(2.1)(12)$

$= 50.4\pi$

$= 158 \text{ cm}^2$ (to 3 s.f.)

(iii) Total surface area of the pipe

$= 2(1.84\pi) + 50.4\pi + 2\pi(2.5)(12)$

$= 3.68\pi + 50.4\pi + 60\pi$

$= 114.08\pi$

$= 358 \text{ cm}^2$ (to 3 s.f.)

Practise Now 10

1. **(i)** Volume of the container

$= 20 \times 9 \times 14 + \frac{1}{4} \times \pi(14)^2(20)$

$= 2520 + 980\pi$

$= 5600 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the container

$= 2\left[\frac{1}{4} \times \pi(14)^2\right] + 2(9 \times 14) + 2(20 \times 9) + 2(14 \times 20)$

$+ \frac{1}{4} \times 2\pi(14)(20)$

$= 98\pi + 252 + 360 + 560 + 140\pi$

$= 238\pi + 1172$

$= 1920 \text{ cm}^2$ (to 3 s.f.)

2. **(i)** Volume of the solid

$= 6 \times 12 \times 8 - \frac{1}{2} \times \pi(3)^2(12)$

$= 576 - 54\pi$

$= 406 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the solid

$= 2\left[8 \times 6 - \frac{1}{2} \times \pi(3)^2\right] + 2(8 \times 12) + \frac{1}{2} \times 2\pi(3)(12) + 6 \times 12$

$= 96 - 9\pi + 192 + 36\pi + 72$

$= 360 + 27\pi$

$= 445 \text{ cm}^2$ (to 3 s.f.)

Exercise 14A

1. **(a)** **(i)** $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$

$4 \text{ m}^3 = 4 \times 1\,000\,000 \text{ cm}^3$

$= 4\,000\,000 \text{ cm}^3$

(ii) $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$

$0.5 \text{ m}^3 = 0.5 \times 1\,000\,000 \text{ cm}^3$

$= 500\,000 \text{ cm}^3$

(b) **(i)** $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$

$250\,000 \text{ cm}^3 = \frac{250\,000}{1\,000\,000} \text{ m}^3$

$= 0.25 \text{ m}^3$

(ii) $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$

$67\,800 \text{ cm}^3 = \frac{67\,800}{1\,000\,000} \text{ m}^3$

$= 0.0678 \text{ m}^3$

2. **(a)** **(i)** $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$

$0.84 \text{ m}^3 = 0.84 \times 1\,000\,000 \text{ cm}^3$

$= 840\,000 \text{ cm}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$

$840\,000 \text{ cm}^3 = 840\,000 \text{ m}l$

(b) **(i)** $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$

$2560 \text{ cm}^3 = \frac{2560}{1\,000\,000} \text{ m}^3$

$= 0.002\,56 \text{ m}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$

$2560 \text{ cm}^3 = 2560 \text{ m}l$

$= \frac{2560}{1000}\ l$

$= 2.56\ l$

3. **(a)** **(i)** Volume of the cuboid $= 6 \times 8 \times 10$

$= 480 \text{ cm}^3$

(ii) Surface area of the cuboid $= 2(6 \times 8 + 8 \times 10 + 6 \times 10)$

$= 376 \text{ cm}^2$

(b) **(i)** Volume of the cuboid $= 7 \times 12 \times 5$

$= 420 \text{ cm}^3$

(ii) Surface area of the cuboid $= 2(7 \times 12 + 5 \times 7 + 5 \times 12)$

$= 358 \text{ cm}^2$

(c) **(i)** Volume of the cuboid $= 120 \times 10 \times 96$

$= 115\,200 \text{ mm}^3$

(ii) Surface area of the cuboid

$= 2(120 \times 10 + 96 \times 10 + 120 \times 96)$

$= 27\,360 \text{ mm}^2$

(d) **(i)** Volume of the cuboid $= 1\frac{1}{2} \times \frac{1}{2} \times 10$

$= 7\frac{1}{2} \text{ cm}^3$

(ii) Surface area of the cuboid

$= 2\left(1\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 10 + 1\frac{1}{2} \times 10\right)$

$= 41\frac{1}{2} \text{ cm}^2$

(e) **(i)** Volume of the cuboid $= 1\frac{2}{5} \times \frac{3}{8} \times \frac{5}{8}$

$= \frac{21}{64}$ cm^3

(ii) Surface area of the cuboid

$= 2\left(1\frac{2}{5} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8} + 1\frac{2}{5} \times \frac{5}{8}\right)$

$= 3\frac{43}{160}$ cm^2

(f) **(i)** Volume of the cuboid $= 3.9 \times 0.7 \times 1.5$

$= 4.095$ cm^3

(ii) Surface area of the cuboid

$= 2(3.9 \times 0.7 + 0.7 \times 1.5 + 3.9 \times 1.5)$

$= 19.26$ cm^2

4.

	Length	Breadth	Height	Volume	Total surface area
(a)	24 mm	18 mm	5 mm	2160 mm^3	1284 mm^2
(b)	5 cm	3 cm	8 cm	120 cm^3	158 cm^2
(c)	2.5 cm	6 cm	3.5 cm	52.5 cm^3	89.5 cm^2
(d)	12 m	8 m	6 m	576 m^3	432 m^2

(a) Volume $= 24 \times 18 \times 5$

$= 2160$ mm^3

Surface area $= 2(24 \times 18 + 24 \times 5 + 18 \times 5)$

$= 1284$ mm^2

(b) Let the height of the cuboid be h cm.

Volume $= 5 \times 3 \times h = 120$ cm^3

$\therefore h = \frac{120}{5 \times 3} = 8$ cm

Surface area $= 2(5 \times 3 \times 5 \times 8 + 3 \times 8)$

$= 158$ cm^2

(c) Let the length of the cuboid be l cm.

Volume $= l \times 6 \times 3.5 = 52.5$ cm^3

$\therefore l = \frac{52.5}{6 \times 3.5} = 2.5$ cm

Surface area $= 2(2.5 \times 6 + 6 \times 3.5 + 2.5 \times 3.5)$

$= 89.5$ cm^2

(d) Let the breadth of the cuboid be b m.

Volume $= 12 \times b \times 6 = 576$ m^3

$\therefore b = \frac{576}{12 \times 6} = 8$ m

Surface area $= 2(12 \times 8 + 6 \times 8 + 12 \times 6)$

$= 432$ m^2

5. **(i)** Volume of the cuboid $= 28 \times b \times 15 = 6720$ cm^3

$\therefore b = \frac{6720}{28 \times 15}$

$= 16$

$\therefore$ Breadth = 16 cm

(ii) Volume of each small cube $= 4 \times 4 \times 4 = 64$ cm^3

Number of cubes to be obtained

$= \frac{6720}{64}$

$= 105$

6. Volume of the rectangular block of metal

$= 0.24 \times 0.19 \times 0.15$

$= 0.00684$ m^3

Let the length of the cube be l cm.

Volume of each small cube $= l \times l \times l = 0.00684$ m^3

$l^3 = 0.00684$

$l = \sqrt[3]{0.00684}$

$= 0.190$ (to 3 s.f.)

$\therefore$ Length of each side = 0.190 m

7. Volume of the open rectangular tank

$= 4 \times 2 \times 4.8$

$= 38.4$ m^3

Volume of water in the open rectangular tank initally

$= \frac{3}{4} \times 38.4$

$= 28.8$ m^3

$4000\ l = 4000 \times 1000$ ml

$= 4\ 000\ 000$ ml

$= 4\ 000\ 000$ cm^3

$= \frac{4\ 000\ 000}{1\ 000\ 000}$ m^3

$= 4$ m^3

Total volume of water in the open rectangular tank after 4000 litres of water are added to it

$= 28.8 + 4$

$= 32.8$ m^3

Let the depth of water in the tank be d m.

$4 \times 2 \times d = 32.8$

$8d = 32.8$

$d = 4.1$

$\therefore$ Depth = 4.1 m

8. External volume $= (3.2 + 0.2 + 0.2) \times (2.2 + 0.2 + 0.2) \times (1.5 + 0.2)$

$= 3.6 \times 2.6 \times 1.7$

$= 15.912$ m^3

Internal volume $= 3.2 \times 2.2 \times 1.5$

$= 10.56$ m^3

Volume of wood used $= 15.912 - 10.56$

$= 5.352$ m^3

9. External volume $= 15 \times 10 \times 45$

$= 6750$ cm^3

Internal volume $= 3 \times 2 \times 45$

$= 270$ cm^3

Volume of the hollow glass structure $= 6750 - 270$

$= 6480$ cm^3

10. **(i)** Volume of water in the tank

$= 0.2 \times 0.15 \times 0.16$

$= 0.0048$ m^3

$= 0.0048 \times 1\ 000\ 000$ cm^3

$= 4800$ cm^3

$= 4800$ ml

$= \frac{4800}{1000}\ l$

$= 4.8\ l$

(ii) Surface area of the tank that is in contact with the water
$= (0.2 \times 0.15) + 2(0.2 \times 0.16 + 0.15 \times 0.16)$
$= 0.142 \text{ m}^2$

11. (i) Volume of water in the tank
$= 80 \times 40 \times 35$
$= 112\,000 \text{ cm}^3$
$= 112\,000 \text{ m}l$
$= \dfrac{112\,000}{1000}\ l$
$= 112\ l$

(ii) Surface area of the tank that is in contact with the water
$= (80 \times 40) + 2(80 \times 35 + 40 \times 35)$
$= 11\,600 \text{ cm}^2$
$= \dfrac{11\,600}{10\,000} \text{ m}^2$
$= 1.16 \text{ m}^2$

12. Let the length of the cube be l cm.
$l \times l \times l = 64 \text{ cm}^3$
$l^3 = 64$
$l = \sqrt[3]{64}$
$= 4$
Total area of the faces that will be coated with paint
$= 6(4 \times 4)$
$= 96 \text{ cm}^2$

13. Let the length of the cube be l cm.
$6(l \times l) = 433.5$
$6l^2 = 433.5$
$l^2 = 72.25$
$l = \sqrt{72.25}$
$= 8.5$
Volume of the cube
$= 8.5 \times 8.5 \times 8.5$
$= 614.125 \text{ cm}^3$

14. (i) Number of trips required to fill the entire quarry
$= \dfrac{2.85 \times 1\,000\,000}{6.25}$
$= 456\,000$

(ii) Cost to fill the quarry
$= 456\,000 \times \$55$
$= \$25\,080\,000$

(iii) 3 hectares = $30\,000 \text{ m}^2$
Cost to fill 1 m^2 of the land
$= \$\dfrac{25\,080\,000}{30\,000}$
$= \$836$

15. Volume of wood used to make this trough
$= (185 \times 45 \times 28) - [(185 - 2.5 - 2.5) \times (45 - 2.5 - 2.5) \times (28 - 2.5)]$
$= (185 \times 45 \times 28) - (180 \times 40 \times 25.5)$
$= 233\,100 - 183\,600$
$= 49\,500 \text{ cm}^3$
$= \dfrac{49\,500}{1\,000\,000} \text{ m}^3$
$= 0.0495 \text{ m}^3$

16. In one minute, the water will flow through $22 \times 60 = 1320$ cm along the drain.
Amount of water that will flow through in one minute
$= 30 \times 3.5 \times 1320$
$= 138\,600 \text{ cm}^3$
$= 138\,600 \text{ m}l$
$= \dfrac{138\,600}{1000}\ l$
$= 138.6\ l$

17. (i) Let the height of the cuboid be h cm.
Surface area of the cuboid $= 2(12 \times 9 + 12 \times h + 9 \times h)$
$= 426 \text{ cm}^2$
$2(108 + 12h + 9h) = 426$
$2(108 + 21h) = 426$
$108 + 21h = 213$
$21h = 213 - 108$
$21h = 105$
$h = 5$
$\therefore$ Height of cuboid = 5 cm

(ii) Volume of the cuboid
$= 12 \times 9 \times 5$
$= 540 \text{ cm}^3$

18. (i) Floor area of Room A $= 26 \times 1$
$= 26 \text{ m}^2$
Volume of Room A $= 26 \times 1 \times 3$
$= 78 \text{ m}^3$
Floor area of Room B $= 5 \times 5$
$= 25 \text{ m}^2$
Volume of Room B $= 5 \times 5 \times 3$
$= 75 \text{ m}^3$
Floor area of Room C $= 6 \times 6$
$= 36 \text{ m}^2$
Volume of Room C $= 6 \times 6 \times 1.8$
$= 64.8 \text{ m}^3$

(ii) No. If both rooms, A and B, have the same height, then we will use the floor area as the gauge. If the rooms do not have the same height, then we will use the volume to decide.

Exercise 14B

1. (a) Volume of the prism = base area × height
$= \left[\dfrac{1}{2} \times (75 + 59) \times 46\right] \times 120$
$= 3028 \times 120$
$= 369\,840 \text{ cm}^3$

(b) Volume of the prism
= base area × height
$= \left[\dfrac{1}{2} \times (16 + 28) \times (18 - 7) + 7 \times 28\right] \times 38$
$= 438 \times 38$
$= 16\,644 \text{ cm}^3$

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(c) Volume of the prism = base area × height

$= [9 \times 5 + 9 \times 3 + (16 - 8) \times (9 - 6)] \times 10$

$= 96 \times 10$

$= 960 \text{ cm}^3$

(d) Volume of the prism = base area × height

$= \left[\frac{1}{2} \times (14 + 18) \times 6\right] \times 12$

$= 96 \times 12$

$= 1152 \text{ cm}^3$

(e) Volume of the prism = base area × height

$= \left[\frac{1}{2} \times 6 \times 8 + 13 \times 10\right] \times 5$

$= 154 \times 5$

$= 770 \text{ cm}^3$

(f) Volume of the prism = base area × height

$= \left[\frac{1}{2} \times 18 \times (12 - 3) + 3 \times 18\right] \times 35$

$= 135 \times 35$

$= 4725 \text{ cm}^3$

2.

	AB	***BC***	***BC***	**Area of $\triangle ABC$**	**Volume of prism**
(a)	3 cm	4 cm	7 cm	6 cm^2	42 cm^3
(b)	9 cm	14 cm	11 cm	63 cm^2	693 cm^3
(c)	32 cm	15 cm	300 cm	240 cm^2	$72\,000 \text{ cm}^3$
(d)	24.6 cm	7.8 cm	400 cm	95.94 cm^2	$38\,376 \text{ cm}^3$

(a) Area of $\triangle ABC$

$= \frac{1}{2} \times 4 \times 3$

$= 6 \text{ cm}^2$

Volume of prism

$= 6 \times 7$

$= 42 \text{ cm}^3$

(b) Area of $\triangle ABC$

$= \frac{1}{2} \times BC \times 9 = 63$

$4.5BC = 63$

$BC = 14 \text{ cm}$

Volume of prism

$= 63 \times 11$

$= 693 \text{ cm}^3$

(c) Volume of prism

$= \text{Area of } \triangle ABC \times 300 = 72\,000$

$\text{Area of } \triangle ABC = 240 \text{ cm}^2$

Area of $\triangle ABC$

$= \frac{1}{2} \times 15 \times AB = 240$

$7.5AB = 240$

$AB = 32 \text{ cm}$

(d) Area of $\triangle ABC$

$= \frac{1}{2} \times 7.8 \times 24.6$

$= 95.94 \text{ cm}^2$

Volume of prism

$= 95.94 \times CD = 38\,376$

$CD = 400 \text{ cm}$

3. Air space in the hall = Volume of the prism

= base area × height

$= \left[\frac{1}{2} \times 42 \times (38 - 23) + 42 \times 23\right] \times 80$

$= 1281 \times 80$

$= 102\,480 \text{ m}^3$

4. (a) (i) Volume of the prism = base area × height

$= \left(\frac{1}{2} \times 6 \times 4\right) \times 15$

$= 12 \times 15$

$= 180 \text{ cm}^3$

(ii) Total surface area of the prism

= perimeter of the base × height + 2 × base area

$= (5 + 5 + 6) \times 15 + 2 \times 12$

$= 264 \text{ cm}^2$

(b) (i) Volume of the prism = base area × height

$= [2 \times 7 + (5 - 2) \times (7 - 6)] \times 9$

$= 17 \times 9$

$= 153 \text{ cm}^3$

(ii) Total surface area of the prism

= perimeter of the base × height + 2 × base area

$= (7 + 2 + 6 + 3 + 1 + 5) \times 9 + 2 \times 17$

$= 250 \text{ cm}^2$

5. (i) Volume of water in the pool when it is full

= Volume of the prism

= base area × height

$= \left[\frac{1}{2} \times (1.2 + 2) \times 50\right] \times 25$

$= 80 \times 25$

$= 2000 \text{ m}^3$

(ii) Area of the pool which is in contact with the water

$= [(1.2 + 50 + 2 + 50.01) \times 25 + 2 \times 80] - (25 \times 50)$

$= 1490.25 \text{ cm}^2$

Exercise 14C

1. (a) (i) Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(7)^2(12)$

$= 1850 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(7)^2 + 2\pi(7)(12)$

$= 98\pi + 168\pi$

$= 266\pi$

$= 836 \text{ cm}^2$ (to 3 s.f.)

(b) Base radius = $1.2 \div 2 = 0.6$ m

(i) Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(0.6)^2(4)$

$= 4.52$ m^3 (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(0.6)^2 + 2\pi(0.6)(4)$

$= 0.72\pi + 4.8\pi$

$= 5.52\pi$

$= 17.3$ m^2 (to 3 s.f.)

(c) **(i)** Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(15)^2(63)$

$= 44\ 500$ mm^3 (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(15)^2 + 2\pi(15)(63)$

$= 450\pi + 1890\pi$

$= 2340\pi$

$= 7350$ mm^2 (to 3 s.f.)

2.

	Diameter	**Radius**	**Height**	**Volume**	**Total surface area**
(a)	8.00 cm	4.00 cm	14 cm	704 cm^3	453 cm^2
(b)	28.0 cm	14.0 cm	20 cm	12 320 cm^3	2990 cm^2
(c)	4 cm	2 cm	42.0 cm	528 cm^3	553 cm^2
(d)	8 m	4 m	21.0 m	1056 m^3	629 m^2

(a) Volume = 704 cm^3

$\pi r^2(14) = 704$

$r^2 = \dfrac{704}{14\pi}$

$r = \sqrt{\dfrac{704}{14\pi}}$

$= 4.00$ cm (to 3 s.f.)

$\therefore\ d = 2 \times 4.00 = 8.00$ cm (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4.001)^2 + 2\pi(4.001)(14)$

$= 453$ cm^2

(b) Volume = 12 320 cm^3

$\pi r^2(20) = 12\ 320$

$r^2 = \dfrac{12\ 320}{20\pi}$

$r = \sqrt{\dfrac{12\ 320}{20\pi}}$

$= 14.0$ cm (to 3 s.f.)

$\therefore\ d = 2 \times 14.0 = 28.0$ cm (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(14.00)^2 + 2\pi(14.00)(20)$

$= 2990$ cm^2 (to 3 s.f.)

(c) $r = 4 \div 2 = 2$ cm

Volume = 528 cm^3

$\pi(2)^2 h = 528$

$h = \dfrac{528}{4\pi}$

$= 42.0$ cm (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(2)^2 + 2\pi(2)(42.02)$

$= 553$ cm^2 (to 3 s.f.)

(d) $d = 4 \times 2 = 8$ m

Volume = 1056 m^3

$\pi(4)^2 h = 1056$

$h = \dfrac{1056}{16\pi}$

$= 21.0$ m (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(21.01)$

$= 629$ m^2 (to 3 s.f.)

3. Base radius = $0.4 \div 2 = 0.2$ m

Height of the cylinder = $\dfrac{3}{4} \times 0.2 = 0.15$ m

Volume of the cylinder $= \pi r^2 h$

$= \pi(0.2)^2(0.15)$

$= 0.006\pi$ m^3

$= 6000\pi$ cm^3

$= \dfrac{6000\pi}{1000}$ l

$= 18.8$ l

4. Let the depth of water in the drum be d cm.

Base radius = $48 \div 2 = 24$ cm

150 l = 150 000 ml = 150 000 cm^3

Volume of water in the drum $= \pi r^2 d = 150\ 000$ cm^3

$\pi(24)^2 d = 150\ 000$

$d = \dfrac{150\ 000}{\pi(24)^2}$

$= 82.9$

$\therefore$ Depth of water = 82.9 cm

5. Base radius = $15 \div 2 = 7.5$ cm

Capacity of the drink trough

$= \dfrac{1}{2} \times \pi \times (7.5)^2 \times 84$

$= 7420$ cm^3 (to 3 s.f.)

$= 7.42$ l

6. 35 mm = $35 \div 10 = 3.5$ cm

Base radius = $3.5 \div 2 = 1.75$ cm

Total surface area that need to be painted for 1 wooden closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(1.75)^2 + 2\pi(1.75)(7)$

$= 6.125\pi + 24.5\pi$

$= 30.625\pi$

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Total surface area that need to be painted for 200 wooden closed cylinders
$= 200 \times 30.625\pi$
$= 19\ 200$ cm^2 (to 3 s.f.)

7. Base radius $= 2.4 \div 2 = 1.2$ m
Volume of the tank $= \pi r^2 h$
$= \pi(1.2)^2(6.4)$
$= 9.216\pi$ m^3
$= 9\ 216\ 000\pi$ cm^3
Volume of the cylinder container $= \pi r^2 h$
$= \pi(8.2)^2(28)$
$= 1882.72\pi$ cm^3
Number of completed cylindrical container which can be filled by the oil in the tank
$= \dfrac{9\ 216\ 000\pi}{1882.72\pi}$
$= 4895$ (to the nearest whole number)

8. External base radius $= 28 \div 2 = 14$ mm $= 1.4$ cm
Internal base radius $= 20 \div 2 = 10$ mm $= 1$ cm
Volume of the metal used in making the pipe
$= \pi(1.4)^2(35) - \pi(1)^2(35)$
$= 68.6\pi - 35\pi$
$= 33.6\pi$
$= 106$ cm^3 (to 3 s.f.)

9. Base radius of the copper cylindrical rod $= 14 \div 2 = 7$ cm
Volume of the copper cylindrical rod
$= \pi r^2 h$
$= \pi(7)^2(47)$
$= 2303\pi$
Let the length of the wire be l.
Base radius of the wire $= 8 \div 2 = 4$ mm $= 0.4$ cm
Volume of the wire $= \pi(0.4)^2\, l = 2303\pi$
$(0.4)^2\, l = 2303$
$l = \dfrac{2303}{0.16}$
$= 14\ 400$ cm (to 3 s.f.)
$= 144$ m

10. Base radius $= 2.4 \div 2 = 1.2$ cm
Since water is discharged through the pipe at a rate of 2.8 m/s, i.e. 280 cm/s, in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 280 cm.
In 1 second, volume of water discharged
= volume of pipe of length 280 cm
$= \pi r^2 h$
$= \pi(1.2)^2(280)$
$= 403.2\pi$ cm^3
Half an hour = 30 minutes
In 30 minutes, volume of water discharged
$= 403.2\pi \times 30 \times 60$
$= 725\ 760\pi$ cm^3
$= 2\ 280\ 000$ cm^3 (to 3 s.f.)
$= 2280\ l$

11. Base radius of the pipe $= 64 \div 2 = 32$ mm
Since water is discharged through the pipe at a rate of 2.05 mm/s, i.e. in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 2.05 mm.
In 1 second, volume of water discharged
= volume of pipe of length 2.05 mm
$= \pi r^2 h$
$= \pi(32)^2(2.05)$
$= 2099.2\pi$ mm^3
Base radius of the cylindrical tank $= 7.6 \div 2 = 3.8$ cm $= 38$ mm
2.3 m = 230 cm = 2300 mm
Volume of the cylindrical tank
$= \pi r^2 h$
$= \pi(38)^2(2300)$
$= 3\ 321\ 200\pi$ cm^3
Time required to fill the tank
$= \dfrac{3\ 321\ 200\pi}{2099.2\pi}$
$= 1582\dfrac{83}{656}$ s
$= 26$ min (to the nearest minute)

12. **(i)** Volume of water in the tank $= 18 \times 16 \times 13$
$= 3744$ cm^3

(ii) Let the height of water in the cylindrical container be h.
Base radius of the cylindrical container $= 17 \div 2 = 8.5$ cm
Volume of water in the cylindrical container $= \pi(8.5)^2 h$
$= 3744$
$h = \dfrac{3744}{\pi(8.5)^2}$
$= 16.5$
$\therefore$ Height of water $= 16.5$ cm

(iii) Surface area of the cylindrical container that is in contact with the water
$= \pi r^2 + 2\pi rh$
$= \pi(8.5)^2 + 2\pi(8.5)(16.49)$
$= 72.25\pi + 280.33\pi$
$= 352.58\pi$
$= 1110$ cm^2 (to 3 s.f.)

13. **(i)** Base radius $= 186 \div 2 = 93$ mm $= 9.3$ cm
Height $= \dfrac{1}{3} \times 93 = 31$ mm $= 3.1$ cm
Total surface area of the container
$= 2\pi r^2 + 2\pi rh$
$= 2\pi(9.3)^2 + 2\pi(9.3)(3.1)$
$= 172.98\pi + 57.66\pi$
$= 230.64\pi$
$= 725$ cm^2 (to 3 s.f.)

(ii) Area of the container that is painted

$= \pi r^2 + 2\pi rh$ (An open cylinder has only one base and a curved surface)

$= \pi(9.3)^2 + 2\pi r(9.3)(3.1)$

$= 86.49\pi + 57.66\pi$

$= 144.15\pi$

Fraction

$= \dfrac{144.15\pi}{230.64\pi}$

$= \dfrac{5}{8}$

14. (i) Base radius $= 23 \div 2 = 11.5$ mm $= 1.15$ cm

Height $= 4$ mm $= 0.4$ cm

Volume of water and metal discs in the tank

$= (32 \times 28 \times 19) + 2580[\pi \times (1.15)^2 \times 0.4]$

$= (17\,024 + 1364.82\pi)$ cm^3

Let the new height in the tank be h.

$$\begin{aligned} \text{Volume in the tank} &= 32 \times 28 \times h \\ &= 17\,024 + 1364.82\pi \\ 896h &= 17\,024 + 1364.82\pi \\ h &= \frac{17\,024 + 1364.82\pi}{896} \\ &= 23.8 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore$ New height of water in the tank = 23.8 cm

(ii) Surface area of the tank that is in contact with the water after the discs have been added

$= 2(32 \times 23.79 + 28 \times 23.79) + 32 \times 28$

$= 3750$ cm^2 (to 3 s.f.)

15. Total surface area of the pipe

$= 2[\pi(3.8 + 0.8)^2 - \pi(3.8)^2] + 2\pi(3.8 + 0.8)(15) + 2\pi(3.8)(15)$

$= 13.44\pi + 138\pi + 114\pi$

$= 265.44\pi$

$= 834$ cm^2 (to 3 s.f.)

16. 124 mm = 12.4 cm = 0.124 m

28 km^2 = 28 000 000 m^2

Volume of the rain

$= 28\,000\,000 \times 0.124$

$= 3\,472\,000$ m^3

Volume of each channel

$= 18 \times 26.4$

$= 475.2$ m^3

Time required for the channels to drain off the rain

$= \dfrac{3\,472\,000}{475.2 \times 2}$

$= 3653\dfrac{59}{297}$ s

= 61 minutes (to the nearest minute)

Exercise 14D

1. (i) Volume of the solid

$= 7 \times 3 \times 2 + 12 \times 8 \times 5$

$= 42 + 480$

$= 522$ cm^3

(ii) Total surface area of the solid

$= 2(5 \times 12) + 2(5 \times 8) + 2(12 \times 8) + 2(2 \times 7) + 2(3 \times 2)$

$= 120 + 80 + 192 + 28 + 12$

$= 432$ cm^2

2. (i) Volume of the solid

$= \pi(2.5)^2(8) + 7 \times 11 \times 3$

$= 50\pi + 231$

$= 388$ cm^3 (to 3 s.f.)

(ii) Total surface area of the solid

$= 7 \times 11 + 2\pi(2.5)(8) + 2(3 \times 7) + 2(3 \times 11) + (7 \times 11)$

$= 77 + 40\pi + 42 + 66 + 77$

$= 262 + 40\pi$

$= 388$ cm^2 (to 3 s.f.)

3. (i) Volume of the solid

$= \pi(5)^2(3) + \pi(12.5)^2(2)$

$= 75\pi + 312.5\pi$

$= 387.5\pi$

$= 1220$ cm^3 (to 3 s.f.)

(ii) Total surface area of the solid

$= \pi(12.5)^2 + 2\pi(5)(3) + 2\pi(12.5)(2) + \pi(12.5)^2$

$= 156.25\pi + 30\pi + 50\pi + 156.25\pi$

$= 392.5\pi$

$= 1230$ cm^2 (to 3 s.f.)

4. (i) Volume of the glass block

$= \dfrac{1}{4} \times \pi(24)^2(56) + 24 \times 56 \times 40$

$= 8064\pi + 53\,760$

$= 79\,100$ cm^3 (to 3 s.f.)

(ii) Total surface area of the glass block

$= 2\left[\dfrac{1}{4} \times \pi(24)^2\right] + 2(40 \times 24) + 2(40 \times 56) + 2(24 \times 56)$

$\quad + \dfrac{1}{4} \times 2\pi(24)(56)$

$= 288\pi + 1920 + 4480 + 2688 + 672\pi$

$= 960\pi + 9088$

$= 12\,100$ cm^2 (to 3 s.f.)

5. (i) Volume of the solid

$= 10 \times 12 \times 7 - \dfrac{1}{2} \times \pi(2)^2(12)$

$= 840 - 24\pi$

$= 765$ cm^3 (to 3 s.f.)

(ii) Total surface area of the solid

$= 2\left[7 \times 10 - \dfrac{1}{2} \times \pi(2)^2\right] + 2(7 \times 12) + 2(3 \times 12)$

$\quad + \dfrac{1}{2} \times 2\pi(2)(12) + 10 \times 12$

$= 140 - 4\pi + 168 + 72 + 24\pi + 120$

$= 500 + 20\pi$

$= 563$ cm^2 (to 3 s.f.)

6. (i) Volume of the remaining solid

$= \pi(12)^2(32) - \pi(5)^2(14)$

$= 4608\pi - 350\pi$

$= 4258\pi$

$= 13\,400$ cm^3 (to 3 s.f.)

(ii) Area that will covered in paint
$= 2\pi(12)(32) + 2\pi(5)(14) + 2[\pi(12)^2]$
$= 768\pi + 140\pi + 288\pi$
$= 1196\pi$
$= 3760 \text{ cm}^2$ (to 3 s.f.)

7. **(i)** Volume of the solid
$= \left[\frac{1}{2} \times (40 + 88) \times 70\right] \times 25 - \pi(15)^2(25)$
$= 112\,000 - 5625\pi$
$= 94\,300 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the solid
$= (74 + 40 + 74 + 88) \times 25 + 2\left[\frac{1}{2} \times (40 + 88) \times 70 - \pi(15)^2\right]$
$+ 2\pi(15)(25)$
$= 6900 + 8960 - 450\pi + 750\pi$
$= 15\,860 + 300\pi$
$= 16\,800 \text{ cm}^2$ (to 3 s.f.)

8. **(i)** Volume of the solid
$= \frac{1}{2} \times [\pi(6 + 1.5)^2 - \pi(6)^2] \times 8$
$= 4(56.25\pi - 36\pi)$
$= 4(20.25\pi)$
$= 81\pi$
$= 254 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the solid
$= 2 \times \frac{1}{2} \times [\pi(7.5)^2 - \pi(6)^2] + \frac{1}{2} \times 2\pi(7.5) \times 8$
$+ \frac{1}{2} \times 2\pi(6) \times 8 + 2(1.5 \times 8)$
$= 20.25\pi + 60\pi + 48\pi + 24$
$= 24 + 128.25\,\pi$
$= 427 \text{ cm}^2$ (to 3 s.f.)

Review Exercise 14

1. **(a)** **(i)** Volume of the solid
$= 6 \times 3 \times 2 + 12 \times 2 \times 3$
$= 36 + 72$
$= 108 \text{ cm}^3$

(ii) Total surface area of the solid
$= 2(2 \times 12) + 2(3 \times 2) + 2(3 \times 3) + 2(2 \times 6) + 2(3 \times 2)$
$+ 3 \times 6 + 3 \times 12$
$= 48 + 12 + 18 + 24 + 12 + 18 + 36$
$= 168 \text{ cm}^2$

(b) **(i)** Volume of the solid
$= 6 \times 8 \times 2 - 2 \times 2 \times 2$
$= 96 - 8$
$= 88 \text{ cm}^3$

(ii) Total surface area of the solid
$= 2(2 \times 6) + 2 \times 8 + 2(3 \times 2) + 3(2 \times 2) + 2(6 \times 8 - 2 \times 2)$
$= 24 + 16 + 12 + 12 + 88$
$= 152 \text{ cm}^2$

(c) **(i)** Volume of the solid
$= 4 \times 5 \times 1 - 2(1 \times 3)$
$= 20 - 6$
$= 14 \text{ cm}^3$

(ii) Total surface area of the solid
$= 2(1 \times 4) + 8(1 \times 1) + 2(1 \times 3) + 2[4 \times 5 - 2(1 \times 3)]$
$= 8 + 8 + 6 + 40 - 12$
$= 50 \text{ cm}^2$

(d) **(i)** Volume of the solid
$= 1 \times 1 \times 5 + 2 \times 4 \times 1 + 1 \times 1 \times 3$
$= 5 + 8 + 3$
$= 16 \text{ cm}^3$

(ii) Total surface area of the solid
$= 2(1 \times 5) + 2(1 \times 1) + 2(1 \times 3) + 2(1 \times 4)$
$+ 2[1 \times 5 + 2 \times 3 + 1 \times 5]$
$= 10 + 2 + 6 + 8 + 32$
$= 58 \text{ cm}^2$

2. 4.5 m = 450 cm, 3.6 m = 360 cm
Number of bricks required
$= \frac{450}{18} \times \frac{18}{9} \times \frac{360}{6}$
$= 3000$

3. Volume of the rectangular block of metal
$= 256 \times 152 \times 81$
$= 3\,151\,872 \text{ mm}^3$
Let the length of the cube be l mm.
$l^3 = 3\,151\,872$
$l = \sqrt[3]{3\,151\,872}$
$= 147$ (to 3 s.f.)
$\therefore$ Length of each side = 147 mm

4. Let the length of the cube be l cm.
$l^3 = 343$
$l = \sqrt[3]{343}$
$= 7$
Total surface area of a cube
$= 6l^2$
$= 6(7)^2$
$= 294 \text{ cm}^2$

5. **(i)** Its volume
$= \frac{1}{2} \times (20 + 22.5) \times 17 \times 45.5$
$= 16\,436.875 \text{ cm}^3$

(ii) Volume of a gold bar with a mass of 200 g
$= \frac{16\,436.875}{250\,000} \times 200$
$= 13.1495 \text{ cm}^3$
$= 13.1495 \times 1000 \text{ mm}^3$
$= 13\,149.5 \text{ mm}^3$

(iii) Volume of the gold bar weighing 200 g = 13 149.5 mm^3

$$\frac{1}{2} \times (20 + x) \times 15 \times 50 = 13\,149.5$$

$$375(20 + x) = 13\,149.5$$

$$20 + x = \frac{13\,149.5}{375}$$

$$x = \frac{13\,149.5}{375} - 20$$

$$\therefore x = 15\frac{49}{750}$$

6. Base radius of the cylindrical barrel = 70 ÷ 2 = 35 cm

Volume of water in the cylindrical barrel that is drained away

$= \pi(35)^2(6)$

$= 7350\pi$ cm^3

0.2 l = 200 ml = 200 cm^3

Time taken for the water level in the barrel to drop by 6 cm

$= \dfrac{7350\pi}{200}$

= 115 minutes (to 3 s.f.)

7. (i) Volume of water in the pail

$= \pi(32)^2(25)$

$= 25\,600\pi$

= 80 400 cm^3 (to 3 s.f.)

(ii) Volume of water in the pail after 2000 metal cubes are added to it

$= 25\,600\pi + 2000(2 \times 2 \times 2)$

$= 25\,600\pi + 16\,000$

Let the new height of water in the pail be h cm.

$\pi(32)^2h = 25\,600\pi + 16\,000$

$$h = \frac{25600\pi + 16000}{\pi(32)^2}$$

= 30.0 (to 3 s.f.)

∴ New height = 30.0 cm

8. (i) Internal radius = 4.2 ÷ 2 = 2.1 cm

External radius = 5 ÷ 2 = 2.5 cm

Volume of metal used in making the pipe

$= [\pi(2.5)^2 - \pi(2.1)^2] \times 8.9$

$= 16.376\pi$

= 51.4 cm^3 (to 3 s.f.)

(ii) 51.45 cm^3 = 0.00 005 145 m^3

Cost of the pipe = 0.00 005 145 × 2700 × $8

= $1.11

9. (i) Volume of the solid

$= \pi(6)^2(14) + 22 \times 18 \times 8$

$= 504\pi + 3168$

= 4750 cm^3 (to 3 s.f.)

(ii) Total surface area of the solid

$= 2(22 \times 18) + 2\pi(6)(14) + 2(8 \times 22) + 2(8 \times 18)$

$= 792 + 168\pi + 352 + 288$

$= 1432 + 168\pi$

= 1960 cm^2 (to 3 s.f.)

10. (i) Volume of the remaining solid

$= 15 \times 24 \times 16 - \pi(4)^2(7)$

$= 5760 - 112\pi$

= 5410 cm^3 (to 3 s.f.)

(ii) Area that will be covered in paint

$= 2\pi(4)(7) + 2(15 \times 24) + 2(16 \times 24) + 2(16 \times 15)$

$= 56\pi + 720 + 768 + 480$

$= 56\pi + 1968$

= 2140 cm^2 (to 3 s.f.)

Challenge Yourself

(i) Volume of the solid

$= 50 \times 70 \times 30 - 10 \times 10 \times 70 - 2(10 \times 10 \times 10) - 2(10 \times 10 \times 20)$

$= 105\,000 - 7000 - 4000 - 2000$

= 92 000 cm^3

(ii) Total surface area of the solid

$= 2(30 \times 70 - 10 \times 10) + 2(50 \times 30 - 10 \times 10) + 2(50 \times 70 - 10 \times 10)$
$+ 4(10 \times 60) + 8(10 \times 10) + 8(10 \times 20)$

$= 4000 + 2800 + 6800 + 2400 + 800 + 1600$

= 18 400 cm^2

Chapter 15 Statistical Data Handling

TEACHING NOTES

Suggested Approach

In primary school, students have learnt statistical diagrams such as pictograms, bar graphs, pie charts and line graphs. Here, students revisit what they have learnt and they are expected to know and appreciate the advantages and disadvantages of each diagram. With such knowledge, students can choose the most appropriate diagram given a certain situation. Teachers may want to give more examples when introducing the various stages of a statistical study and engage with students in evaluating and discussing the issues involved in each stage. Knowledge from past chapters may be required (i.e. percentage).

Section 15.1: Introduction to Statistics

Teachers should define statistics as the collection, organisation, display and interpretation of data. Teachers may want to briefly cover each stage of a statistical study and give real-life examples for discussion with students, in the later sections. Students are expected to solve problems involving various statistical diagrams.

Section 15.2: Pictograms and Bar Graphs

Using the example in the textbook, teachers can show how each stage is involved in a statistical study, where the data is displayed in the form of a pictogram and bar graph. Students should appreciate what happens in each stage, cumulating in the conclusion through the interpretation of the data. Through the example, students should also learn to read, interpret and solve problems using information presented in these statistical diagrams.

Students should know the characteristics of pictograms and bar graphs and take note of the merits and limitations of pictograms and bar graphs (see Attention on page 370 and Thinking Time on page 371).

Section 15.3: Pie Charts

Some students may still be unfamiliar with calculating the size of the angle of each sector in a pie chart. As such, teachers may wish to illustrate how this is done. Students need to recall the characteristics of a pie chart (see Attention on page 376).

Other than the examples given in the textbooks, teachers may give more examples where a data set is represented by a pie chart, such as students' views on recent current affairs.

Section 15.4: Line Graphs

Teachers may want to recap how line graphs are drawn. Students need to know the advantage, disadvantage and the cases line graphs are best used in. (see Attention on page 378).

Teachers can discuss some situations where pictograms, bar graphs, pie charts or line graphs are most suitable and assess students' understanding of statistical diagrams (see Class Discussion: Comparison of Various Statistical Diagrams).

Section 15.5: Statistics in Real-World Contexts

Teachers can use the examples given in the textbooks and further illustrate in detail how each stage in a statistical study is carried out using real-life examples.

Teachers can get the students to discuss and think of more ways to collect data besides conducting questionnaires. Other ways can include telephone interviews, emails, online surveys etc.

Teachers may want to assign small-scale projects for students where they conduct their own statistical studies. Such projects allow students to apply what they have learnt about statistical data handling in real-world contexts.

Section 15.6: Evaluation of Statistics

Teachers should go through the various examples in the textbook and discuss with students the potential issues that can arise at each stage of a statistical study. The importance of not engaging in any unethical behaviors, ensuring objectivity and providing the complete picture without omitting any forms of misrepresentation need to be inculcated into students.

WORKED SOLUTIONS

Thinking Time (Page 371)

1. Michael is correct. In a pictogram, each icon represents the same number. Hence, since there are 3 buses and 4 cars, more students travel to school by car than by bus.
2. To avoid a misinterpretation of the data, we can replace each bus and each car in the pictogram with a standard icon. Alternatively, we can draw the buses and the cars to be of the same size.

Class Discussion (Comparison of Various Statistical Diagrams)

1.

Statistical Diagram	Advantages	Disadvantages
Pictogram	• It is more colourful and appealing. • It is easy to read.	• It is difficult to use icons to represent exact values. • If the sizes of the icons are inconsistent, the data may easily be misinterpreted. • If the data has many categories, it is not desirable to use a pictogram to display it as it is quite tedious to draw so many icons.
Bar graph	• The data sets with the lowest and the highest frequencies can be easily identified. • It can be used to compare data across many categories. • Two or more sets of data with many categories can be easily compared.	• If the frequency axis does not start from 0, the displayed data may be misleading. • The categories can be rearranged to highlight certain results.
Pie chart	• The relative size of each data set in proportion to the entire set of data can be easily observed. • It can be used to display data with many categories. • It is visually appealing.	• The exact numerical value of each data set cannot be determined directly. • The sum of the angles of all the sectors may not be 360° due to rounding errors in the calculation of the individual angles. • It is not easy to compare across the categories of two or more sets of data.
Line graph	• Intermediate values can be easily obtained. • It can better display trends over time as compared to most of the other graphs. • The trends of two or more sets of data can be easily compared.	• Intermediate values may not be meaningful. • If the frequency axis does not start from 0, the displayed data may be misleading. • It is less visually appealing as compared to most of the other graphs.

2. **(a)** A bar graph should be used to display the data as we need to compare data across 12 categories. The categories with the lowest and the highest frequencies can also be easily identified.

(b) A line graph should be used to display the data as we need to display the trend of the change in the population of Singapore from the year 2004 to the year 2013.

(c) A pie chart cannot be used to display the data as we will not be able to directly determine the exact number of Secondary 1 students who travel to school by each of the 4 modes of transport. A line graph is inappropriate as it is used to display trends over time. Hence, a pictogram or a bar graph should be used to display the data. Since there are only 4 categories, we may wish to use a pictogram instead of a bar graph as it is more visually appealing and is easier to read.

(d) A pie chart should be used to display the data as it is easier to compare the relative proportions of Secondary 1 students who prefer the different drinks.

Performance Task (Page 381)

1. **Collection of Data**
   Guiding Questions:
   - What are the types of food that are sold in your current school canteen?
   - What other types of food would students like to be sold in the school canteen? How many choices would you like to include in the questionnaire?
   - What should be the sample size? How do you ensure that the sample chosen is representative of the entire school?
   - How many choices would you like each student surveyed to select?
2. **Organisation of Data**
   Guiding Questions:
   - How can you consolidate the data collected and present it in a table?
   - How should you organise the data such that it is easy to understand?
3. **Display of Data**
   Guiding Question:
   - Which statistical diagram, i.e. pictogram, bar graph, pie chart or line graph, is the most suitable to display the data obtained?
4. **Interpretation of Data**
   Guiding Questions:
   - How many more food stalls can your school canteen accommodate?
   - What is the conclusion of your survey, i.e. based on the statistical diagram drawn, which types of food stalls should your school engage for the school canteen?

Teachers may wish to refer students to pages 380 and 381 of the textbook for an example on how they can present their report.

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Class Discussion (Evaluation of Statistics)

Part I: Collection of Data

1. *Teachers to conduct poll to find out the number of students who know Zidane, Beckenbauer and Cruyff. It is most likely that some students will know who Zidane is, but most (if not all) students will not know who Beckenbauer and Cruyff are.*
2. It is stated in the article that the poll was conducted on the UEFA website. As such, the voters who took part in the poll were most likely to belong to the younger generation who are more computer-savvy and hence, the voters were unlikely to be representative of all football fans.
3. As shown in the article, the number of votes for the three footballers were close, with 123 582 votes for Zidane, 122 569 votes for Beckenbauer and 119 332 votes for Cruyff. This is despite the fact that most of the younger generation, who were most likely to have voted in the poll, may not know who Beckenbauer and Cruyff are as they were at the peak of their careers in the 1970s. Hence, if older football fans were to participate in the poll, Zidane would probably not have come in first place.
4. The choice of a sample is important as if the sample chosen for collection of data is not representative of the whole population, the figures that are obtained may be misleading. Hence, a representative sample should be chosen whenever possible.

Part II: Organisation of Data

1. Banks and insurance firms, timeshare companies and motor vehicle companies received the most number of complaints.
2. The article states that banks and insurance firms, which were grouped together, received the most number of complaints. If banks and insurance firms were not grouped together, it is possible that timeshare companies received the most number of companies. For example, if the 1416 complaints were split equally between banks and insurance firms, they would have received 708 complaints each, then the number of complaints received by timeshare companies, i.e. 1238 complaints, would have been the greatest.
3. This shows that when organising data, it is important to consider whether to group separate entities as doing so might mislead consumers and result in inaccurate conclusions.

Part III: Display of Data

1. Although the height of the bar for Company E appears to be twice that of the bar for Company C, Company E's claim is not valid as the bars do not start from 0. By reading off the bar graph, Company E sold 160 light bulbs in a week, which is not twice as many as the 130 light bulbs sold by Company C in a week.
2. For bar graphs, if the vertical axis does not start from 0, the height of each bar will not be proportional to its corresponding frequency, i.e. number of light bulbs sold by each company in a week. Such display of statistical data may mislead consumers.

Part IV: Interpretation of Data

1. The conclusion was obtained based on a simple majority, i.e. since more than 50% of the employees were satisfied with working in the company, the survey concluded that the employees were satisfied with the company and that the company was a good place to work in.
2. $40\% \times 300 = \dfrac{40}{100} \times 300$

   $= 120$ employees

   It is stated in the article that 40% of the employees, i.e. 120 employees were not satisfied with working in the company. As such, even though a simple majority of the employees was satisfied with working in the company, it cannot be concluded that most of the employees were satisfied. This shows that we should not use simple majorities to arrive at conclusions or make decisions.
3. The amendment of the constitution of a country is a very serious matter where the agreement of a simple majority is insufficient, therefore there is a need for a greater percentage of elected Members of Parliament (MPs) to agree before the constitution can be amended. As a result, the Singapore government requires the agreement of at least a two-third majority before the constitution can be amended.

   Teachers may wish to take this opportunity to get students to search on the Internet for some laws that have been passed in the Singapore Parliament that resulted in a constitutional amendment.
4. It is important to have a basis or contention in order to decide on an issue, and that in some occasions, it is insufficient to make decisions based on a simple majority.

   Teachers may wish to ask students whether a simple majority, i.e. more than 50% of the votes, is necessary to decide on an issue. For example, in the 2011 Singapore Presidential Elections, Dr Tony Tan was elected President of the Republic of Singapore with 35.2% of the total valid votes cast.

Part V: Ethical Issues

It is unethical to use statistics to mislead others as it is essentially a form of misrepresentation and people may arrive at the wrong conclusions or make the wrong decisions.

The rationale for teaching students to be aware of how statistics can be used to mislead others is so that the students will be more discerning when they encounter statistics and will not be misled by others. Teachers should also impress upon students that they should not use statistics to mislead others because it is unethical to do so.

Practise Now (Page 371)

1. **(a)** **(i)** Profit earned by the company in 2010 $= 5.5 \times \$1\ 000\ 000$

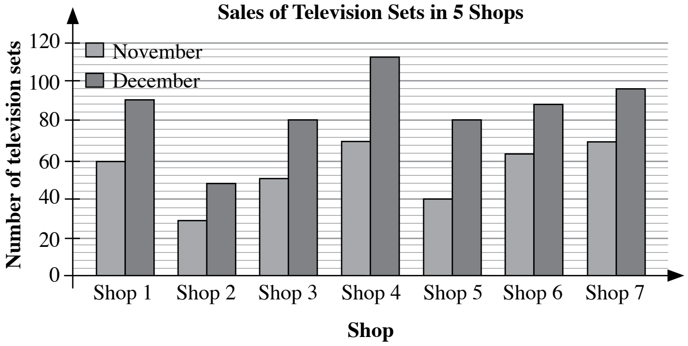
   $= \$5\ 500\ 000$

   (ii) Profit earned by the company in 2012 $= 7 \times \$1\ 000\ 000$

   $= \$7\ 000\ 000$

   (b) The company earned the least profit in 2009. The profit decreased by $1.5 \times \$1\ 000\ 000 = \$1\ 500\ 000$ in 2009 as compared to 2008.

2. **(a)**



(b) **(i)** Total number of television sets sold in the seven shops in November
$= 60 + 30 + 50 + 70 + 40 + 64 + 70$
$= 384$

(ii) Total number of television sets sold in the seven shops in December
$= 90 + 48 + 80 + 112 + 80 + 88 + 96$
$= 594$

(c) Required percentage $= \dfrac{384}{384 + 594} \times 100\%$
$= \dfrac{384}{978} \times 100\%$
$= 39\dfrac{43}{163}\,\%$

(d) **(i)** Required percentage $= \dfrac{70 + 96}{978} \times 100\%$
$= \dfrac{166}{978} \times 100\%$
$= 16\dfrac{476}{489}\,\%$

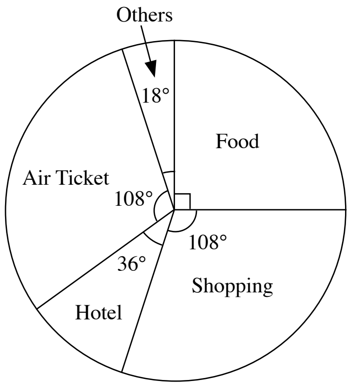
(ii) No, I do not agree with the manager. Since Shop 2 sold the least number of televsion sets in November and December, it should be closed down.

(e) The company performed better in terms of sales in December. This could be due to the fact that Christmas is in December when people buy television sets as gifts for others.

Practise Now (Page 376)

Farhan's total expenditure on the holiday
$= \$1000 + \$1200 + \$400 + \$1200 + \$200$
$= \$4000$

Item	Angle of sector
Food	$\dfrac{\$1000}{\$4000} \times 360° = 90°$
Shopping	$\dfrac{\$1200}{\$4000} \times 360° = 108°$
Hotel	$\dfrac{\$400}{\$4000} \times 360° = 36°$
Air Ticket	$\dfrac{\$1200}{\$4000} \times 360° = 108°$
Others	$\dfrac{\$200}{\$4000} \times 360° = 18°$



Practise Now 1

1. **(i)** $4x° + 2x° + 237.6° = 360°$ ($\angle$s at a point)
$4x° + 2x° = 360° - 237.6°$
$6x° = 122.4°$
$x° = 20.4°$
$\therefore x = 20.4$

(ii) Required percentage $= \dfrac{4(20.4°)}{360°} \times 100\%$
$= \dfrac{81.6°}{360°} \times 100\%$
$= 22\dfrac{2}{3}\,\%$

(iii) Amount of fruit punch in the jar $= \dfrac{360°}{237.6°} \times 759$ ml
$= 1150$ ml

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2. (i) The least popular colour is black.

(ii) Total number of cars sold

$= 2000 + 3500 + 5000 + 6000 + 1500$

$= 18\,000$

Angle of sector that represents number of blue cars sold

$= \dfrac{2000}{18\,000} \times 360°$

$= 40°$

Angle of sector that represents number of grey cars sold

$= \dfrac{3500}{18\,000} \times 360°$

$= 70°$

Angle of sector that represents number of white cars sold

$= \dfrac{5000}{18\,000} \times 360°$

$= 100°$

Angle of sector that represents number of red cars sold

$= \dfrac{6000}{18\,000} \times 360°$

$= 120°$

Angle of sector that represents number of black cars sold

$= \dfrac{1500}{18\,000} \times 360°$

$= 30°$

(iii) No, I do not agree with her. This is because the number of cars indicated on the *y*-axis is in thousands, thus 3500 grey cars and 1500 black cars are sold.

Practise Now 2

(i) The number of fatal road casualties was the highest in 2008.

(ii)

Year	2005	2006	2007	2008	2009
Number of fatal road casualties	173	190	214	221	183

(iii) Percentage decrease in number of fatal road casualties from 2008 to 2009

$= \dfrac{221 - 183}{221} \times 100\%$

$= \dfrac{38}{221} \times 100\%$

$= 17\dfrac{43}{221}\,\%$

(iv) There are traffic cameras installed along more roads.

Exercise 15A

1. (i) The greatest number of buses registered was in 2012.

Number of buses registered in 2012 $\approx 6.5 \times 40\,000$

$= 260\,000$

(ii) Total number of buses registered from 2008 to 2012

$\approx 24 \times 40\,000$

$= 960\,000$

(iii) Total amount the Registry of Vehicles collected in 2010

$\approx 4.5 \times 40\,000 \times \$1000$

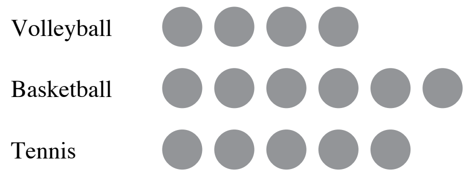
$= \$180\,000\,000$

(iv) Percentage increase in number of buses registered from 2011 to 2012

$= \dfrac{1}{5.5} \times 100\%$

$= 18\dfrac{2}{11}\,\%$

2. (i) **Students who Play Volleyball, Basketball or Tennis**



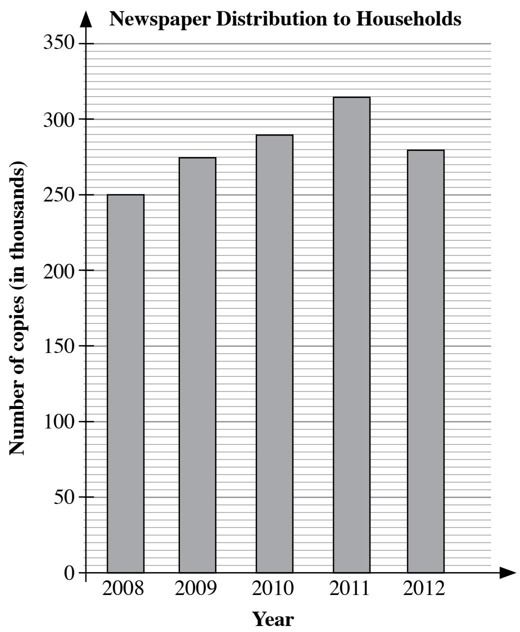
Each circle represents 10 students.

(ii) Required ratio = 4 : 5

(iii) Required percentage $= \dfrac{5}{6} \times 100\%$

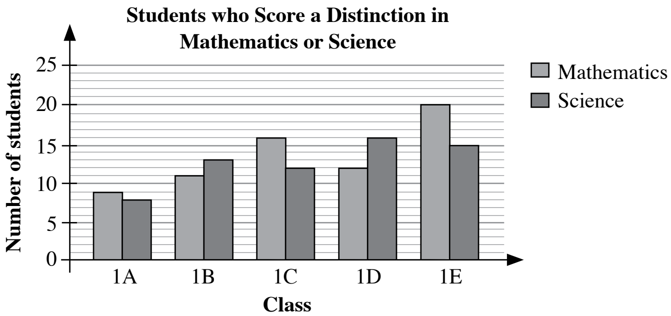
$= 83\dfrac{1}{3}\,\%$

3.



4. (a)

Class	Class 1A	Class 1B	Class 1C	Class 1D	Class 1E
Number of students who score a distinction in Mathematics	9	11	16	12	20
Number of students who score a distinction in Science	8	13	12	16	15



(b) (i) Total number of students in the 5 classes who score a distinction in Mathematics
$= 9 + 11 + 16 + 12 + 20$
$= 68$

(ii) Total number of students in the 5 classes who score a distinction in Science
$= 8 + 13 + 12 + 16 + 15$
$= 64$

(c) Required percentage $= \dfrac{12}{68} \times 100\%$
$= 17\dfrac{11}{17}\ \%$

(d) Percentage of students in Class 1D who score a distinction in Science
$= \dfrac{16}{40} \times 100\%$
$= 40\%$

(e) No, Jun Wei is not correct to say that there are 35 students in Class 1E. There may be students in the class who do not score distinctions in both Mathematics and Science. There may also be students in the class who score distinctions in both Mathematics and Science.

5. (i) Number of candidates who sat for the examination in 2009
$= 950$

(ii) Number of candidates who failed the examination in 2012
$= 500$

(iii) Total number of candidates who failed the examination in the six years
$= 400 + 350 + 350 + 400 + 450 + 500$
$= 2450$

$\therefore$ Required percentage $= \dfrac{500}{2450} \times 100\%$
$= 20\dfrac{20}{49}\ \%$

(iv) The percentage of successful candidates increases over the six years as they practise past-year papers and learn from their mistakes.

6. (i) Total number of workers employed in the housing estate
$= 4 \times 1 + 6 \times 2 + 5 \times 3 + 3 \times 4 + 2 \times 5$
$= 4 + 12 + 15 + 12 + 10$
$= 53$

(ii) Total number of shops in the housing estate
$= 4 + 6 + 5 + 3 + 2$
$= 20$

Number of shops hiring 3 or more workers $= 5 + 3 + 2$
$= 10$

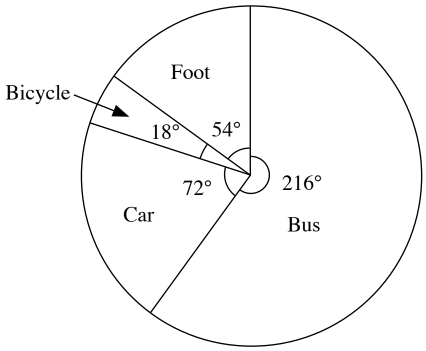
$\therefore$ Required percentage $= \dfrac{10}{20} \times 100\%$
$= 50\%$

(iii) Some shops have more customers as they are located at places with higher human traffic, thus they need to employ more workers.

Exercise 15B

1. Total number of students surveyed $= 768 + 256 + 64 + 192$
$= 1280$

Mode of transport	Angle of sector
Bus	$\dfrac{768}{1280} \times 360° = 216°$
Car	$\dfrac{256}{1280} \times 360° = 72°$
Bicycle	$\dfrac{64}{1280} \times 360° = 18°$
Foot	$\dfrac{192}{1280} \times 360° = 54°$



2. (i) Angle of sector that represents number of students who prefer yam $= 90°$

(ii) Angle of sector that represents number of students who prefer vanilla
$= 360° - 120° - 90° - 50°$ ($\angle$s at a point)
$= 100°$

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(iii) Required percentage $= \frac{100°}{360°} \times 100\%$

$= 27\frac{7}{9}\%$

(iv) Total number of students in the class $= \frac{360°}{50°} \times 5$

$= 36$

3. (i) Required percentage $= \frac{180°}{360°} \times 100\%$

$= 50\%$

(ii) Required percentage $= \frac{72°}{360°} \times 100\%$

$= 20\%$

(iii) $x° = \frac{17\frac{1}{2}}{100} \times 360°$

$= 63°$

$\therefore x = 63$

4. (i) Total number of cars in the survey $= 20 + 25 + 20 + 30 + 25$

$= 120$

(ii) Total number of people in all the cars

$= 20 \times 1 + 25 \times 2 + 20 \times 3 + 30 \times 4 + 25 \times 5$

$= 20 + 50 + 60 + 120 + 125$

$= 375$

(iii) Number of cars with 4 or more people $= 30 + 25$

$= 55$

$\therefore$ Required percentage $= \frac{55}{120} \times 100\%$

$= 45\frac{5}{6}\%$

(iv) Angle of sector that represents number of cars with 1 people

$= \frac{20}{120} \times 360°$

$= 60°$

Angle of sector that represents number of cars with 2 people

$= \frac{25}{120} \times 360°$

$= 75°$

Angle of sector that represents number of cars with 3 people

$= \frac{20}{120} \times 360°$

$= 60°$

Angle of sector that represents number of cars with 4 people

$= \frac{30}{120} \times 360°$

$= 90°$

Angle of sector that represents number of cars with 5 people

$= \frac{25}{120} \times 360°$

$= 75°$

5. (i)

Month	0	1	2	3	4	5	6
Mass (kg)	3.2	3.4	3.8	4	4.2	4.4	5

(ii) Percentage increase in mass of the baby from the 4th to 6th month

$= \frac{5 - 4.2}{4.2} \times 100\%$

$= \frac{0.8}{4.2} \times 100\%$

$= 19\frac{1}{21}\%$

6. (a) Total angle of sectors that represent number of female students and teachers in the school

$= 360° - 240°$ ($\angle$s at a point)

$= 120°$

Angle of sector that represents number of teachers in the school

$= \frac{1}{6} \times 120°$

$= 20°$

(b) (i) Number of female students in the school $= 5 \times 45$

$= 225$

(ii) Number of male students in the school $= \frac{240°}{20°} \times 45$

$= 540$

(c) Total school population $= 45 + 225 + 540$

$= 810$

Number of female teachers in the school $= \frac{2}{3} \times 45$

$= 30$

Number of females in the school $= 225 + 30$

$= 255$

$\therefore$ Required percentage $= \frac{255}{810} \times 100\%$

$= 31\frac{13}{27}\%$

7. $\frac{5}{1 + x + 5} \times 360° = 120°$

$\frac{5}{6 + x} = \frac{120°}{360°}$

$\frac{5}{6 + x} = \frac{1}{3}$

$15 = 6 + x$

$\therefore x = 9$

8. (i) The town had the greatest increase in the number of people from 2011 to 2012.

(ii)

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Number of people (in thousands)	8	6	9	9.5	12	14	15	16	18	19	25

(iii) Percentage increase in number of people in the town from 2009 to 2012

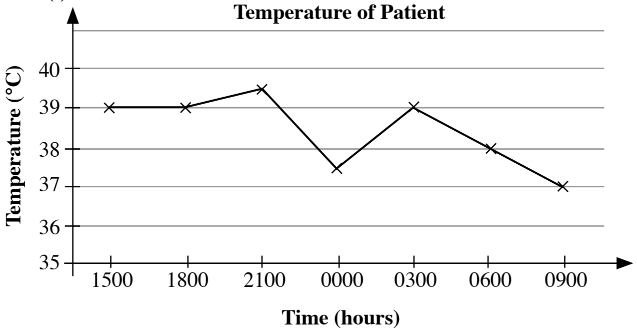
$= \frac{25\,000 - 16\,000}{16\,000} \times 100\%$

$= \frac{9000}{16\,000} \times 100\%$

$= 56\frac{1}{4}\%$

(iv) There are more new immigrants in the town.

9. **(i)**



(ii) Temperature of the patient at 1700 hours ≈ 39 °C

Temperature of the patient at 0100 hours ≈ 38 °C

10. The majority of the respondents in Kate's survey are most likely females while those in Khairul's survey are most likely males. Kate and Khairul may have conducted each of their surveys at a different location, e.g. Kate may have conducted her survey at Orchard Road while Khairul may have conducted his survey at a housing estate.

11. No, I do not agree with Nora. The temperatures in both countries range from 24 °C to 35 °C. The temperatures in Country X seem to change more drastically than those in Country Y because the vertical axis of the line graph which shows the temperatures of Country X starts from 23 °C instead of 0 °C.

12. **(i)** Based on the 3-dimensional pie chart, Raj spends the most on luxury goods.

(ii) Based on the 2-dimensional pie chart, Raj spends the most on rent and luxury goods.

(iii) In a 3-dimensional pie chart, the sizes of the sectors will look distorted. The sectors towards the back of the pie chart will appear smaller than those towards the front.

13. No, I do not agree with Amirah. As there are more cars than motorcycles in Singapore, it is not surprising that there are more accidents involving cars than motorcycles. Moreover, there may be a higher chance of accidents involving motorcycles occurring due to the nature of the vehicle.

Review Exercise 15

1. **(i)** Required ratio = 6 : 3

= 2 : 1

(ii) Required percentage = $\frac{7}{4} \times 100\%$

= 175%

2. **(i)** Total number of books read by the students in the class in a month

$= 2 \times 0 + 5 \times 1 + 9 \times 2 + 8 \times 3 + 6 \times 4 + 5 \times 5 + 1 \times 6$

$= 0 + 5 + 18 + 24 + 24 + 25 + 6$

$= 102$

(ii) Number of students who read more than 4 books = 5 + 1

= 6

Total number of students in the class = 2 + 5 + 9 + 8 + 6 + 5 + 1

= 36

$\therefore$ Required percentage = $\frac{6}{36} \times 100\%$

$= 16\frac{2}{3}\%$

(iii) Number of students who read fewer than 3 books = 2 + 5 + 9

= 16

Angle of sector that represents number of students who read fewer than 3 books

$= \frac{16}{36} \times 360°$

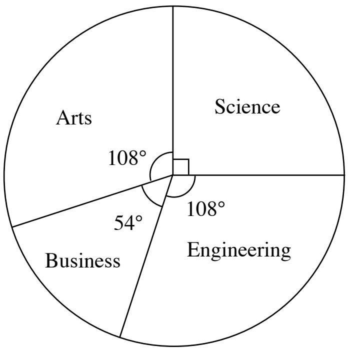
$= 160°$

3. Percentage of students who are enrolled in the Arts course

= 100% – 25% – 30% – 15%

= 30%

Type of course	Angle of sector
Science	$\frac{25}{100} \times 360° = 90°$
Engineering	$\frac{30}{100} \times 360° = 108°$
Business	$\frac{15}{100} \times 360° = 54°$
Arts	$\frac{30}{100} \times 360° = 108°$



4. **(i)** Total angle of sectors that represent amount Devi spends on clothes and food

= 360° – 36° – 90° – 90° (∠s at a point)

= 144°

Angle of sector that represents amount Devi spends on food

$= \frac{1}{4} \times 144°$

$= 36°$

$\therefore$ Required percentage = $\frac{36°}{90°} \times 100\%$

= 40%

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(ii) Devi's monthly income $= \frac{360°}{36°} \times \$400$

$= \$4000$

Devi's annual income $= 12 \times \$4000$

$= \$48\ 000$

5. (i)

Year	**2008**	**2009**	**2010**	**2011**	**2012**
Number of laptops	70	30	44	90	26

(ii) Percentage decrease in number of laptops purchased by the company from 2008 to 2009

$= \frac{70 - 30}{70} \times 100\%$

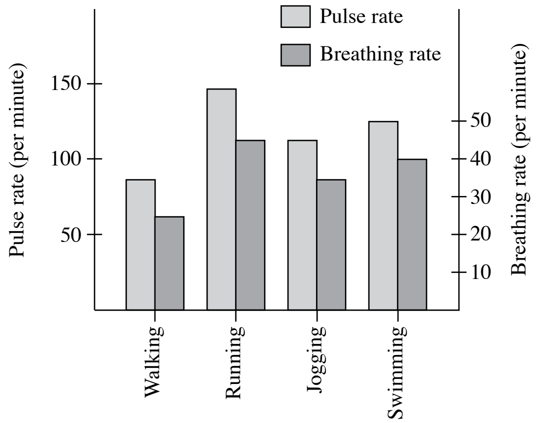
$= \frac{40}{70} \times 100\%$

$= 57\frac{1}{7}\%$

(iii) The company might have had a tighter budget in 2009.

Challenge Yourself

The better way to display the data using a bar graph is as follows:



Revision Exercise D1

1. Let the radius of the quadrant be x cm.

$$\frac{1}{4} \times 2\pi x + 2x = 71.4$$
$$\frac{1}{2}\pi x + 2x = 71.4$$
$$x\left(\frac{1}{2}\pi + 2\right) = 71.4$$
$$\therefore x = \frac{71.4}{\frac{1}{2}\pi + 2}$$
$$= 20.00 \text{ (to 4 s.f.)}$$

Area of quadrant $= \frac{1}{4} \times \pi(20.00)^2$
$= 314 \text{ cm}^2$ (to 3 s.f.)

2. Perimeter of shaded region $= \frac{1}{2} \times 2\pi(12) + 2\pi\left(\frac{12}{2}\right)$
$= 12\pi + 2\pi(6)$
$= 12\pi + 12\pi$
$= 24\pi$
$= 75.4$ cm (to 3 s.f.)

Area of shaded region = area of big semicircle
$= \frac{1}{2} \times \pi(12)^2$
$= 72\pi$
$= 226 \text{ cm}^2$ (to 3 s.f.)

3. Area of trapezium $ABEF$
= area of rectangle $ACEF$ – area of $\triangle BCE$
= area of rectangle $ACEF$ – $\frac{1}{2} \times$ area of $\triangle BDE$
$= 12 \times 8 - \frac{1}{2} \times 24$
$= 96 - 12$
$= 84 \text{ cm}^2$

4. Volume of solid = base area × height
$= [12 \times 12 + (18 - 12) \times 3] \times 6$
$= (144 + 6 \times 3) \times 6$
$= (144 + 18) \times 6$
$= 162 \times 6$
$= 972 \text{ cm}^3$

Total surface area of solid
= perimeter of base × height + 2 × base area
$= [12 + 12 + 18 + 3 + 6 + (12 - 3)] \times 6 + 2 \times 162$
$= (12 + 12 + 18 + 3 + 6 + 9) \times 6 + 324$
$= 60 \times 6 + 324$
$= 360 + 324$
$= 684 \text{ cm}^2$

5. **(i)** Capacity of box
$= (72 - 1.5 - 1.5) \times (54 - 1.5 - 1.5) \times (48 - 1.5 - 1.5)$
$= 69 \times 51 \times 45$
$= 158\ 355 \text{ cm}^3$
$= 158.355\ l$

(ii) Volume of wood used $= 72 \times 54 \times 48 - 158\ 355$
$= 186\ 624 - 158\ 355$
$= 28\ 269 \text{ cm}^3$

(iii) Mass of box $= 0.9 \times 28\ 269$
$= 25\ 442.1$ g

6. **(a)** **(i)** Required percentage $= \frac{150°}{360°} \times 100\%$
$= 41\frac{2}{3}\%$

(ii) Required percentage $= \frac{72°}{360°} \times 100\%$
$= 20\%$

(b) $x° = \frac{15}{100} \times 360°$
$= 54°$
$\therefore x = 54$

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Revision Exercise D2

1. Area of photograph $= 40 \times 25$

$= 1000 \text{ cm}^2$

Area of margin $= (40 + 4 + 4) \times (25 + 4 + 4) - 1000$

$= 48 \times 33 - 1000$

$= 1584 - 1000$

$= 584 \text{ cm}^2$

2. **(i)** Perimeter of figure

$= 24 + 15 + (24 - 10) + \frac{1}{4} \times 2\pi(10) + (15 - 10)$

$= 24 + 15 + 14 + 5\pi + 5$

$= 58 + 5\pi$

$= 73.7$ cm (to 3 s.f.)

(ii) Area of figure = area of rectangle – area of quadrant

$= 24 \times 15 - \frac{1}{4} \times \pi(10)^2$

$= 360 - 25\pi$

$= 281 \text{ cm}^2$ (to 3 s.f.)

3. Area of parallelogram $= PQ \times ST = QR \times SU$

$10 \times ST = 7 \times 9$

$10 \times ST = 63$

$ST = 6.3$

Length of $ST = 6.3$ cm

4. Volume of prism = base area × height

$= \left[\frac{1}{2} \times (8 + 3 + 8 + 3) \times 4\right] \times 20$

$= \left(\frac{1}{2} \times 22 \times 4\right) \times 20$

$= 44 \times 20$

$= 880 \text{ cm}^3$

Total surface area of prism

= perimeter of base × height + 2 × base area

$= (8 + 5 + 3 + 8 + 3 + 5) \times 20 + 2 \times 44$

$= 32 \times 20 + 88$

$= 640 + 88$

$= 728 \text{ cm}^2$

5. Volume of cylinder $= \pi(6^2 - 5^2)(2.4 \times 100)$

$= \pi(36 - 25)(240)$

$= \pi(11)(240)$

$= 2640\pi \text{ cm}^3$

Mass of cylinder $= 7.6 \times 2640\pi$

$= 20\,064\pi$

$= 63\,000$ g (to 3 s.f.)

6. **(i)** The attendance was the greatest in the 4th week.

(ii) The Drama Club stopped its weekly meeting in the 9th week.

(iii) Required percentage $= \frac{45 - 15}{45} \times 100\%$

$= \frac{30}{45} \times 100\%$

$= 66\frac{2}{3}\%$

(iv) Most of the Drama Club members were busy preparing for the school examination.

Problems in Real-World Contexts

1. (i) A suitable unit for the measurements in the floor plan is the millimetre (mm).

(ii) Length of $AB = (800 + 4800 + 3200 + 1600 + 1400 + 2500) - (4400 + 1775 + 400)$

$= 14\,300 - 6575$

$= 7725$ mm

(iii) Price per square metre $= \dfrac{\$500\,000}{110}$

$= \$4545$ (to the nearest dollar)

(iv) 1 foot ≈ 0.3048 m

1 square foot $= 0.092\,90$ m^2 (to 4 s.f.)

$\therefore$ \$1000 psf $= \$1000 \div 0.092\,90$

$= \$10\,760$ per square metre (to 4 s.f.)

The condominium unit is $\$10\,760 \div \$4545 = 2.37$ (to 3 s.f.) times as expensive as the flat that Mr Lee is interested to purchase.

2. (i) Volume of water the diving cylinder can contain

$= \pi(6.75)^2(85)$

$= 3872.8125\pi$ cm^3

$= 3.872\,812\,5\pi\ l$

$= 12.2\ l$ (to 3 s.f.)

(ii) Volume of gas that the diving cylinder can hold

$= \dfrac{\text{volume of cylinder} \times \text{pressure in cylinder}}{\text{atmospheric pressure}}$

$= \dfrac{3.872\,812\,5\pi \times 200}{1.01}$

$= 2409\ l$ (to 4 s.f.)

$= 2410\ l$ (to 3 s.f.)

(iii) Duration the diver can stay underwater

$= \dfrac{\text{volume of gas consumed}}{\text{breathing rate} \times \text{ambient pressure}}$

$= \dfrac{2409}{20 \times \left(1.01 + \dfrac{15}{10}\right)}$

$= \dfrac{2409}{20 \times (1.01 + 1.5)}$

$= \dfrac{2409}{20 \times 2.51}$

$= 48.0$ minutes (to 3 s.f.)

Teachers may wish to ask students to state an assumption that they have made in their calculations, e.g. the volume of gas consumed by the diver as he descends to a depth of 15 m is negligible.

(iv) Since the amount of time the diver can stay underwater if he uses the diving cylinder is 48.0 minutes, which is less than one hour, the diving cylinder is not suitable for the diver.

Diving cylinders are usually made of aluminium or steel. Teachers may wish to ask students to find out an advantage and a disadvantage of a diving cylinder made up of aluminium and of steel.
For example, aluminium cylinders are easier to maintain as aluminium is more resistant to corrosion. However, as aluminium is a soft metal, diving cylinders made of aluminium are more prone to physical damage. On the other hand, as steel is a tough metal, diving cylinders made of steel are more durable and are less prone to physical damage. However, steel comprises of iron, which is more susceptible to corrosion, thus steel diving cylinders are more difficult to maintain.

3. (a) Percentage increase in the annual mean surface temperature in Singapore from 1948 to 2011

$= \dfrac{27.6 - 26.8}{26.8} \times 100\%$

$= \dfrac{0.8}{26.8} \times 100\%$

$= 2.99\%$ (to 2 d.p.)

(b) (i) Majority of Singapore's emissions in 2020 is expected to come from the industry sector.

Amount of emissions contributed by the industry sector

$= 60.3\% \times 77.2$ MT

$= 46.5516$ MT

(ii) Reasons for the likely increase in emissions from 2005 to 2020:

- Due to rapid urbanisation, there will be an increase in the demand for expansion of petrochemicals and manufactured products from Asian countries such as Singapore.
- The growth of the population and the economy results in an increased use of transportation. In addition, the expansion of port activities causes an increase in the emissions from domestic maritime transport.
- There will be an increase in the demand for commercial spaces as well as a greater intensity in the usage of space.
- Due to increases in population and household income, there will be an increase in the demand for electrical appliances.

Teachers may wish to note that the list is not exhaustive.

(c) Measures that have been put in place by the Singapore government to reduce emissions and to mitigate the effects of climate change:

- The Energy Conservation Act, which was implemented in 2013, mandates companies in the industry and transport sectors to adopt energy efficient technologies and processes.
- The implementation of a Carbon Emissions-based Vehicle (CEV) Scheme in 2013 seeks to encourage consumers to buy low carbon emissions cars. Moreover, in the next few years, the rail network will be increased to about 280 km. This encourages people to use public transport, which is more carbon efficient.
- All new buildings and existing ones undergoing extensive renovation are required to adhere to the Green Mark standards, i.e. the buildings should be sustainable and environmentally-friendly.

- The Minimum Energy Performance Standards Scheme which was implemented in 2011 restricts the sale of energy inefficient appliances.
- The Singapore government is moving away from the disposal of waste in landfills to the incineration of waste.

Teachers may wish to note that the list is not exhaustive.

4. The Mathematical Modelling Process consists of the following steps:

A. Formulating

- Students are required to understand the information given in the question and a discussion may be carried out to help them comprehend the problem. Some guiding questions are as follows:
  (i) Based on the information in the table given, what does your classmates' choice of plan depend on? Should your classmates choose a plan based on the price alone, i.e. would you advise your classmates to choose Plan A because the monthly subscription fee is the lowest?
  (ii) Other than the monthly subscription fee, what are some other factors that may affect the amount a user has to pay each month? How can you simplify the problem so that it is easier to carry out a comparison?
- Some examples of assumptions that can be made to simplify the problem are as follows:
  (i) The subscription contract spans a period of two years and within the two years, the monthly subscription fees, as well as the terms and conditions of the price plans, do not change.
  (ii) The charges for overseas incoming calls and overseas outgoing calls are the same for all three price plans.
  (iii) Other mobile services, e.g. caller ID, multimedia messaging service (MMS) etc., provided are set at the same price.
  (iv) The amount of data used each month is the same.
  (v) The average download speeds are the same.

Teachers may wish to ask students to state other assumptions that may be made.

- In this problem, we shall consider data usage to be the only variable.

Teachers may wish to ask students what is meant by 'data usage is the only variable' and get them to give some other examples of variables. They may also wish to guide students to classify the information into three categories, i.e. when data usage does not exceed 12 GB, when data usage exceeds 12 GB but does not reach the cap of $30 and when data usage reaches the cap of $30.

B. Solving

- How do you solve the problem mathematically?
- What are the different ways that you can use to present the results from your calculations? Which way should you use to present the results to your classmates so that they will be able to make an informed decision?

C. Interpreting

Students should be able to interpret the mathematical solution in the context of the real world, i.e. they should be able to advise their classmates to choose one of the three price plans based on the mathematical solution that they have obtained.

D. Reflecting

- After finding the most value-for-money plan for their classmates, students should check whether there are any other issues that their classmates may have to take into consideration before subscribing to a plan.
- Students should review the chosen plan to determine whether it is the ideal plan for their classmates.
- Students should also review the method that they have used and consider whether there are other methods that can be used to solve the problem.

For higher-ability students, teachers may wish to remove one assumption from the assumptions made, and get them to come up with another model for the problem.

5. *For this problem, teachers may wish to first set the budget and the fundraising target for the students.*

The Mathematical Modelling Process consists of the following steps:

A. Formulating

- Students are required to understand the information given in the question and a discussion may be carried out to help them comprehend the problem. Some guiding questions are as follows:
  (i) How many cookies do you estimate your class will be able to sell? How many cookies should your class make? Does the number of cookies your class makes have to be a multiple of 48? How would this affect the ingredients required?
  (ii) Based on the number of cookies your class decides to make, what is the total amount of money required to purchase the ingredients?

  Teachers may wish to ask students whether they are able to buy the exact quantities of ingredients required to bake the cookies, e.g. 20 teaspoons of baking soda, and how this may affect the total cost.

  (iii) What is the budget for the fundraising event? Are there other costs that need to be taken into consideration? Does the total cost lie within the budget? If not, what can be done to ensure that the total cost lies within the budget?

(iv) What is the fundraising target that needs to be met? Besides the fundraising target, what are some other factors which need to be considered before pricing the cookies? How should you price your cookies?

- Some examples of assumptions that can be made to simplify the problem are as follows:
  (i) Miscellaneous costs, e.g. transport costs for travelling to buy the ingredients and cost of electricity used to bake the cookies, are not taken into consideration when calculating the total cost.
  (ii) There is no wastage of ingredients.

  Teachers may wish to ask students to state other assumptions that may be made.

B. Solving
- How do you solve the problem mathematically?
- How should you present the results from your calculations?

C. Interpreting

Students should be able to interpret the mathematical solution in the context of the real world, i.e. they should be able to advise the class on the number of cookies that need to be made and the price at which they should sell the cookies in order to maximise their profit based on the mathematical solution that they have obtained.

D. Reflecting
- After finding the number of cookies that need to be made and the price at which they should sell the cookies in order to maximise their profit, students should check whether there are any other factors that may affect the profit made, e.g. wastage at the end of the day.
- Students should also review the method that they have used and consider whether there are other methods that can be used to solve the problem.

For higher-ability students, teachers may wish to remove one assumption from the assumptions made, and get them to come up with another model for the problem.

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NOTES